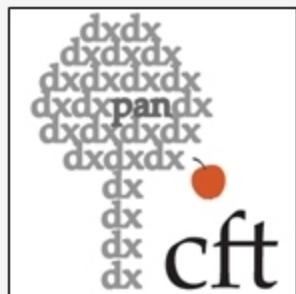




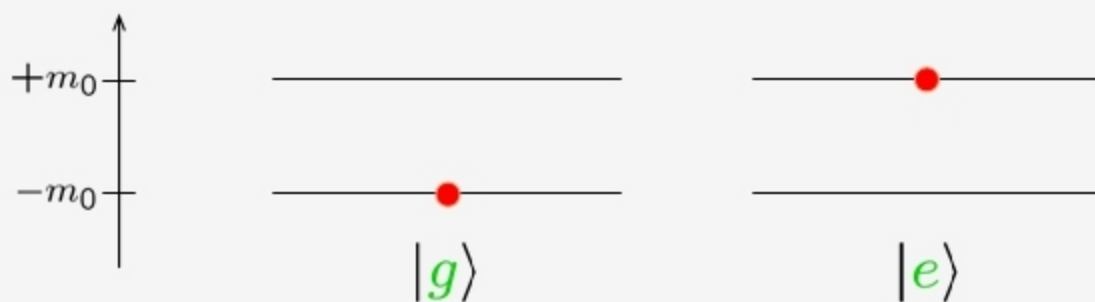
Zastosowanie metod kwantowej teorii pola do opisu qubitów

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Standardowy opis qubitu



$$\mathcal{H}_0 = m_0 \sigma_z \quad \Phi_k = \frac{1}{\sqrt{2k}} (a_k + a_k^\dagger)$$

$$\mathcal{H}_I = \int dk g(k) \Phi_k \sigma_x$$

Atom dwupoziomowy

$$\mathcal{H}_I = \int dk g(k) \vec{\Phi}_k \cdot \vec{\sigma}$$

Spin $1/2$

Standardowy opis qubitu

DYGRESJA
Standardowy rachunek zaburzeń

$$\langle \Omega | \hat{O} | \Omega \rangle$$

$$|\Omega\rangle = |g, 0\rangle + \frac{\langle e, k | \mathcal{H}_I | g, 0 \rangle}{2m_0 + \omega_k} |e, k\rangle + \dots$$

$$E_\Omega = E_{g,0} + \frac{|\langle e, k | \mathcal{H}_I | g, 0 \rangle|^2}{2m_0 + \omega_k} + \dots$$

\mathcal{H}_0

\mathcal{H}_I

$$\mathcal{H}_I = \int dk g(k) \Phi_k \cdot \vec{\sigma}$$

$+ a_k^\dagger$)

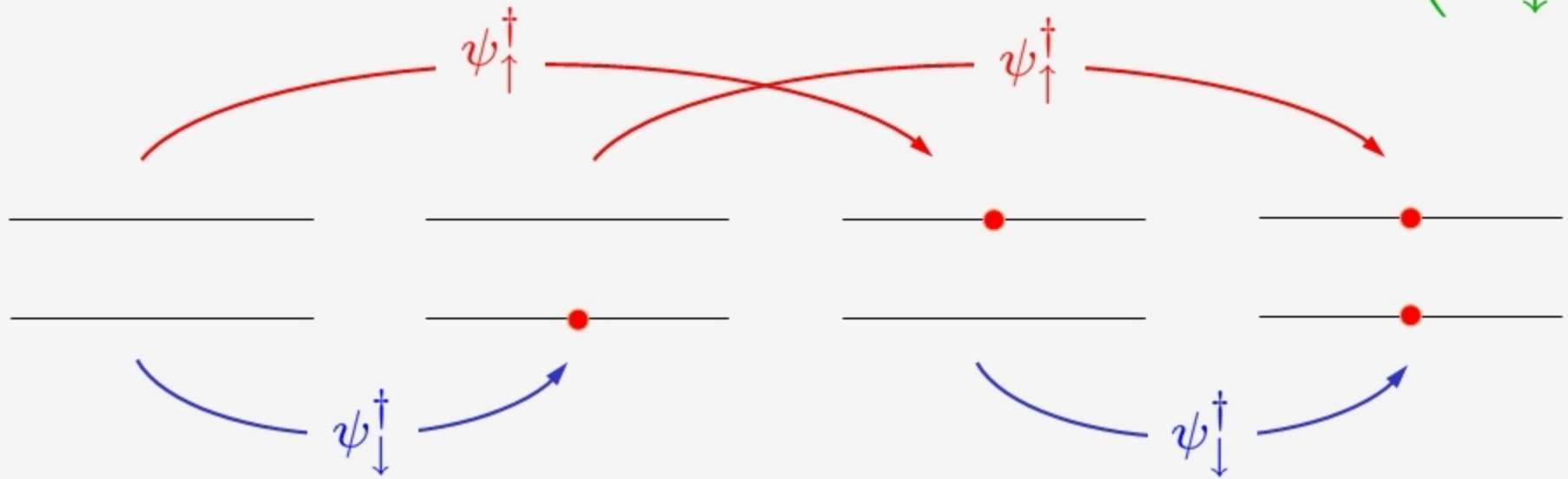
bozomowy

Spin $1/2$

Nowy opis qubitu

Operator pola

$$\Psi = \begin{pmatrix} \psi_{\uparrow} \\ \psi_{\downarrow} \end{pmatrix}$$



$$\mathcal{H}_0 = m_0 \Psi^{\dagger} \sigma_z \Psi$$

$$\mathcal{H}_I = \int dk g(k) \Phi_k \Psi^{\dagger} \sigma_x \Psi$$

$$\mathcal{H}_I = \int dk g(k) \vec{\Phi}_k \cdot \Psi^{\dagger} \vec{\sigma} \Psi$$

Atom dwupoziomowy

Spin $1/2$

Dynamika układu

- **Obraz Heisenberga** $\Upsilon(t) = e^{i\mathcal{H}t} \Upsilon e^{-i\mathcal{H}t}$

$$(i\partial_t - m_0\sigma_z)\psi(t) = \int_0^\infty dk g(k)\vec{\Phi}_k(t) \cdot \vec{\sigma} \psi(t)$$
$$(\partial_t^2 + k^2)\vec{\Phi}_k(t) = -g(k)\psi^\dagger(t) \vec{\sigma} \psi(t)$$

- **Obraz oddziaływania** $\Upsilon(t) = e^{i\mathcal{H}_0 t} \Upsilon e^{-i\mathcal{H}_0 t}$

$$(i\partial_t - m_0\sigma_z)\psi(t) = 0$$
$$(\partial_t^2 + k^2)\vec{\phi}_k(t) = 0$$

Zmiana sposobu
zadawania pytań

$$\langle \Omega | \hat{O}(\Psi, \Phi) | \Omega \rangle$$

Co to daje?

- **Twierdzenie Lowa - Gell-Manna**

$$\langle \Omega | \mathbb{T} \psi \psi \dots \psi^\dagger \psi^\dagger \phi \dots \phi | \Omega \rangle =$$

$$\frac{\langle g | \mathbb{T} \psi \psi \dots \psi^\dagger \psi^\dagger \phi \dots \phi e^{-i \int dt \mathcal{H}_I} | g \rangle}{\langle g | \mathbb{T} e^{-i \int dt \mathcal{H}_I} | g \rangle}$$

- **Twierdzenie Wicka**

$$\langle g | \mathbb{T} \psi \psi \dots \psi^\dagger \psi^\dagger \phi \dots \phi | g \rangle =$$

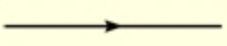
$$\sum_{\sigma} \underbrace{\langle g | \mathbb{T} \psi \psi^\dagger | g \rangle \dots \langle g | \mathbb{T} \psi \psi^\dagger | g \rangle}_{\text{propagatory feynmanowskie}} \underbrace{\langle g | \mathbb{T} \phi \phi | g \rangle \dots \langle g | \mathbb{T} \phi \phi | g \rangle}_{\text{propagatory feynmanowskie}}$$

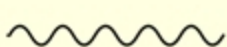
propagatory feynmanowskie

Co to daje?

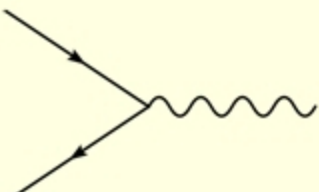
• Tw

Reguły Feynmana

$\langle \Omega |$  $\Rightarrow -i \langle g | \mathbb{T} \psi(t) \psi^\dagger(t') | g \rangle$

 $\Rightarrow -i \langle g | \mathbb{T} \phi_k(t) \phi_{k'}(t') | g \rangle$

• Tw

 $\Rightarrow -ig(k) \sigma_i$

$\langle g | \mathbb{T} \psi \psi \dots \psi' \psi' \phi \dots \phi | g \rangle =$

$\sum_{\sigma} \underbrace{\langle g | \mathbb{T} \psi \psi^\dagger | g \rangle \dots \langle g | \mathbb{T} \psi \psi^\dagger | g \rangle}_{\text{propagatory feynmanowskie}} \underbrace{\langle g | \mathbb{T} \phi \phi | g \rangle \dots \langle g | \mathbb{T} \phi \phi | g \rangle}_{\text{propagatory feynmanowskie}}$

propagatory feynmanowskie

Propagatory swobodne

$$\begin{aligned} \longrightarrow S_F(t, t') &= -i \langle g | \mathbb{T} \psi(t) \psi^\dagger(t') | g \rangle \\ &= \int \frac{dp_0}{2\pi} \frac{\sigma_z}{p_0 \sigma_z - m_0 + i\epsilon} e^{-ip_0(t-t')} \end{aligned}$$

$$\begin{aligned} \sim D_F^{k, k'}(t, t') &= -i \langle g | \mathbb{T} \phi_k(t) \phi_{k'}(t') | g \rangle \\ &= \int \frac{dk_0}{2\pi} \frac{\delta(k - k')}{k_0^2 - k^2 + i\epsilon} e^{-ik_0(t-t')} \end{aligned}$$

Propagator fermionu

$$\Rightarrow \Rightarrow = \mathcal{S}_F(t, t') = \langle \Omega | \mathbb{T} \psi(t) \psi^\dagger(t') | \Omega \rangle$$

$$= \text{---} + \text{---} + \text{---} + \text{---} + \text{---} + \text{---} + \dots$$

$$\square = \text{---} + \text{---} + \text{---} + \text{---} + \dots$$

$$\Rightarrow \Rightarrow = \text{---} + \text{---} \square \text{---} + \text{---} \square \square \text{---} + \dots$$

$$= \frac{1}{(\text{---})^{-1} - \square}$$

"Renormalizacja masy"

Propagator fotonu

$$\begin{aligned}
 \text{~~~~~} &= \mathcal{D}_F(t, t') = \langle \Omega | \mathbb{T} \Phi_k(t) \Phi_{k'}(t') | \Omega \rangle \\
 &= \text{~~~~~} + \text{~~~~~} \circlearrowleft \text{~~~~~} + \text{~~~~~} \circlearrowleft \text{~~~~~} \circlearrowleft \text{~~~~~} + \\
 &\text{~~~~~} \circlearrowright \text{~~~~~} + \text{~~~~~} \circlearrowright \text{~~~~~} + \text{~~~~~} \circlearrowleft \text{~~~~~} \circlearrowright \text{~~~~~} + \dots
 \end{aligned}$$

$$\text{●} = \text{○} + \text{○} \circlearrowleft + \text{○} \circlearrowright + \text{○} \circlearrowleft \circlearrowright + \dots$$

$$\text{~~~~~} = \text{~~~~~} + \text{~~~~~} \text{●} \text{~~~~~} + \text{~~~~~} \text{●} \text{~~~~~} \text{●} \text{~~~~~} + \dots$$

$$= \text{~~~~~} + \text{~~~~~} \left[\frac{1}{1 - \text{●} \text{~~~~~}} \right] \text{~~~~~}$$

Macierz przejścia

Macierz przejścia T (atom dwupoziomowy)

$$T(\omega) = \frac{1}{\text{Diagram}^{-1} - \text{wavy line}} \quad \text{Diagram} = \text{Diagram} + \text{Diagram} + \text{Diagram} + \dots$$

$$\text{wavy line} = P \int dk \frac{g^2(k)}{\omega^2 - k^2} + i\pi \frac{g^2(\omega)}{2\omega} = \Delta(\omega) + i\Gamma(\omega)$$

• **Najniższy rząd rachunku** $\text{Diagram}^{(2)} = \text{Diagram} = \frac{4m}{k_0^2 - 4m^2}$

$$T^{(2)}(\omega) = -\frac{4m}{4m^2 - k_0^2 - 4m[\Delta(k_0) + i\Gamma(k_0)]}$$

• **Czwarty rząd rachunku**

$$\text{Diagram}^{(4)} = \text{Diagram} + \text{Diagram} + \text{Diagram} = \frac{4m(1 - b)}{k_0^2 - 4m^2}$$

Liniowa odpowiedź układu

- **Podatność magnetyczna spinu** $\vec{S}(t) = \Psi^\dagger(t) \vec{\sigma} \Psi(t)$

$$\delta \langle \Omega | \vec{S}(t) | \Omega \rangle = \int_{-\infty}^0 dt' \hat{\chi}(t, t') \cdot \delta \vec{\Phi}(t')$$

$$\hat{\chi}_{ij}(\omega) = -i \int_{-\infty}^0 dt e^{-i\omega t} \langle \Omega | [S_i(0), S_j(t)] | \Omega \rangle$$

- **Polaryzowalność atomu** $P(t) = \Psi^\dagger(t) \sigma_x \Psi(t)$

$$\delta \langle \Omega | D(t) | \Omega \rangle = -\frac{3\hbar}{d^2} \int_{-\infty}^0 dt' \alpha(t, t') \delta \Phi(t')$$

$$\alpha(\omega) = i \frac{d^2}{3\hbar} \int_{-\infty}^0 dt e^{-i\omega t} \langle \Omega | [P(0), P(t)] | \Omega \rangle$$

Polaryzowalność atomu

$$\approx \approx = \approx + \approx \cdot T(\omega) \cdot \approx$$

amplituda
rozpraszania

$$\overset{R}{\approx} = \overset{R}{\approx} + \approx \cdot \alpha(\omega) \cdot \approx$$

polaryzowalność
atomu

$$\operatorname{Re} \alpha(\omega) = \operatorname{Re} T(\omega)$$

$$\operatorname{Im} \alpha(\omega) = \operatorname{sgn}(\omega) \operatorname{Im} T(\omega)$$

Polaryzowalność atomu


$$\approx \approx = \approx + \approx \cdot T(\omega) \cdot \approx$$

$$\overset{R}{\approx} = \overset{R}{\approx} + \approx \cdot \alpha(\omega) \cdot \approx$$


$$\operatorname{Re} \alpha(\omega) = \operatorname{Re} T(\omega)$$

$$\operatorname{Im} \alpha(\omega) = \operatorname{sgn}(\omega) \operatorname{Im} T(\omega)$$

$$T^{(2)}(\omega) \approx - \left(\frac{1}{2m + \Delta - \omega - i\Gamma} + \frac{1}{2m + \Delta + \omega - i\Gamma} \right)$$

 TEN SAM ZNAK

$$\alpha^{(2)}(\omega) \approx - \left(\frac{1}{2m + \Delta - \omega - i\Gamma} + \frac{1}{2m + \Delta + \omega + i\Gamma} \right)$$

 PRZECIWNY ZNAK

Podsumowanie

- ✓ **systematyczny i efektywny sposób prowadzenia rachunku perturbacyjnego (diagramy Feynmana)**
- ✓ **w czwartym rzędzie rachunku zaburzeń:**
 - amplituda rozpraszania fotonów
 - podatność magnetyczna spinu $1/2$
 - polaryzowalność atomu dwupoziomowego (poprawnie)
- ✓ **rozstrzygnięcie sporu o przepis znakowy**