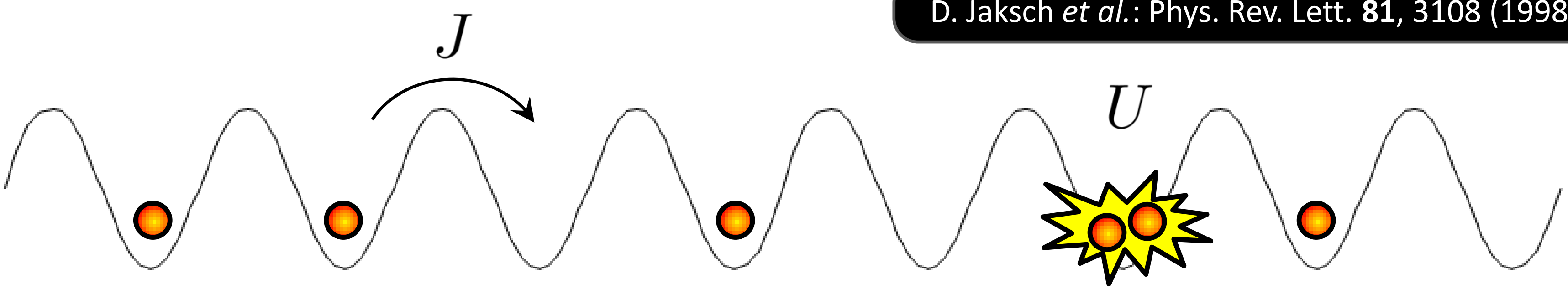
one-dimensional Bose-Hubbard model with local three-body interactions

Tomasz Sowiński

standard Bose-Hubbard model

- **ultra-cold atoms in optical lattice**

D. Jaksch *et al.*: Phys. Rev. Lett. **81**, 3108 (1998)



- the tunneling amplitude **J** is determined by the shape of the lattice potential
- the interaction energy **U** is determined by the shape of the lattice (via Wannier functions) and details of the interaction potential

- **Hamiltonian of the one-dimensional system**

$$\mathcal{H} = -J \sum_{i=1}^{L} \hat{a}_i^\dagger \left(\hat{a}_{i-1} + \hat{a}_{i+1}\right) + \frac{U}{2} \sum_{i=1}^{L} \hat{n}_i(\hat{n}_i - 1)$$

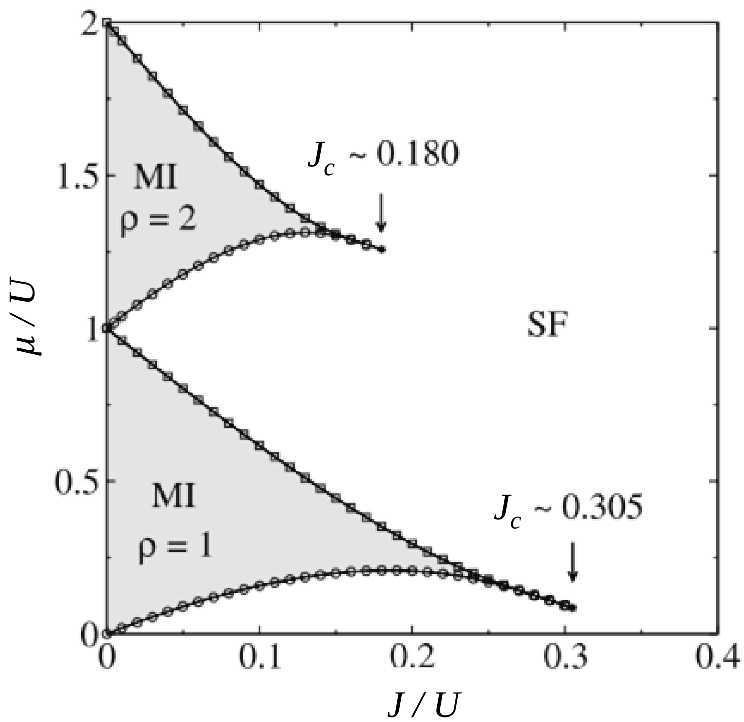
$$\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$$

phase diagram

- **grand canonical ensemble**

$$\mathcal{H} = -J \sum_{i=1}^{L} \hat{a}_i^\dagger \left(\hat{a}_{i-1} + \hat{a}_{i+1}\right) + \frac{U}{2} \sum_{i=1}^{L} \hat{n}_i(\hat{n}_i - 1) - \mu \hat{N}$$

$$\hat{N} = \sum_{i=1}^{L} \hat{n}_i$$



average filling

$$\rho = N/L$$

S. Ejima *et al.*: EPL **93**, 3002 (2011)

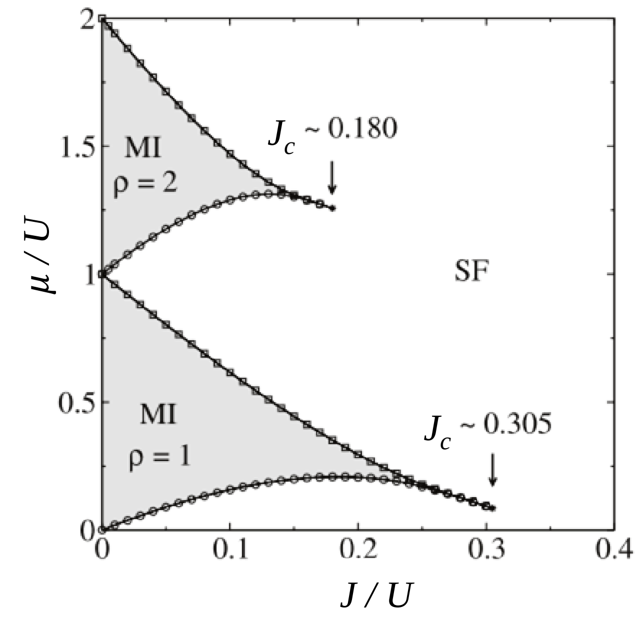
phase diagram

- **grand canonical ensemble**

$$\mathcal{H} = -J\sum_{i=1}^{L}\hat{a}_i^\dagger\,(\hat{a}_{i-1}+\hat{a}_{i+1}) + \frac{U}{2}\sum_{i=1}^{L}\hat{n}_i(\hat{n}_i-1) - \mu\,\hat{N}$$

$$+\frac{W}{6}\sum_{i=1}^{L}\hat{n}_i(\hat{n}_i-1)(\hat{n}_i-2)$$

how the properties of the studied model will change when local three-body interactions are taken into account ????



S. Ejima *et al.*: EPL **93**, 3002 (2011)

origins of three-body interactions

- **Bose-Hubbard model originates in more general theory**

$$\mathcal{H} = \int \mathrm{d}^3\boldsymbol{r}\ \Psi^\dagger(\boldsymbol{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{ext}}(\boldsymbol{r}) \right] \Psi(\boldsymbol{r}) + \frac{1}{2} \int\!\!\int \mathrm{d}^3\boldsymbol{r}\, \mathrm{d}^3\boldsymbol{r}'\ \Psi^\dagger(\boldsymbol{r})\Psi^\dagger(\boldsymbol{r}')\mathcal{V}(\boldsymbol{r}-\boldsymbol{r}')\Psi(\boldsymbol{r}')\Psi(\boldsymbol{r})$$

- **beyond standard approximations**

  - beyond single band approximation

nature

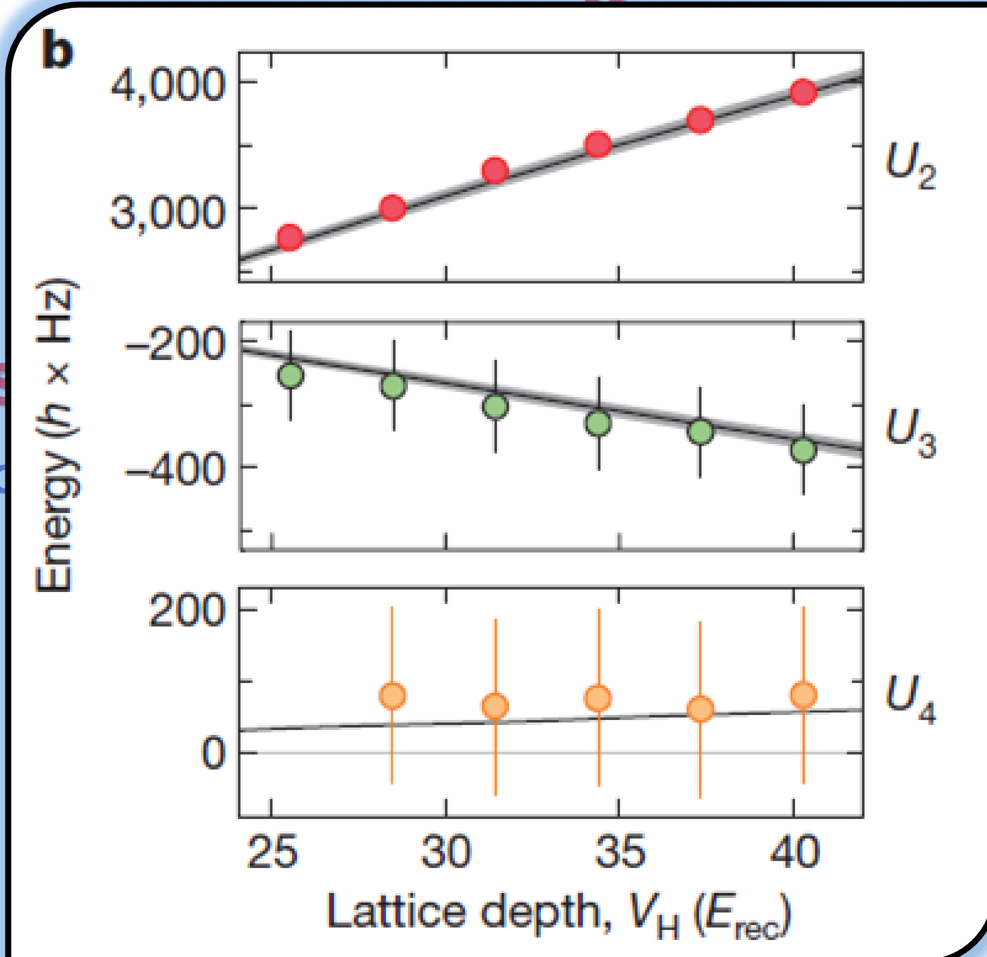
Nature **465**, 197–201 (13 May 2010) | doi:10.1038/nature09036
Received 01 February 2010 | Accepted 17 March 2010

Time-resolved observation of coherent multi-body interactions in quantum phase revivals

Sebastian Will, Thorsten Best, Ulrich Schneider, Lucia Hackermüller, Dirk-Sören Lühmann & Immanuel Bloch

origins of three-body interactions

- **Bose-Hubbard model originates in more general**



$r)\Big]\,\Psi(r)$

$r - r')\Psi(r')\Psi(r)$

- **beyond s**
  - beyond sh

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  - beyond short-range interaction approximation

PHYSICAL REVIEW VOLUME 115, NUMBER 6 SEPTEMBER 15, 1959

Ground State of a Bose System of Hard Spheres

Tai Tsun Wu*
The Institute for Advanced Study, Princeton, New Jersey, and the Bell Telephone Laboratories, Murray Hill, New Jersey

(Received April 3, 1959)

It is shown that the pseudopotential method can be extended to yield further terms in the low-density expansion of the ground-state energy of a system of Boltzmann or Bose particles with hard-sphere interaction. Two terms beyond the known result are found, and the expansion is no longer a power series in $(a^3\rho)^{\frac{1}{2}}$. Other related properties of the system are discussed.

nature physics

Nature Physics 3, 726 - 731 (2007)
Published online: 22 July 2007 | doi:10.1038/nphys678

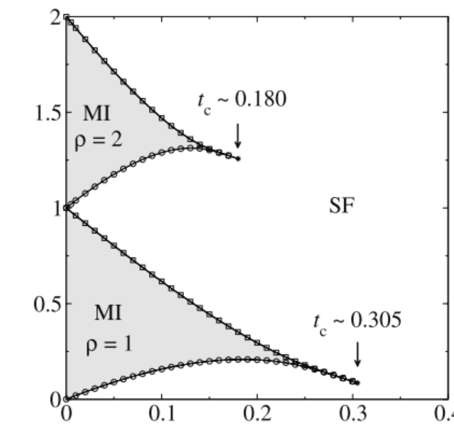
Three-body interactions with cold polar molecules

H. P. Büchler, A. Micheli & P. Zoller

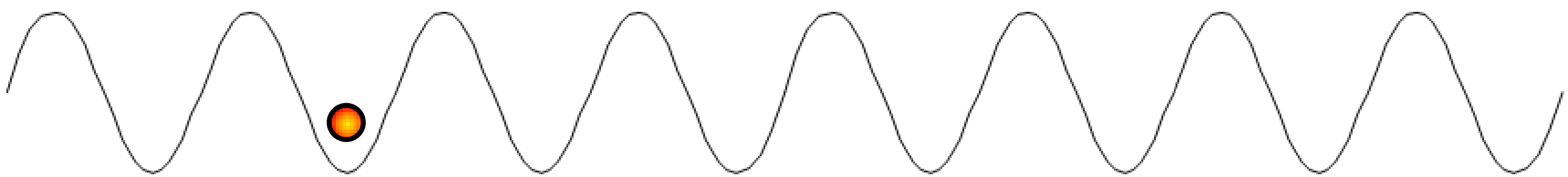
local three-body interactions

$$\mathcal{H} = -J\sum_{i=1}^{L}\hat{a}_i^\dagger\left(\hat{a}_{i-1}+\hat{a}_{i+1}\right)+\frac{U}{2}\sum_{i=1}^{L}\hat{n}_i(\hat{n}_i-1)-\mu\,\hat{N}$$

$$+\frac{W}{6}\sum_{i=1}^{L}\hat{n}_i(\hat{n}_i-1)(\hat{n}_i-2)$$



- **energy of local configurations**
  (limit J → 0)



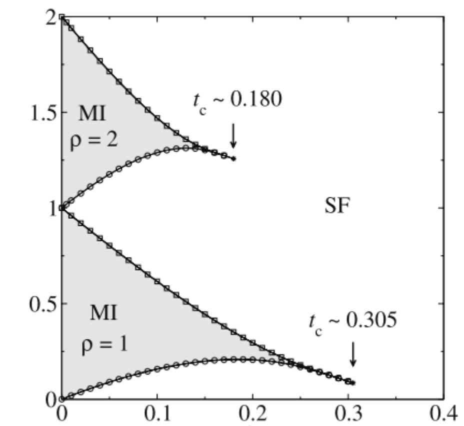
$-\mu$

$\mu > 0$

local three-body interactions

$$\mathcal{H} = -J \sum_{i=1}^{L} \hat{a}_i^\dagger (\hat{a}_{i-1} + \hat{a}_{i+1}) + \frac{U}{2} \sum_{i=1}^{L} \hat{n}_i(\hat{n}_i - 1) - \mu \hat{N}$$

$$+ \frac{W}{6} \sum_{i=1}^{L} \hat{n}_i(\hat{n}_i - 1)(\hat{n}_i - 2)$$



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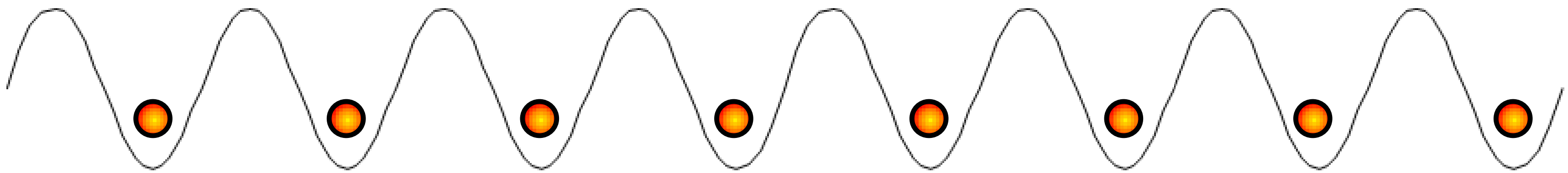

$-\mu L$

$\mu > 0$

local three-body interactions

$$\mathcal{H} = -J\sum_{i=1}^{L}\hat{a}_i^\dagger\,(\hat{a}_{i-1}+\hat{a}_{i+1}) + \frac{U}{2}\sum_{i=1}^{L}\hat{n}_i(\hat{n}_i-1) - \mu\,\hat{N}$$

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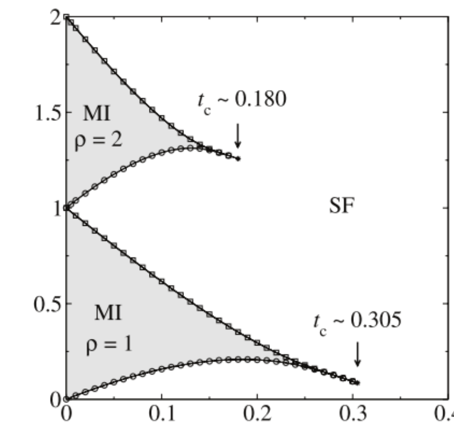

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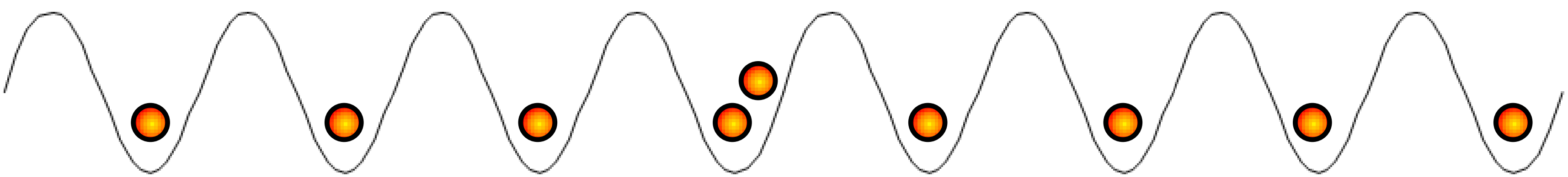

$-\mu L$ $\qquad -\mu(L+1)+U$

$\mu > U$

$-\mu + U$

local three-body interactions

$$\mathcal{H} = -J \sum_{i=1}^{L} \hat{a}_i^\dagger (\hat{a}_{i-1} + \hat{a}_{i+1}) + \frac{U}{2} \sum_{i=1}^{L} \hat{n}_i(\hat{n}_i - 1) - \mu \, \hat{N}$$

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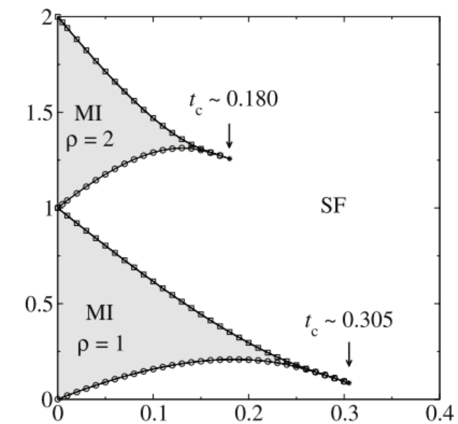

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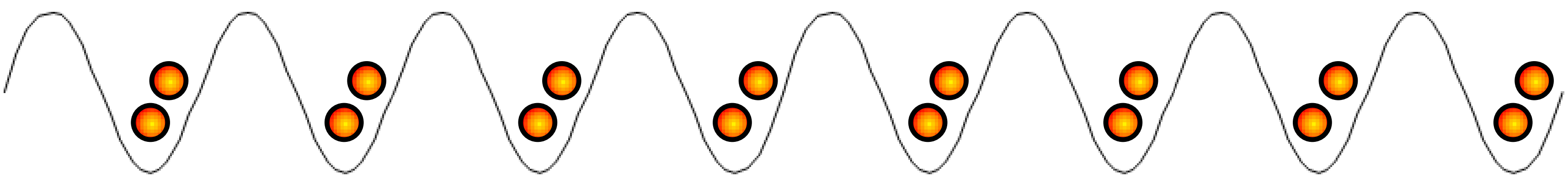

$(-2\mu + U)L$

$\mu > U$

local three-body interactions

$$\mathcal{H} = -J \sum_{i=1}^{L} \hat{a}_i^\dagger \left(\hat{a}_{i-1} + \hat{a}_{i+1}\right) + \frac{U}{2} \sum_{i=1}^{L} \hat{n}_i(\hat{n}_i - 1) - \mu \hat{N}$$

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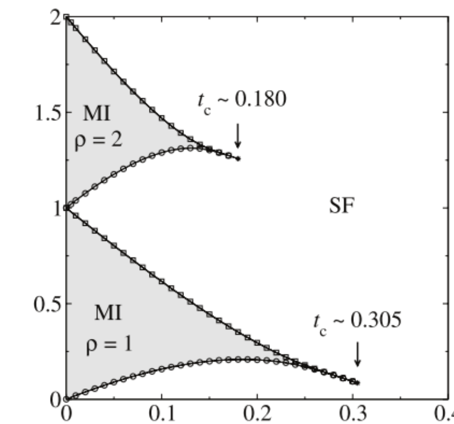
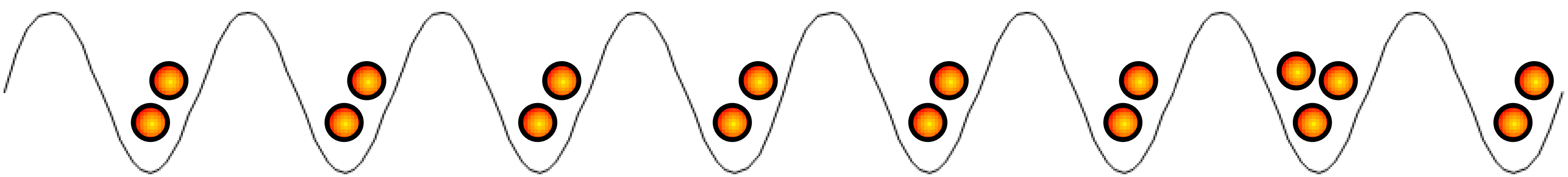


- **energy of local configurations**
  (limit J → 0)

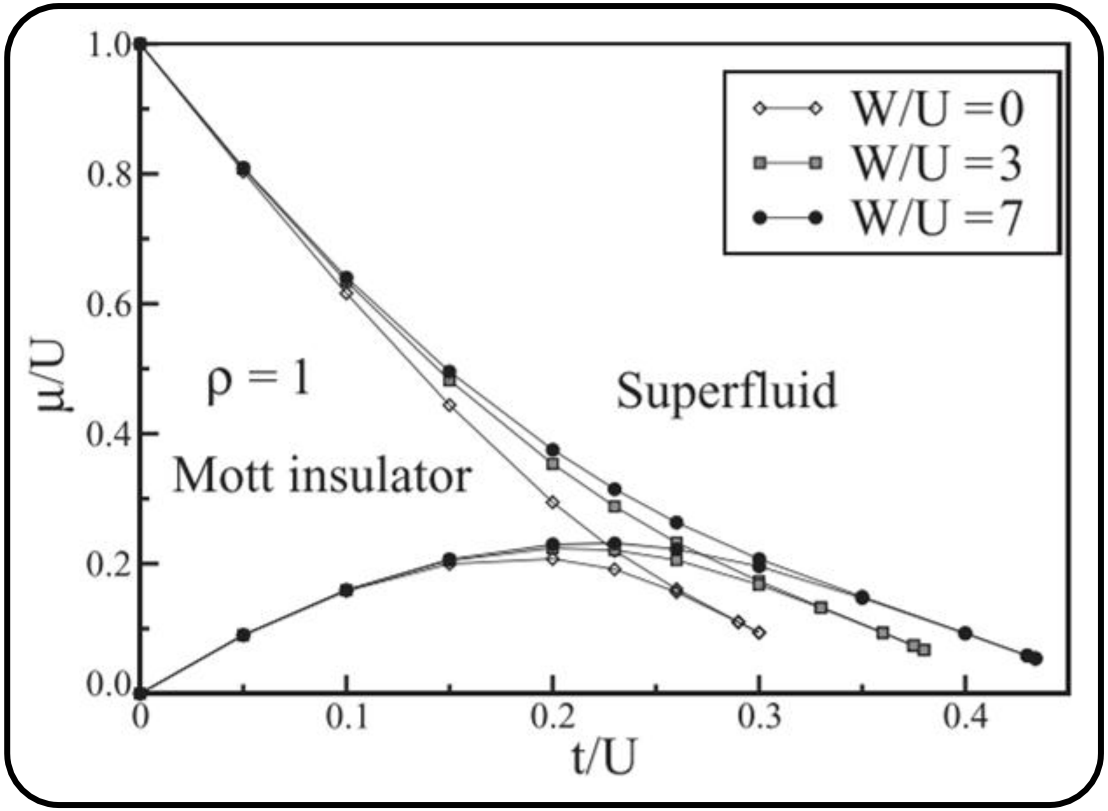


$$(-2\mu + U)L \qquad -(2\mu + U)L - \mu + 2U + W$$

$$-\mu + 2U + W$$

first insulating lobe

in the presence of three-body interactions
the first insulating lobe remains almost unchanged



DMRG with L up to 512

J. Silva-Valencia, A. Souza: Phys. Rev. A **84**, 065601 (2011)

estimation of the boundaries

- **strategy**
  - we exactly diagonalize the Hamiltonian of the system with L sites and N bosons
  - we find the ground state $|G\rangle$ and its energy $E(L,N)$
  - we calculate the upper/lower boundary of the **insulating phase** as the energy cost of adding/substracting one particle to the system

$\rho = 1$

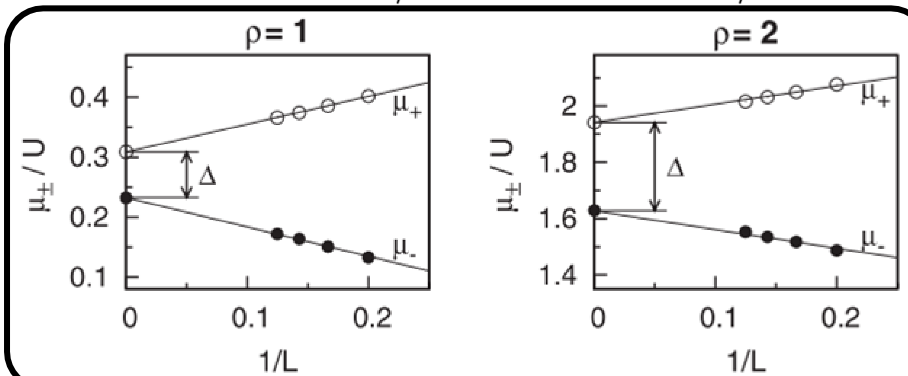
$$\mu_+(L) = E(L, L+1) - E(L, L)$$
$$\mu_-(L) = E(L, L) - E(L, L-1)$$

$\rho = 2$

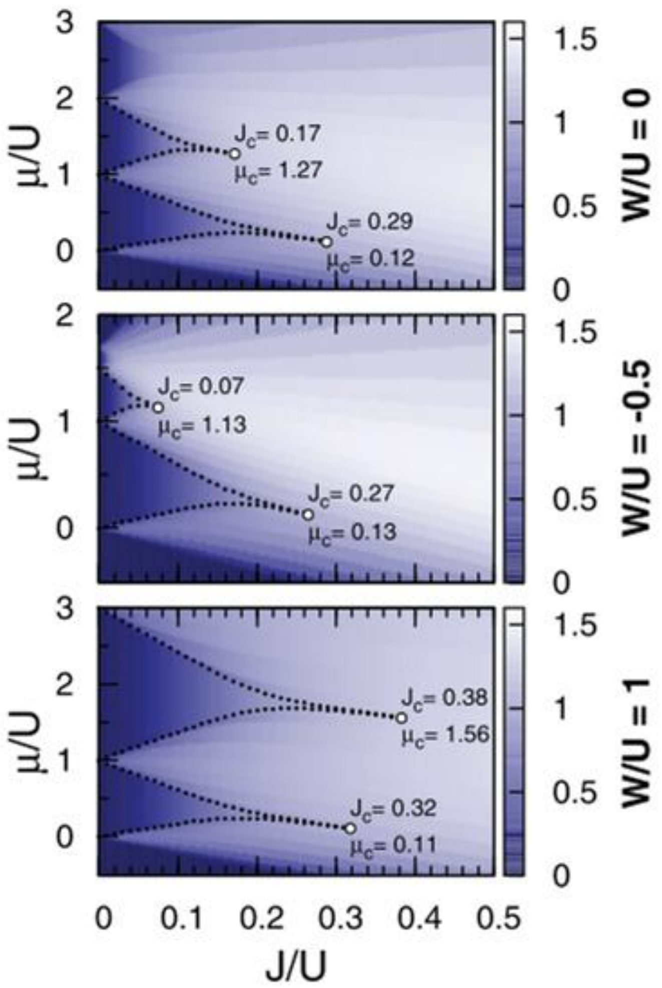
$$\mu_+(L) = E(L, 2L+1) - E(L, 2L)$$
$$\mu_-(L) = E(L, 2L) - E(L, 2L-1)$$

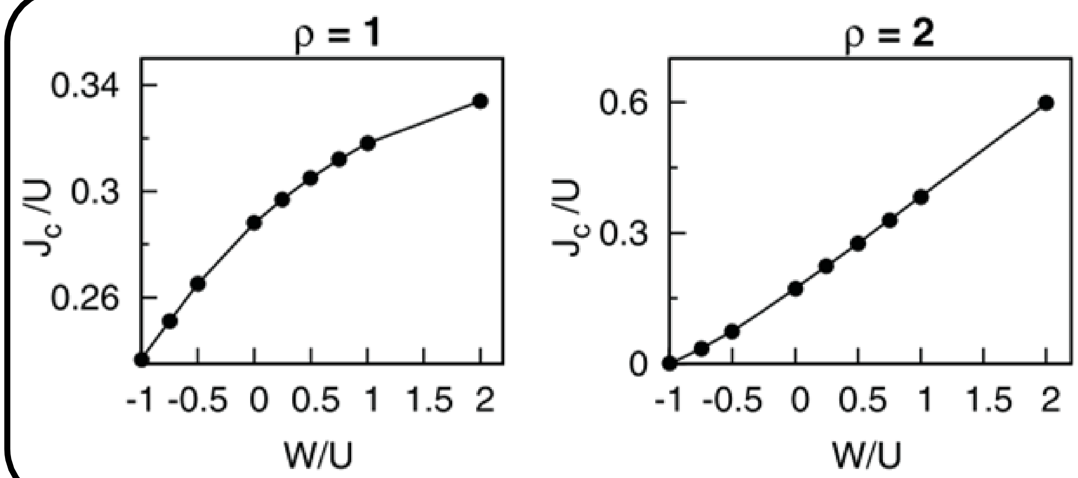
- **example**

$J/U = 0.2 \qquad W/U = 1$



the phase diagram





In the case of attractive three-body interactions (W<0) it is necessary to take into account also four-body repulsive interactions to prevent the system collapsing. However, the four-body interactions do not affect the positions of the critical points of first two insulating lobes.

T. Sowiński: Phys. Rev. A **85**, 065601 (2012)

universality class

- **Kosterlitz-Thouless transition**
  - one-dimensional Bose-Hubbard model belongs to the universality class of the two-dimensional XY spin model
  - the transition from the MI to the SF phase is of the Kosterlitz-Thouless type
  - the correlation length diverges as

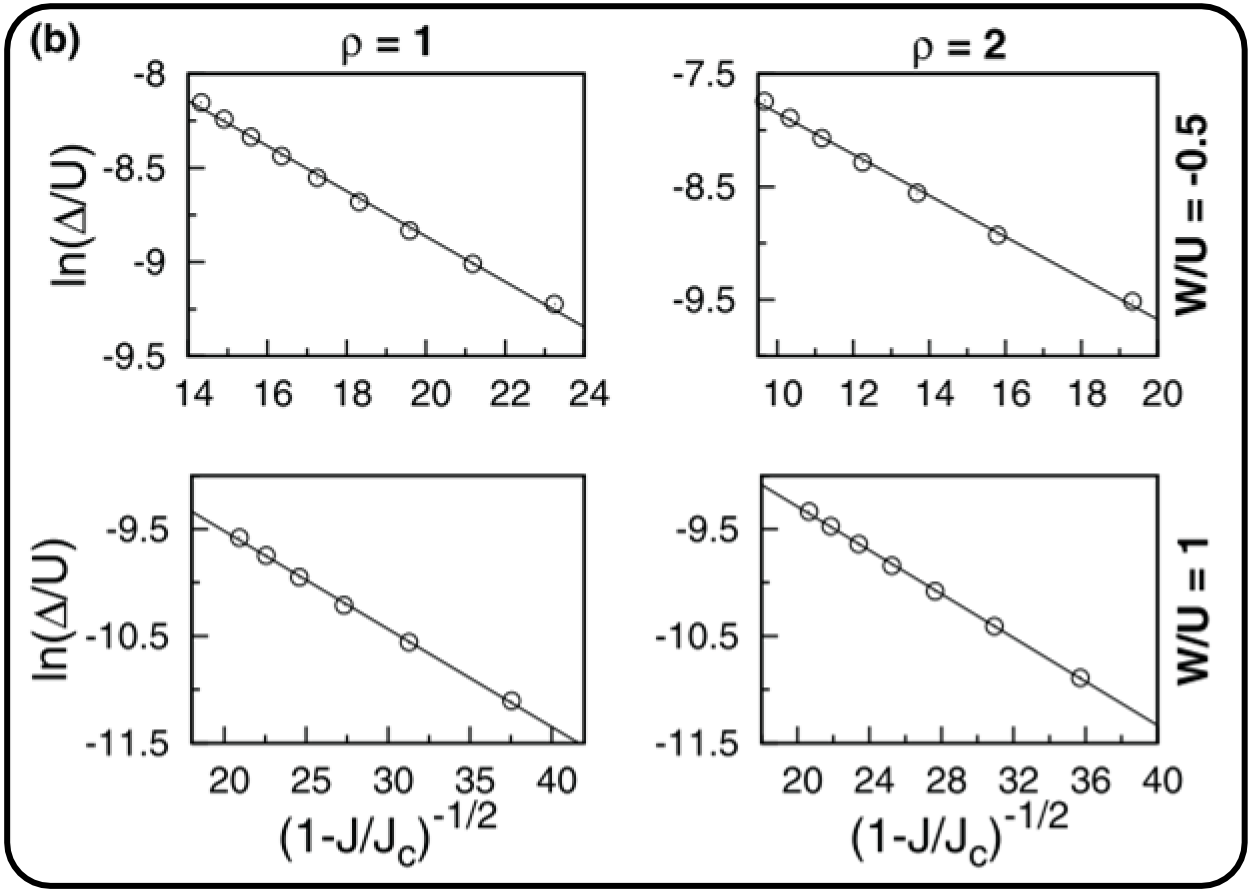
$$\frac{1}{\Delta} \sim \xi \sim \exp\left(\frac{\text{const}}{\sqrt{J_c - J}}\right)$$

- **Question:**

if the local three-body interactions change the critical behaviour of the system?

universality class

T. Sowiński: Phys. Rev. A **85**, 065601 (2012)



the numerical predictions fit almost perfectly
to the theoretical predictions of **Kosterlitz-Thouless** transition