# Few-fermion thermometry

Phys. Rev. A **97**, 063619 (2018)

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FEW-BODY PROBLEMS
THEORY GROUP

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# motivation

Selim Jochim's experimental group in Heidelberg

| $^6$Li | |
|---|---|
| Atomic number (Z) | 3 |
| Nucleons (Z+N) | 6 |
| Total electronic spin (S) | 1/2 |
| Total nuclear spin (I) | 1 |
| Hyperfine states (F=S+I) | 1/2 or 3/2 |

$$|\uparrow\rangle = |F = 3/2, m_F = -3/2\rangle$$
$$|\downarrow\rangle = |F = 1/2, m_F = 1/2\rangle$$

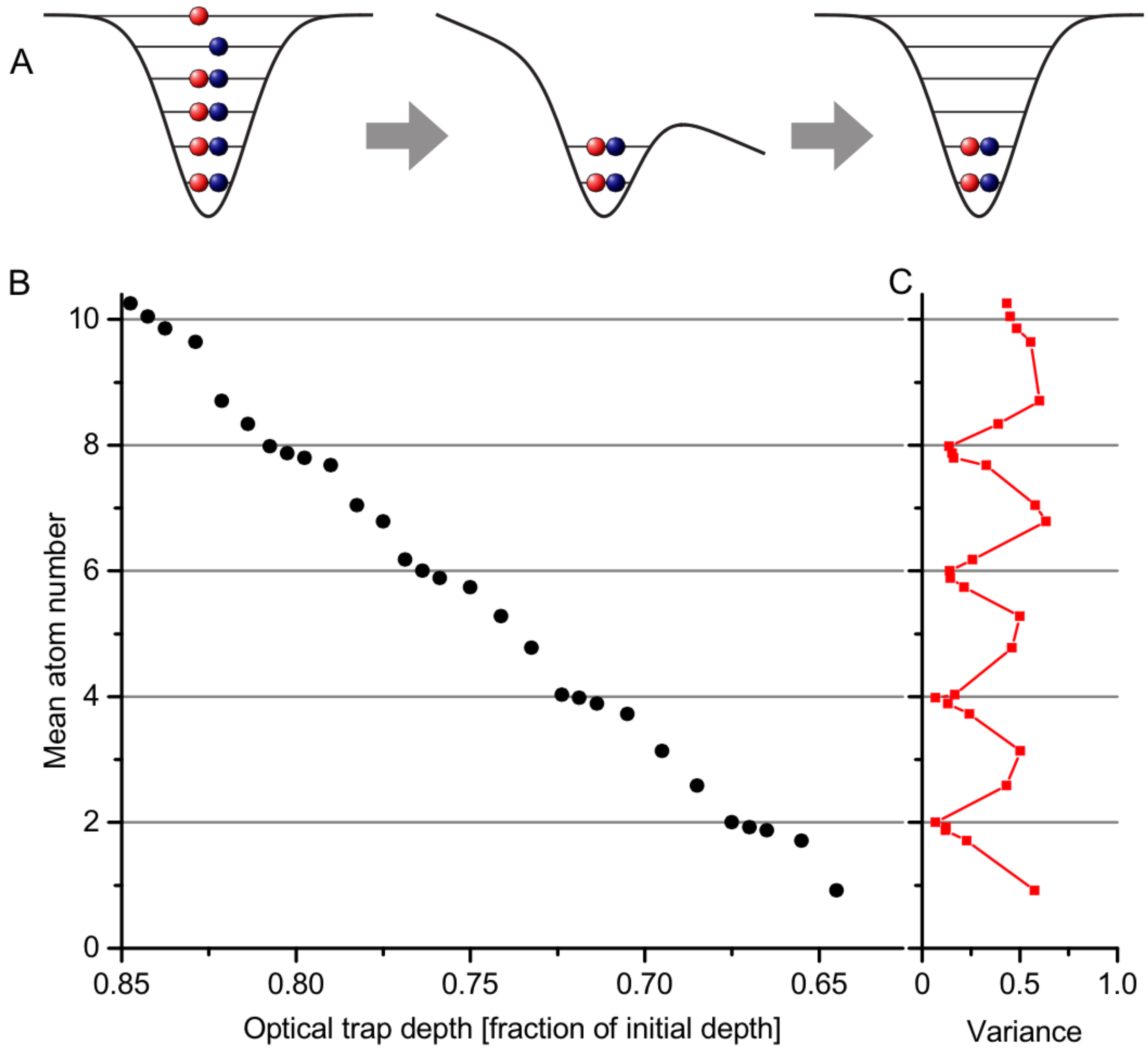
## message from experiments

- two flavours of spinless fermions
- well controlled number of particles
- one-dimensional confinement
- strong interactions between flavours

## ... what for???

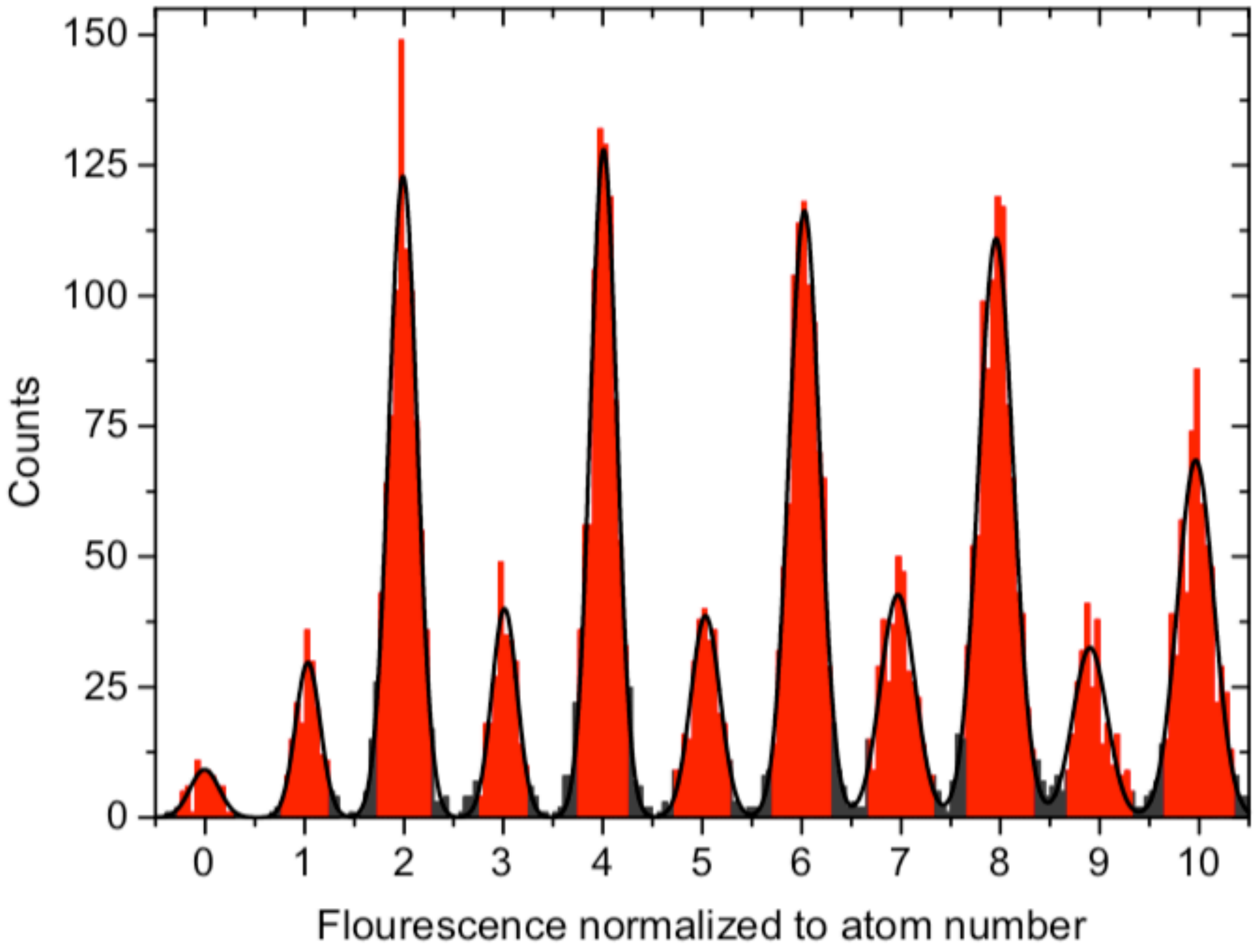
- extreme control on the system
- extreme precision of measurements
- from 'one' to 'many' crossover

# message from experiments




F. Serwane et al., Science **332**, 6027 (2011)

# message from experiments




F. Serwane et al., Science **332**, 6027 (2011)

# the model

- two distinguishable flavours of fermions (↑ and ↓)
- both flavours may have different masses
- both flavours confined in one-dimensional trap
- opposite spins do interact via sort range δ-like potential

$$\hat{\mathcal{H}} = \sum_\sigma \int \mathrm{d}x\, \hat{\Psi}^\dagger_\sigma(x) \left[-\frac{\hbar^2}{2m_\sigma}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V_\sigma(x)\right] \hat{\Psi}_\sigma(x)$$

$$+ g\int \mathrm{d}x\, \hat{\Psi}^\dagger_\downarrow(x)\hat{\Psi}^\dagger_\uparrow(x)\hat{\Psi}_\uparrow(x)\hat{\Psi}_\downarrow(x)$$

## (anti-)commutation relations

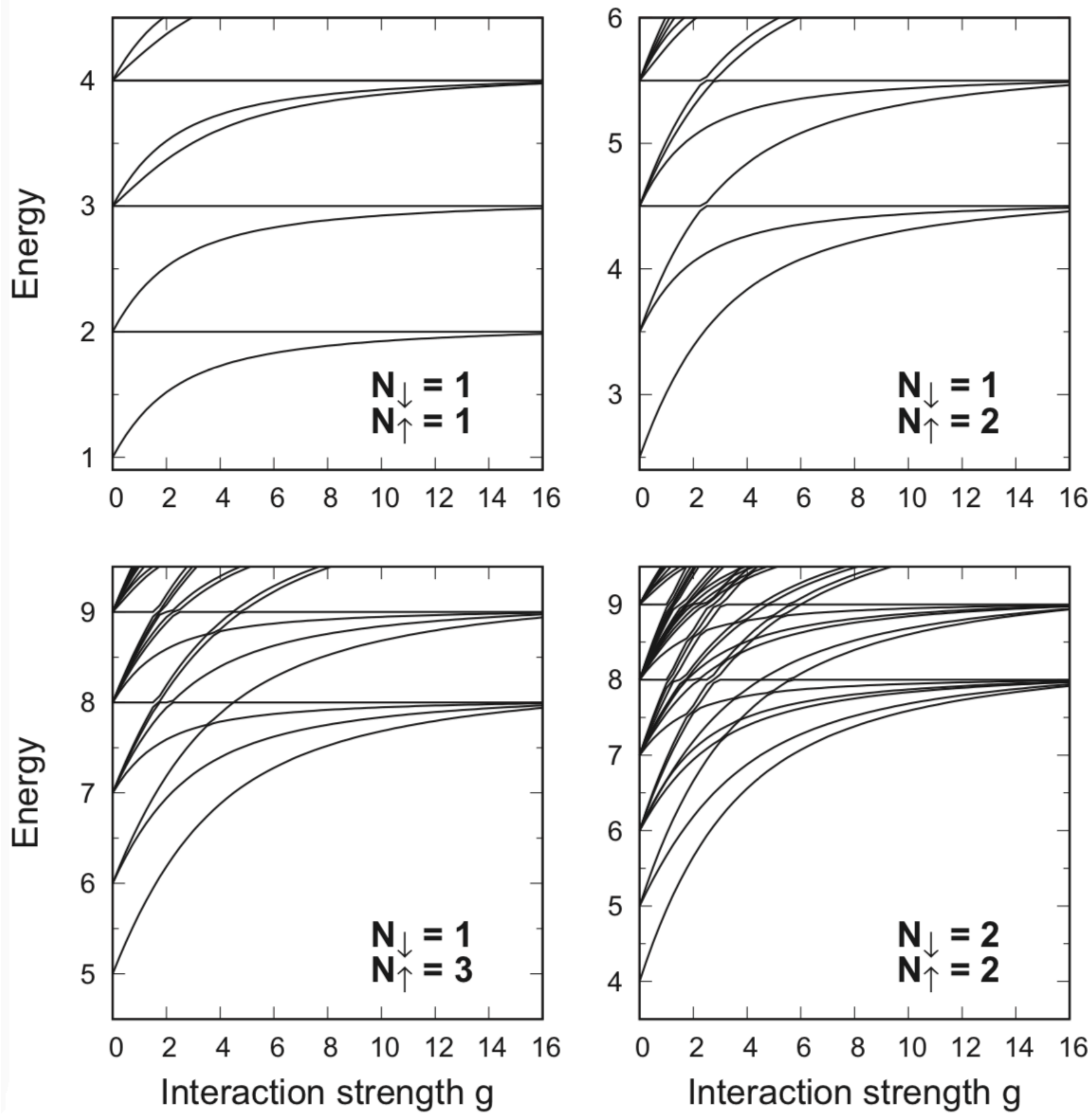
$$\left\{\hat{\Psi}_\sigma(x), \hat{\Psi}^\dagger_\sigma(x')\right\} = \delta(x-x')$$

$$\left\{\hat{\Psi}_\sigma(x), \hat{\Psi}_\sigma(x')\right\} = 0$$

$$\left[\hat{\mathcal{N}}_\uparrow, \hat{\mathcal{H}}\right] = \left[\hat{\mathcal{N}}_\downarrow, \hat{\mathcal{H}}\right] = 0$$

$$\hat{\mathcal{N}}_\sigma = \int \mathrm{d}x\, \hat{\Psi}^\dagger_\sigma(x)\hat{\Psi}_\sigma(x)$$

# many-body spectrum




**Notice** the quasi-degeneracy
of the spectrum for strong repulsions!

**Utilise it**
for precise thermometry!

# thermal sensitivity

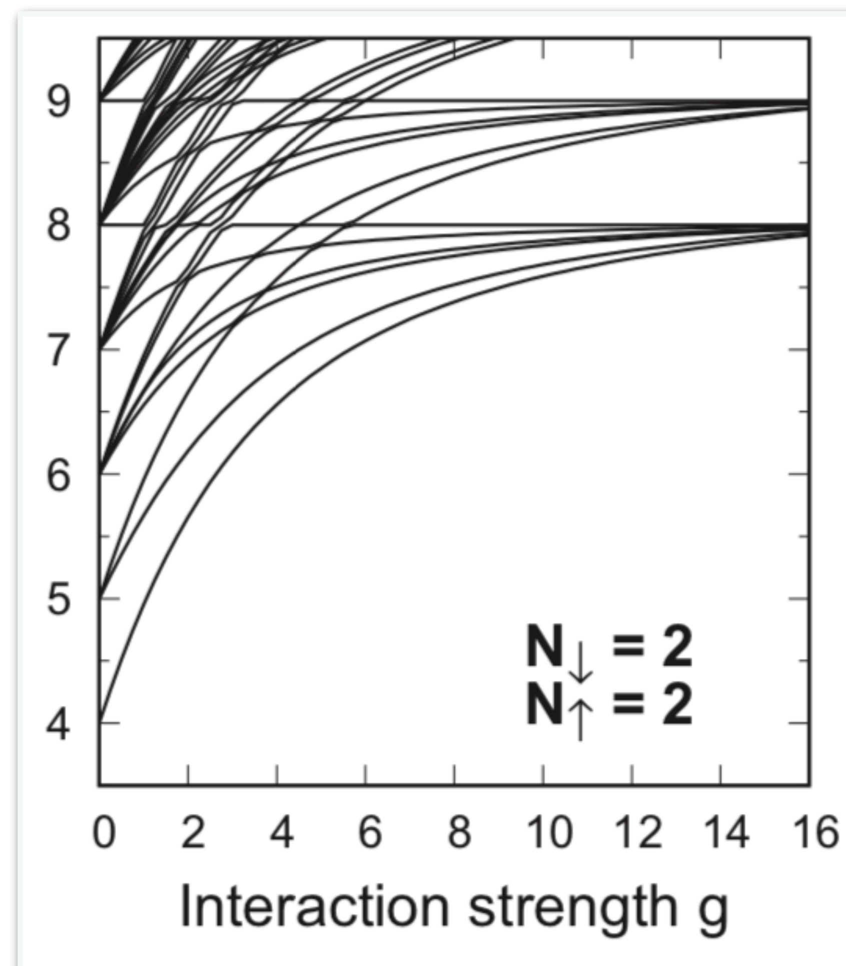
Let us assume that the system is in thermal equilibrium with some other system of unknown temperature **T**

$$P_i = \frac{1}{\mathcal{Z}} \mathrm{e}^{-E_i/k_B T}$$

$$\hat{\rho}_T = \frac{1}{\mathcal{Z}} \mathrm{e}^{-\hat{\mathcal{H}}/k_B T} = \sum_i P_i \, |\mathcal{E}_i\rangle\langle\mathcal{E}_i|$$

**Question:** how precisely one can determine temperature **T** by performing measurements on the system?

**Answer:** it depends on measurements one has in the arsenal!



# thermal sensitivity

$$\hat{\rho}_T = \frac{1}{\mathcal{Z}}\,\mathrm{e}^{-\hat{\mathcal{H}}/k_B T} = \sum_i P_i\,|\mathcal{E}_i\rangle\langle\mathcal{E}_i|$$

**Measurements:** Some set of probabilities as functions of T.
The limit is determined by the Fisher information.

$$\{p_i(T)\} \longrightarrow \mathcal{F}(\{p_i\}) = \sum_i \frac{1}{p_i}\left(\frac{\mathrm{d}p_i}{\mathrm{d}T}\right)^2$$

$$\Delta T^2 \geq \frac{1}{n\,\mathcal{F}(\{p_i\})}$$

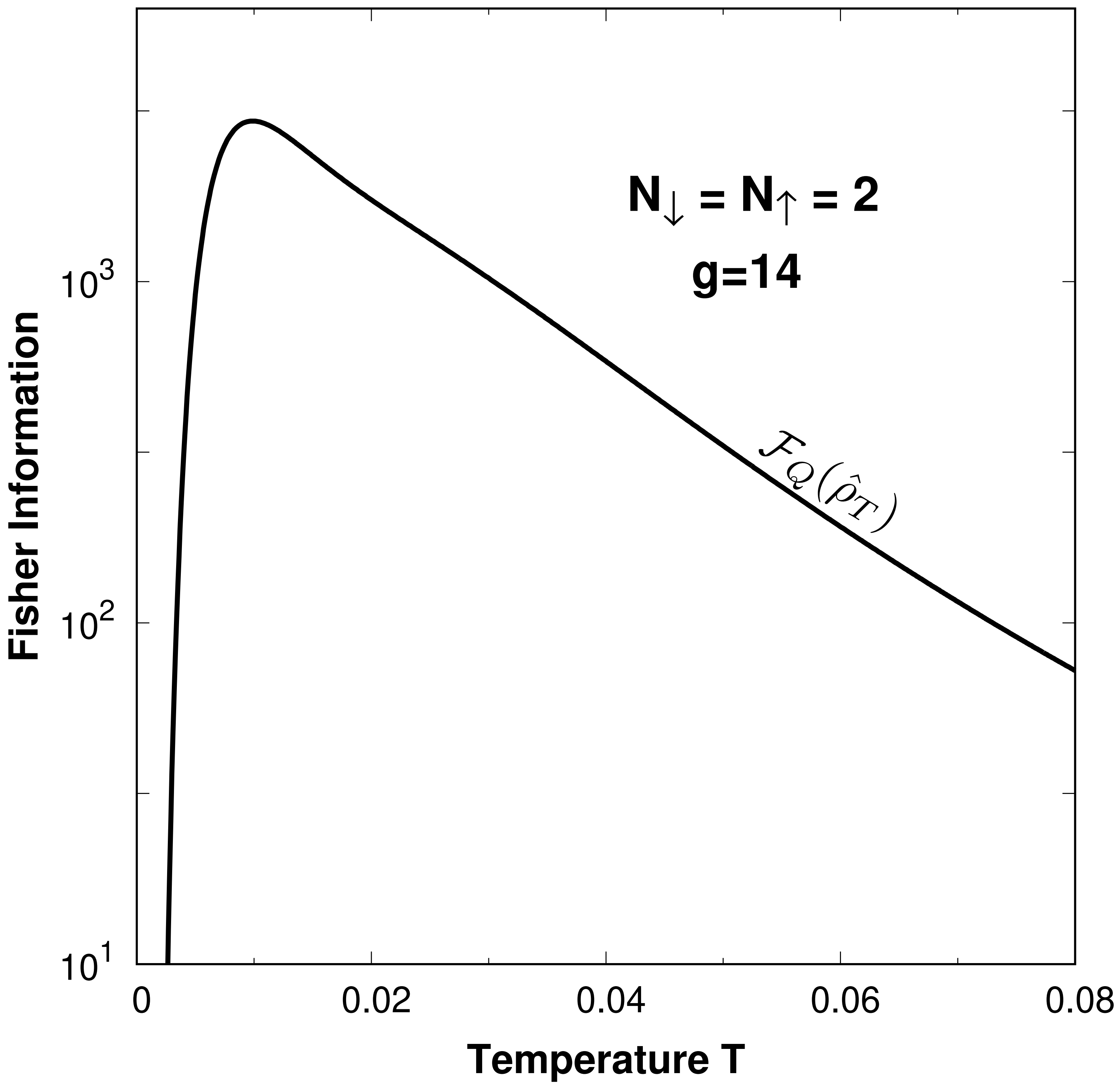
**IF:** One can measure Gibbs probabilities

$$\{p_i(T)\} = \{P_i(T)\}$$

then the Fisher information is the largest possible!

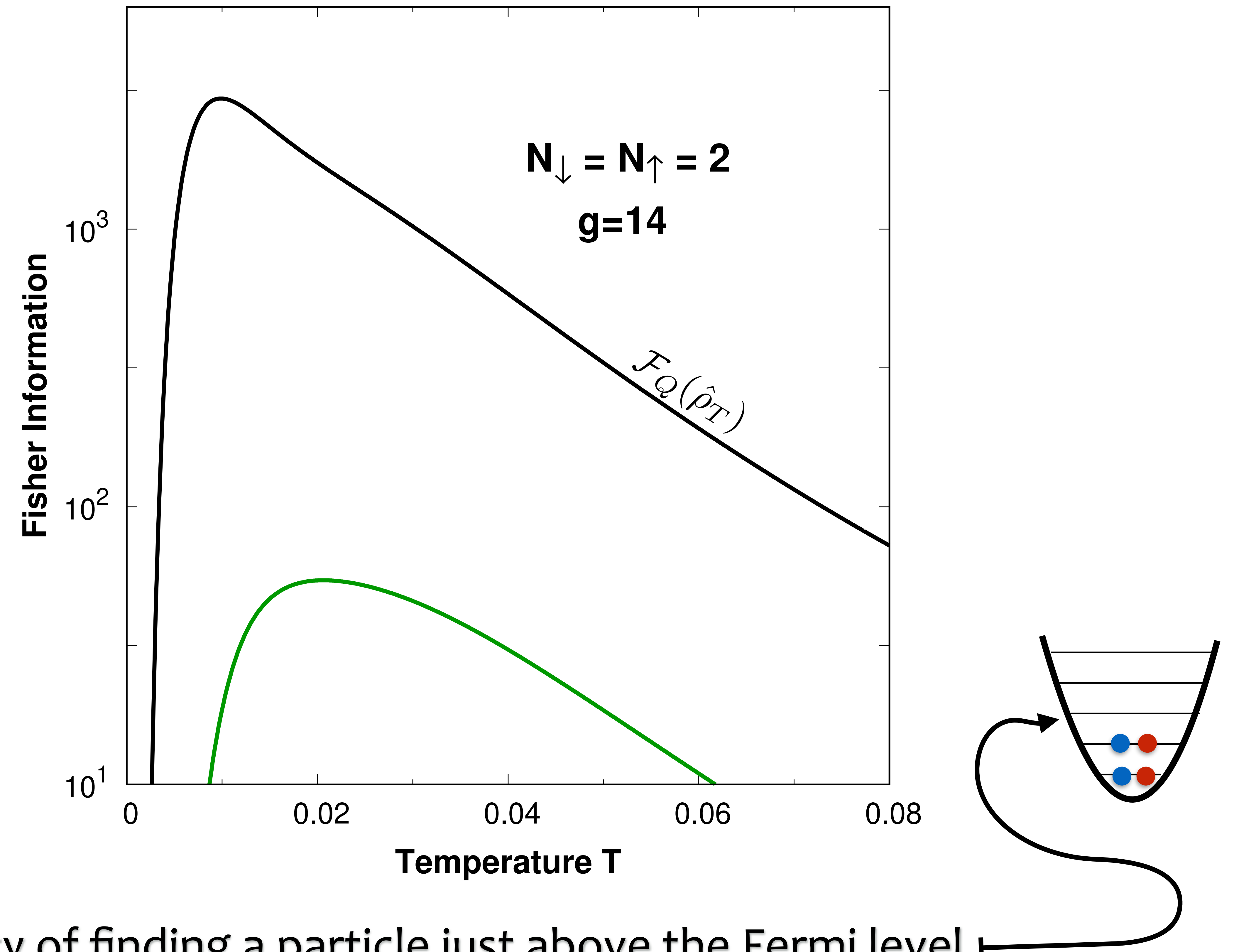
$$\mathcal{F}_Q(\hat{\rho}_T) = \frac{\Delta\hat{\mathcal{H}}^2}{T^4} = \frac{\mathrm{Tr}[\hat{\rho}_T\hat{\mathcal{H}}^2] - \mathrm{Tr}[\hat{\rho}_T\hat{\mathcal{H}}]^2}{T^4}$$

# quantum Fisher information



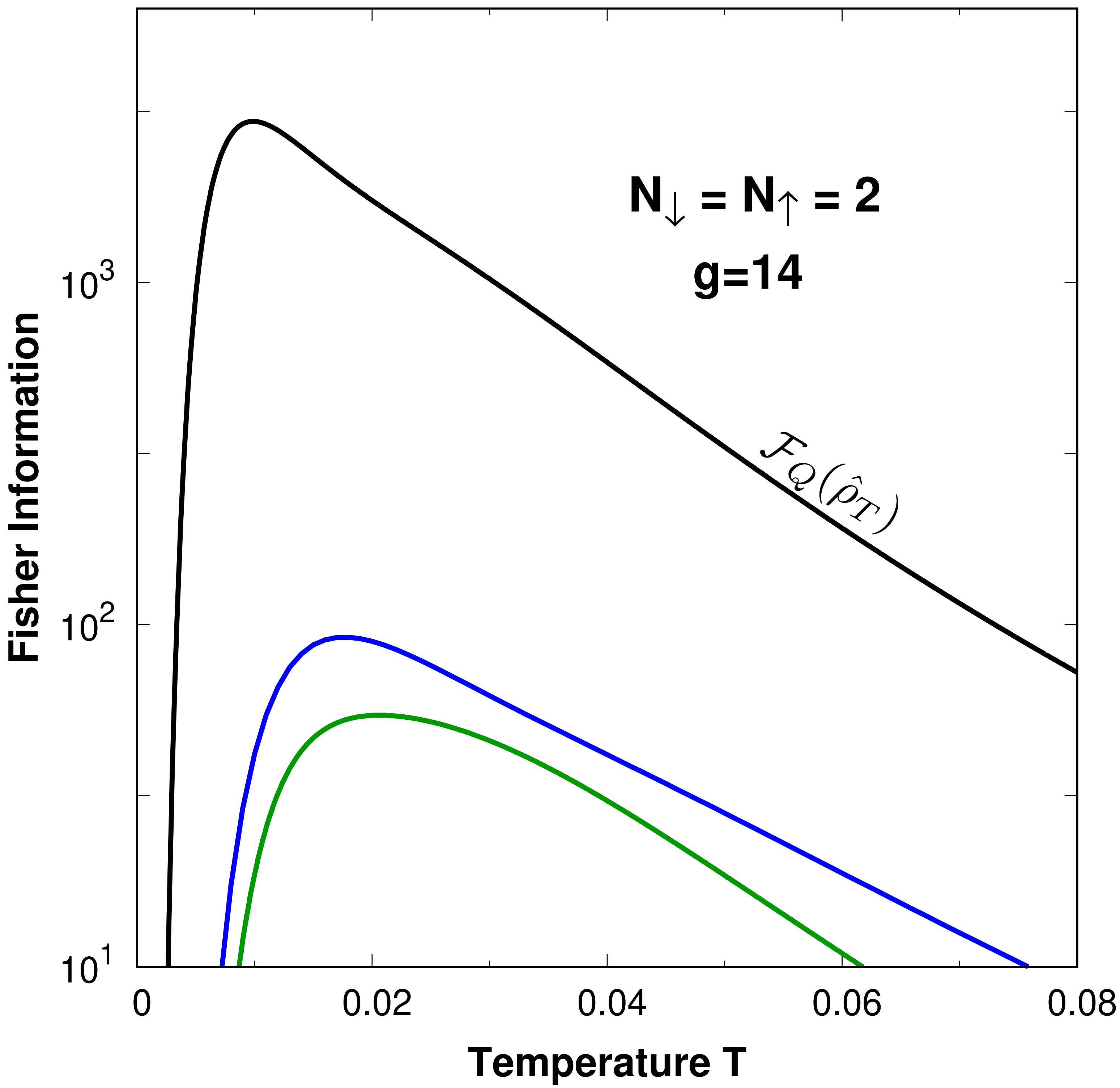

$$\mathcal{F}_Q(\hat{\rho}_T) = \frac{\Delta\hat{\mathcal{H}}^2}{T^4} = \frac{\mathrm{Tr}[\hat{\rho}_T\hat{\mathcal{H}}^2] - \mathrm{Tr}[\hat{\rho}_T\hat{\mathcal{H}}]^2}{T^4}$$

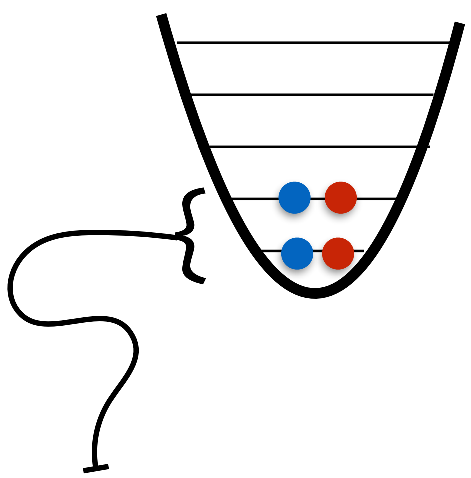
# the simplest scenario




The probability of finding a particle just above the Fermi level

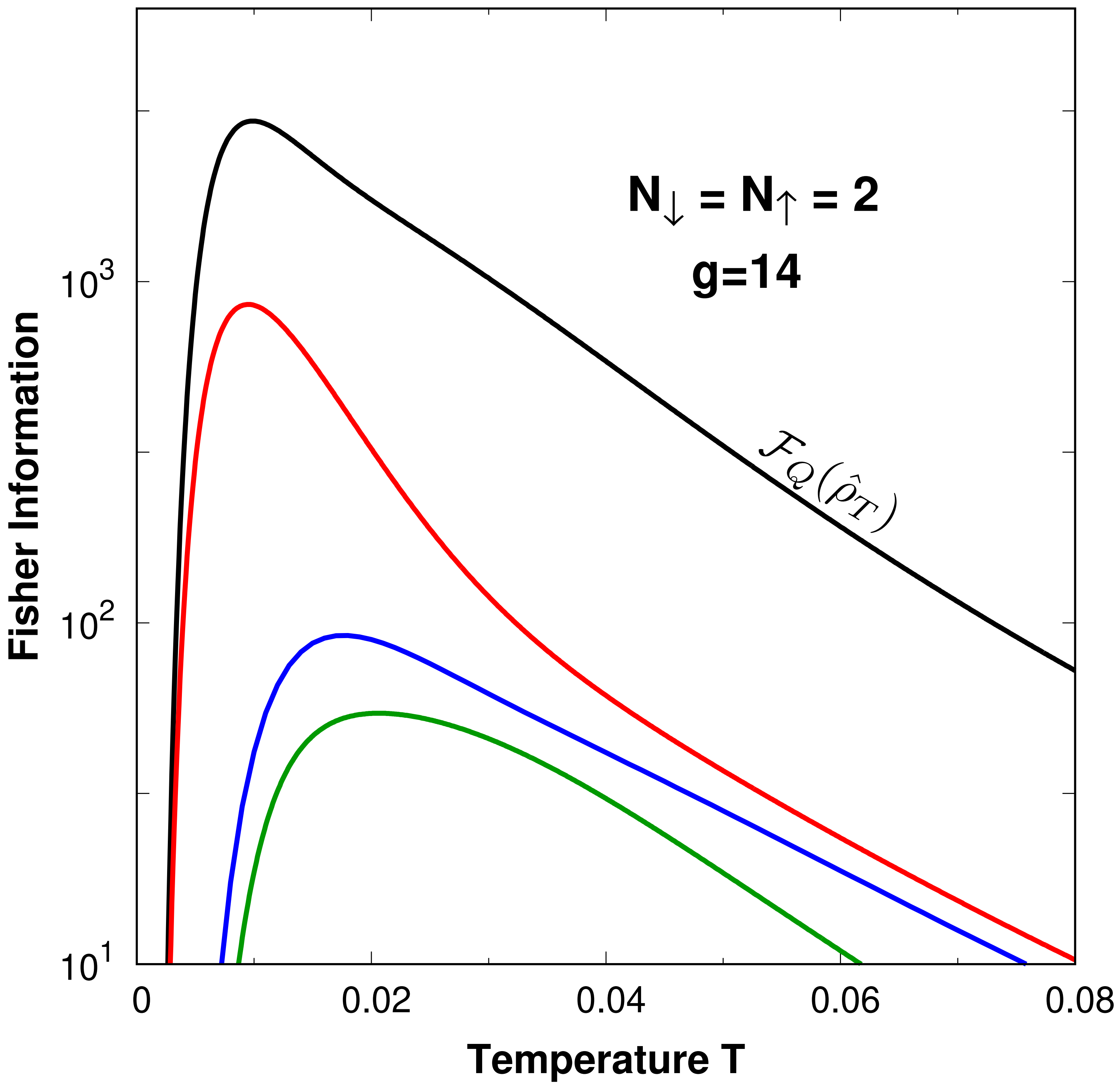
# Occupations of the lowest orbitals

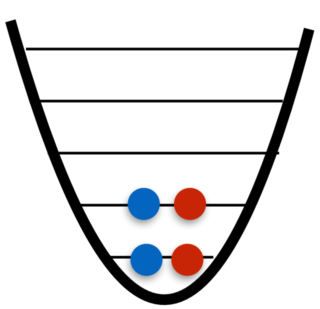


Probabilities **$p_0$, $p_1$, $p_2$, $p_3$, $p_4$** that in two the lowest orbitals one can find **0, 1, 2, 3,** or **4** particles.

# All occupations are known



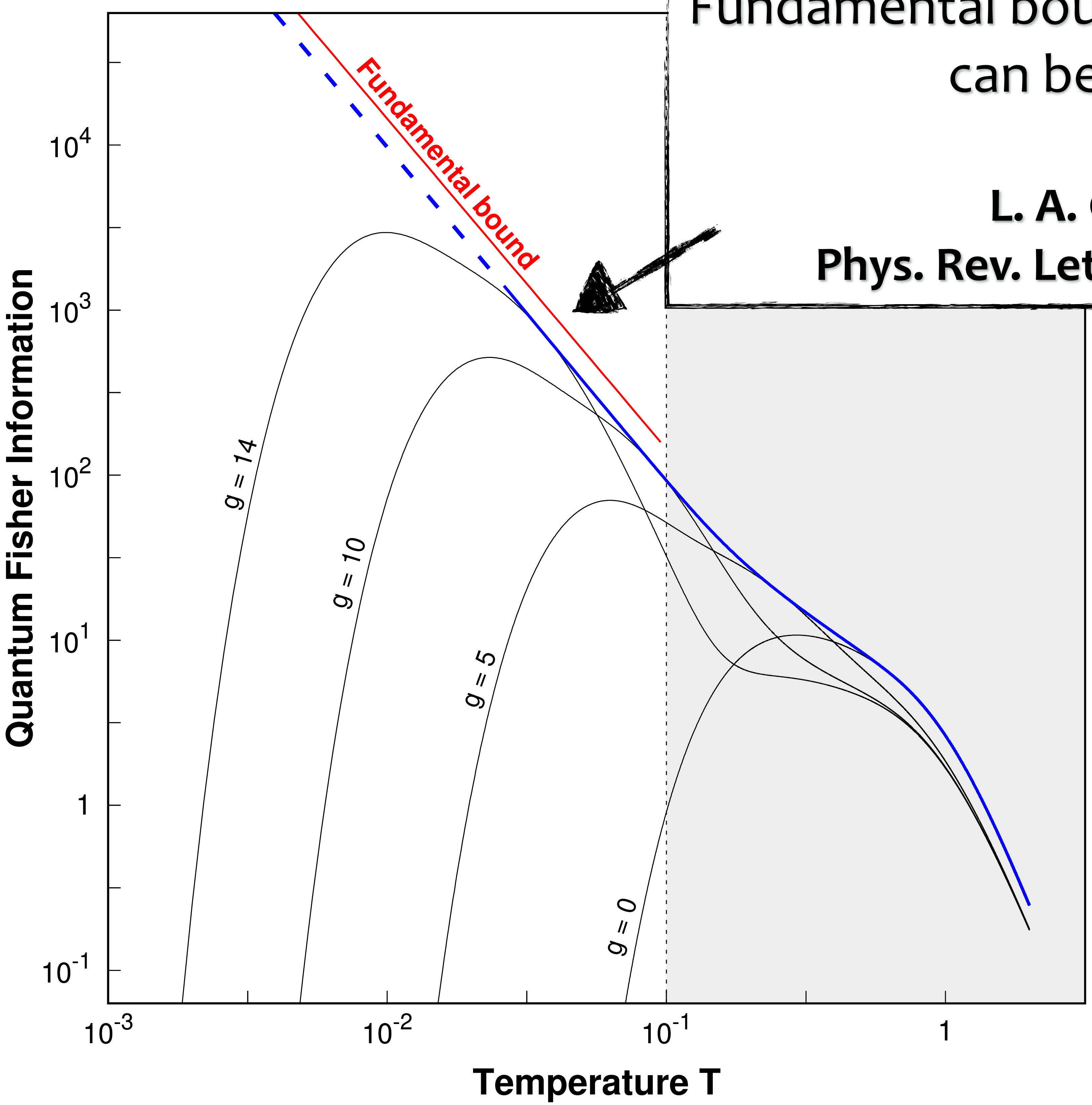



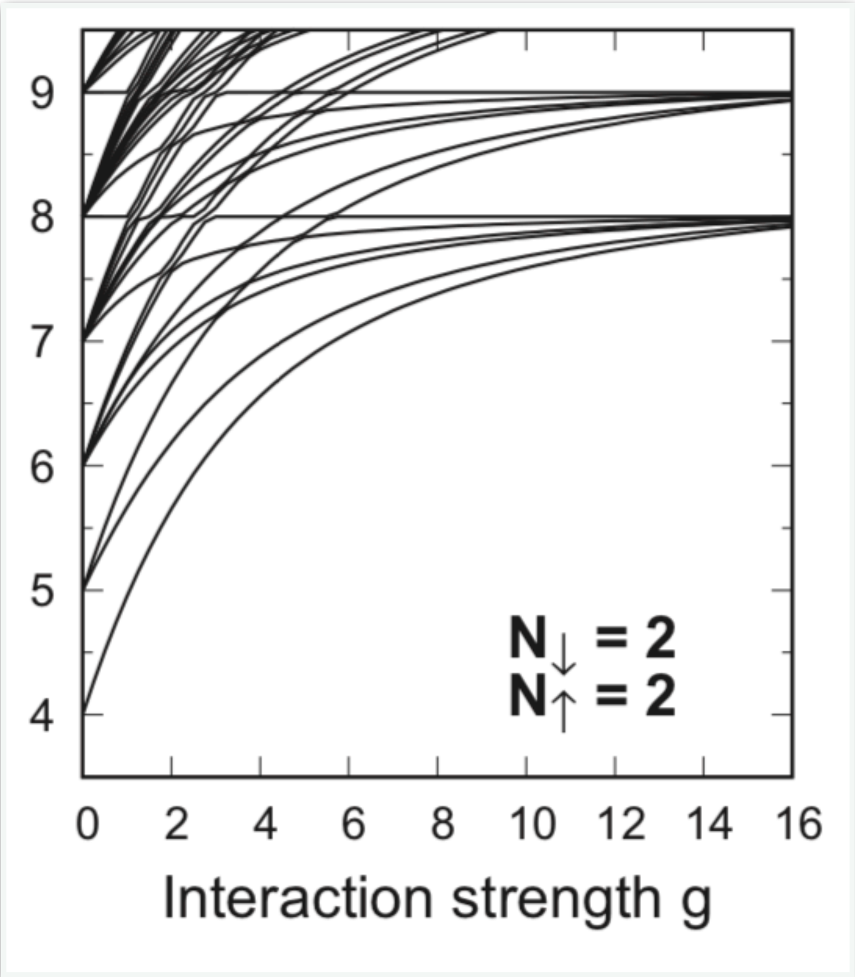
When we know **all** probabilities
of finding **four particles** in different configurations

# the fundamental bound

Fundamental bound when level spacings can be engineered

**L. A. Correa et al.**
**Phys. Rev. Lett. 114, 220405 (2015).**



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