Universality and chaos in XY spin glasses

Marek Cieplak and Mai Suan Li*
Institute of Physics, Polish Academy of Sciences 02-668, Warsaw, Poland

Jayanth R. Banavar
Department of Physics and Materials Research Laboratory, Pennsylvania State University, 104 Davey Laboratory, University Park, Pennsylvania 16802

(Received 20 July 1992)

Chaotic behavior in XY spin-glass models is investigated using the discretized Migdal-Kadanoff renormalizaton-group scheme. The zero-temperature scaling exponent is found to be independent of the degree of discretization. However, the Lyapunov exponent characterizing the chaotic behavior shows a continuous variation reflecting the nonuniversal behavior of the interfacial entropy.

The vortex glass phase[1] has been invoked as an explanation of the unusual magnetic properties of the mixed state of oxide superconductors. A simplified model that is believed to capture the physics of the vortex glass is the random-gauge XY model[2] (RGXY) described by the Hamiltonian

$$H=-\sum_{ij} J_{ij}\cos(\phi_i-\phi_j-A_{ij}) \ , \tag{1}$$

where $J_{ij}=J$ when i and j are nearest neighbors and zero otherwise, A_{ij} is a quenched random variable distributed uniformly between 0 and 2π, and ϕ_i, the spin variable, takes on values between 0 and 2π. It has been demonstrated that the RGXY has a lower critical dimensionality less than three[2-5]—in three dimensions it has a transition from a spin-glass state to a paramagnetic state at a nonzero temperature. Strikingly, this behavior is qualitatively different from the bimodal XY spin glass (BXY) (obtained by restricting the values of A_{ij} to be 0 or π with equal probability) which has a $T=0$ phase transition in three dimensions. These results have been deduced using a powerful yet simple phenomenological scheme called zero-temperature scaling.[6,7] The basic idea is that at long length scales the low-temperature spin glass phase is governed by a zero-temperature fixed point. One of the predictions of this approach is the chaotic nature of the spin-glass phase[8-12]—in the Ising spin-glass context, the relative orientations of spins is increasingly sensitive as the spin separation is increased to small perturbations in externally controlled variables such as temperature. Recently Nifle and Hilhorst[12] have shown that chaos in the critical region in three-dimensional Ising systems is characterized by a new critical exponent leading to new scaling laws. The chaotic nature associated with the flipping of large droplets of Ising spins has been invoked to interpret dynamical spin-glass experiments involving aging[9] and dimensional crossover[13] in layered systems and as a probe of conductance fluctuation in mesoscopic metallic glasses.[14]

While the droplet theory[9] for Ising spin glasses is well developed, much less is known about excitations in systems of spins with continuous symmetry. Indeed, this is a matter of some importance since real materials are expected to have XY or Heisenberg spins possibly with some anisotropy. Indeed, the only study[15] of the chaotic nature of such systems employed a Migdal-Kadanoff renormalization-group (MKRG) approach within a harmonic approximation: The Hamiltonian is expanded about the equilibrium angle between neighboring spins up to quadratic terms with the Gaussian form of the partition function taken to be valid over the entire range of angles. This enables a recursion scheme since the quadratic nature is preserved in a decimation process. Unfortunately this scheme is expected to work only for $T=0$ and even then it is *ad hoc*. Indeed, such a scheme leads to an incorrect result that RGXY has a lower critical dimensionality greater than 3.[16]

Yet another problem of principle arises in extending a simple Imry-Ma[17] argument for Ising spin glasses to continuous spins. The argument for the Ising case is as follows: Consider a system at zero temperature in its ground state. We now add a tiny random perturbation of relative strength ε to each bond. We ask whether the ground state is stable to this perturbation. The cost of a droplet excitation of linear extend L is taken to scale as JL^y where J is a measure of the width of the unperturbed bond distribution and $-y$ is the scaling dimension of the temperature. The possible gain in energy on flipping the droplet could arise at the interface of the domain and scales as $\varepsilon L^{d_s/2}$ where d_s is the fractal dimension of the surface of the droplet. Since $\zeta=d_s/2-y>0$ for Ising spin glasses, the ground state becomes unstable against the perturbation on length scales $L>L^*\sim 1/\varepsilon^{1/\zeta}$. The chaotic dependence on the temperature difference follows from the fact that two identical systems at slightly different temperatures flow to the zero-temperature fixed point but with slightly different renormalized exchange couplings so that the above argument is again valid. In this context, the physical meaning of d_s for a spin possessing continuous symmetry is unclear. Furthermore, the nature of the excitations in such system are probably very different from the Ising case.

In this paper we present the results of the application of a MKRG scheme to the study of the chaotic phase of the RGXY model described by Eq. (1). In addition to the *ad hoc* harmonic approximation approach, we consider a technique introduced recently involving discretization of the XY spins[18]—this scheme is *self-consistent,* it allows us to extend our analysis to nonzero temperature thus permitting the investigation of chaos arising from tiny temperature differences, and it allows us to extrapolate between the RGXY and an Ising spin glass and address important questions of universality.

The MKRG scheme allows one to obtain closed renormalization-group recursion relations by expressing a coupling after renormalization in terms of $2D$ bare couplings consisting of 2^{D-1} parallel strings of two couplings each (D is the spatial dimensionality). The scheme is exact for hierarchical lattices when the spin degrees of freedom are discrete so that the form of the Hamiltonian is preserved under the transformation. The study of systems with continuous symmetry such as those given by Eq. (1) poses a problem since the renormalized Hamiltonian no longer has the simple cosine form. The key idea behind the discretization scheme is that instead of allowing ϕ to be a continuous variable, we allow it to take on one of many discrete values uniformly distributed between 0 and 2π. The values of A_{ij} are also limited to the appropriate set. The Hamiltonian is defined for values of ϕ restricted to be $2\pi k/q$ with $k=0,1,\ldots,(q-1)$. The coupling between neighboring spins in states k_1 and k_2 will be denoted by $J(q,k)$, where $k=|k_2-k_1|$. The continuous symmetry case is recovered in the limit of $q\to\infty$. In the limit of $q=2$, the model reduces to a bimodal Ising spin glass (BSG). Previous studies[18] using the discretized MKRG in three dimensions have shown that the exponent y is the same for both $q=\infty$ and 2. This result does not, however, mean that they are in the same universality class since y is essentially a $T=0$ scaling exponent and is quite unrelated to the exponents at criticality. Indeed, numerical calculations using regular lattices in three dimensions are suggestive that the values of y for the RGXY (Refs. 4 and 5) and BSG (Ref. 7) do *not* coincide.

Our calculations follow the procedures in Refs. 11 and 18. We construct a pool of bonds $J_i(q,k)$, $\varepsilon=1,N$, resulting from the random choices in the gauge factors A_{ij} where N is typically between 2000 and 10 000 (depending on q), chosen initially to represent the Hamiltonian (1). 2^D members of the pool are chosen randomly to construct one renormalized coupling—this procedure is repeated until a renormalized pool is formed. The scaling exponent y is obtained by considering how a characteristic measure of the coupling strength $\tilde{J}(n)$, defined as

$$\frac{1}{N}\sum_{i=1}^{N}\max_k J_i(q,k)$$

(where the iteration index n has been omitted) changes after n iterations $[\tilde{J}(n)\sim 2^{ny}]$. The Lyapunov exponent for chaos is determined by taking two pools that are almost identical and studying how they diverge from each other until they become completely uncorrelated. The characteristic mean-square distance defined by

$$d^2(n)=\sum_{i=j}^{n}\sum_{k=0}^{q-1}[J_i(q,k)-J_i'(q,k)]^2\Big/\sum_{i=1}^{N}\sum_{k=0}^{q-1}[J_i^2(q,k)+J_i'^2(q,k)]\ , \tag{2}$$

approaches 1 (corresponding to uncorrelated pools) with an initial power-law dependence of $d^2(n)$ on a length scale 2^n, $d^2(n)\sim L^{2\zeta}$. We consider two kinds of initial perturbations, one corresponding to small differences in the temperature and the other perturbing randomly and infinitesimally the A_{ij}'s in (1) but holding the temperature at zero.

Our results for y and ζ in $D=3$ are shown in Figs. 1–4. Figure 1 shows that y is the same for the values of q studied confirming the previously stated result that RGXY and BSG have the same y and so does the Gaussian Ising spin glass. However, the exponent ζ controlling the characteristic length scale of the system corresponding to complete decorrelation varies continuously with q and is a striking example of nonuniversal behavior. Indeed the *same* continuous variation of ζ with q is obtained independent of whether one considers minute initial variation in temperature or in the exchange coupling at zero temperature (Figs. 2 and 3). We have also investigated the chaotic behavior for $D=4$ and the results are shown in Fig. 4. As previously found for Ising spin glasses and the Heisenberg spin glass within the harmonic approximation, the value of ζ is substantially independent of D. Figure 5 shows a plot of ζ versus q. The numerical data are described by a fit $\zeta=1-0.309\exp(-0.18q)$. The star in Fig. 5 shows the value of ζ determined using transfer-matrix methods in $D=2$ by Bray and Moore.[9] The arrows at the right represent the values of ζ deduced using the harmonic approximation and by taking the fixed-shape probability distribution to be Gaussian.[20]

We now turn to a discussion of the physical significance of ζ and d_s for continuous symmetry spin systems. Recall that ζ, d_s, and y are related by the equations $\zeta=d_s/2-y$. The exponent ζ determines the characteristic length scale L^* at which two systems at temperatures T and $T+\delta T$ become decorrelated by the expression

$$L^*=(\Upsilon/\sigma\delta T)^{1/\zeta}\ , \tag{3}$$

where Υ and σ are defined by the expressions for interfacial free energy and entropy,

$$F_{\text{int}}=\Upsilon(T)L^y \quad \text{as } L\to\infty \tag{4}$$

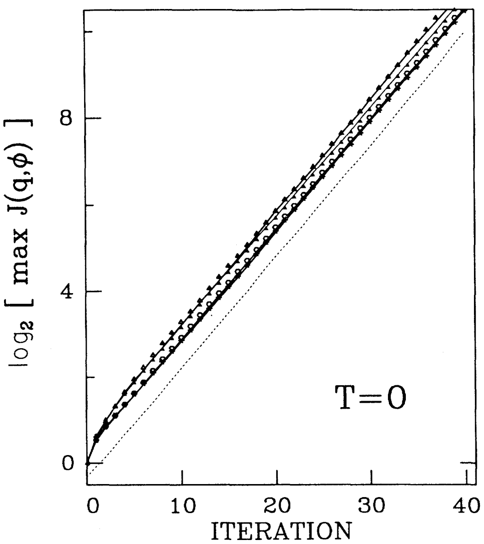

FIG. 1. Scaling of the characteristic coupling strength for $D=3$ and $T=0$. The characteristic value is chosen to be the maximum coupling over the angle ϕ between neighboring spins, or equivalently, over the difference in the discretization index k. The dotted line corresponds to the Gaussian Ising spin glass. The data points are for the RGXY model with various values of q: 2 (large triangles), same as bimodal Ising spin glass; 3 (smaller triangles); 6 (hexagons); and 12 (asterisks). The lines are data points obtained from a second run. The deviations between symbols and their corresponding lines are measures of error bars. The exponent $y\approx 0.26$ for all of the data points.

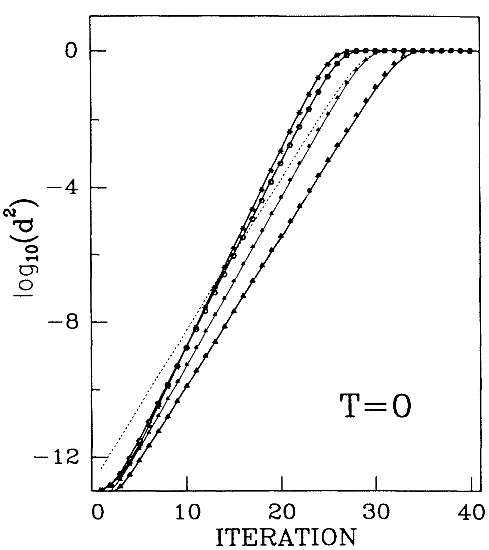

FIG. 2. Scaling of normalized distance between pools for $D=3$ and $T=0$. The chaos is induced by a perturbation in the couplings. The symbols are as in Fig. 1. For $q=2$ the resulting ζ is 0.74 (bimodal and Gaussian Ising spin glass). For $q=3$, 6, and 12 the ζ is 0.82, 0.90, and 0.96, respectively. The error bars are of order 0.01.

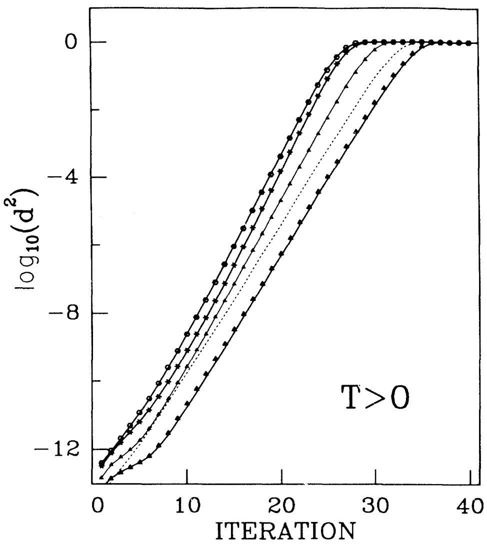

FIG. 3. Same as in Fig. 2 but for $T=0.3J/k_B$. The chaos is induced by a perturbation in temperature of magnitude $\Delta T/T=10^{-6}$.

and

$$S_{\rm int}=\sigma(T)L^{d_s/2} \quad \text{as } L\to\infty\ . \tag{5}$$

Thus the nonuniversal behavior of ζ underscores the nonuniversal behavior of the interfacial entropy on varying the parameter q, the number of discretized states in the model.

As mentioned previously, the RGXY and BXY models have qualitatively different behaviors in $D=3$. We have studied a model with $q=500$ within the discretized MKRG scheme that extrapolates from the BXY to the

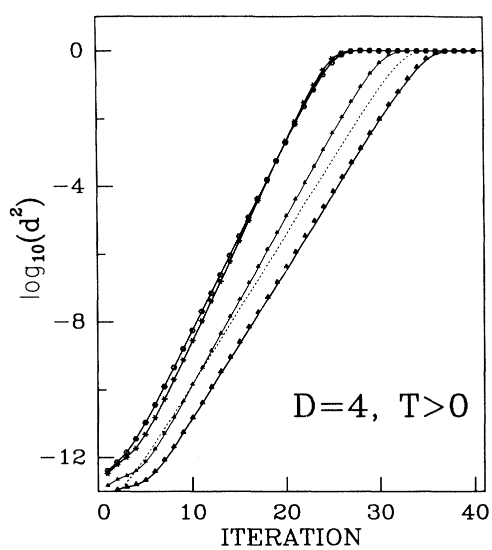

FIG. 4. Same as in Fig. 3, but for $D=4$.

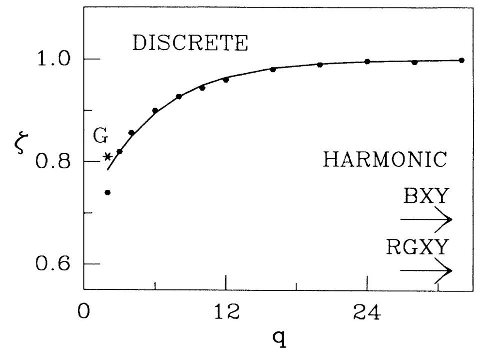

FIG. 5. Dependence of ζ on q. Data points (hexagons) are based on the combined $D=3$ and $D=4$ calculations. The arrows indicate results for ζ in the $q \to \infty$ limit obtained within the harmonic approximation. For the BXY model $\zeta = 0.69$ and for the RGXY model [in the version given by Eq. (6)] $\zeta = 0.59$. The star corresponds to the transfer-matrix result ($\zeta = 0.81$) for the bimodal and Gaussian Ising spin glass.

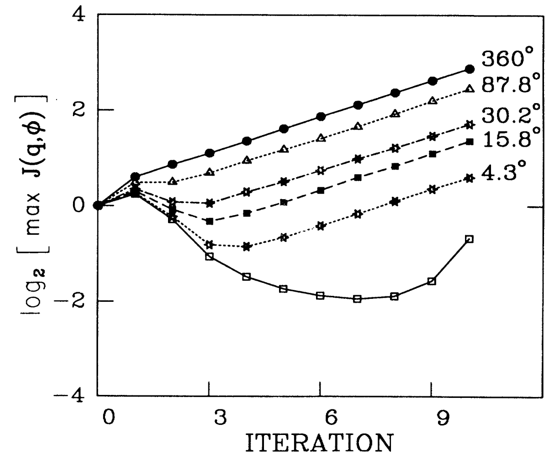

FIG. 6. Scaling of the characteristic coupling strength in the $q=500$ clock model for $D=3$ and $T=0$. The open squares connected by the solid line correspond to the BXY case [A_{ij} randomly equal to 0 or π, so that $J(q,\phi)$ is nonzero only for $\phi=0$ and π]. Increasing q extends the downward trend of the scaling curve. The other data points correspond to the situation in which the $\phi=0$ and π states have a probability of 45% each. The remaining 10% is located in the caps around the poles. The angular extend of each cap, $\Delta\phi$, is half of the number indicated on the right. Rescaling results in spreading of the caps across the whole clock. Varying q for a given $\Delta\phi \neq 0$ does not affect the scaling curves.

RGXY. Note that the BXY model corresponds to the distribution of A_{ij} being either 0 or π with equal probability. Figure 6 demonstrates that spreading this delta function distribution by a tiny amount is a relevant perturbation—at long length scales, the behavior crosses over to that of the RGXY. It would be interesting to verify whether the dependence of the lower critical dimensionality on the distribution of A_{ij} and the continuous variation of ζ with q hold on Euclidean lattices.

We have evaluated the critical behavior of both the RGXY and a Gaussian Dzyaloshinsky-Moriya model[16,19] (GDM) defined by

$$H = \sum_{\langle ij \rangle} J_{ij} \cos(\phi_i - \phi_j) + \sum_{\langle ij \rangle} D_{ij} \sin(\phi_i - \phi_j) \,, \qquad (6)$$

where J_{ij} and D_{ij} are *independent* nearest-neighbor couplings each chosen from a Gaussian distribution. We use the discretized MKRG with $q=100$. Our estimates of T_c and the correlation length exponent ν for the RGXY (and GDM) are 0.46 ± 0.01 (0.50 ± 0.01) and 2.5 ± 0.1 (2.5 ± 0.1), respectively.[13] For the RGXY, the Monte Carlo estimates[5] of T_c are 0.45 ± 0.05 and ≈ 0.6,[2] whereas $\nu = 1.3 \pm 0.4$.[5] The exponent ν has also been obtained experimentally. Olsson *et al.*[20] obtain $\nu = 1.7$ and 2 ± 1, respectively. In recent work, Dekker, Eidelloth, and Koch[21] get $\nu = 1.8 \pm 0.2$ suggesting that the MKRG overestimates the value of ν.

We are indebted to Amil Khurana for many discussions. Grants from the ONR, NSF, the Polish agency KBN, and the Center for Academic Computing at Penn State University are acknowledged gratefully.

*On leave from the Thai-Nguyen Technical Institute, Vietnam.

[1] M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).
[2] D. A. Huse and H. S. Seung, Phys. Rev. B **42**, 1059 (1990).
[3] M. Cieplak, J. R. Banavar, and A. Khurana, J. Phys. A **24**, L145 (1991).
[4] M. J. P. Gingras, Phys. Rev. B **45**, 7547 (1992).
[5] J. D. Reger, T. A. Tokuyasu, A. P. Young, and M. P. A. Fisher, Phys. Rev. B **44**, 7147 (1991).
[6] J. R. Banavar and M. Cieplak, Phys. Rev. Lett. **48**, 832 (1982).
[7] A. J. Bray, Comment. Condens. Matter **14**, 21 (1988); A. J. Bray and M. A. Moore, in *Heidelberg Colloquium on Glossy Dynamics,* edited by L. Van Hemmen and I. Morgenstern (Springer, Berlin, 1987), p. 121.
[8] S. R. McKay, A. N. Berker, and S. Kirkpatrick, Phys. Rev. Lett **48**, 767 (1982); J. Appl. Phys. **53**, 7976 (1982).
[9] D. S. Fisher and D. A. Huse, Phys. Rev. Lett. **56**, 1601 (1986); Phys. Rev. B **38**, 386 (1988).
[10] A. J. Bray and M. A. Moore, Phys. Rev. Lett. **58**, 57 (1987).
[11] J. R. Banavar and A. J. Bray, Phys. Rev. B **35**, 8888 (1987).
[12] M. Nifle and H. J. Hilhorst, Phys. Rev. Lett. **68**, 2992 (1992).
[13] P. Grangerb, J. Mattsson, P. Nordblad, L. Lundgren, R. Stubi, J. Bass, D. L. Leslie-Pelecky, and J. A. Cowen, Phys. Rev. B **44**, 4410 (1991).
[14] S. Feng, A. J. Bray, P. A. Lee, and M. A. Moore, Phys. Rev. B **36**, 5624 (1987); see also, M. Cieplak, B. R. Bulka, and T. Dietl, *ibid.* **44**, 12 337 (1991).
[15] J. R. Banavar and A. J. Bray, Phys. Rev. B **38**, 2564 (1988).
[16] M. J. P. Gingras, Phys. Rev. B **43**, 13 747 (1991).
[17] Y. Imry and S. K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).
[18] M. Cieplak, J. R. Banavar, M. S. Li, and A. Khurana, Phys.

Rev. B **45**, 786 (1992).

[19]In Ref. 18 it was incorrectly reported that the Dzyaloshinsky-Moriya model with a bimodal distribution of J_{ij} and D_{ij} behaved differently from the GDM and RGXY models. This calls into question the role of local gauge invariance in determining the lower critical dimensionality of frustated systems.

[20]H. K. Olsson, R. H. Koch, W. Eidelloth and R. P. Robertazzi, Phys. Rev. Lett. **66**, 2661 (1991).

[21]C. Dekker, W. Eidelloth, and R. H. Koch, Phys. Rev. Lett. **68**, 3347 (1992).