A semiclassical field theory free of the ultraviolet catastrophe

arXiv:1904.06266

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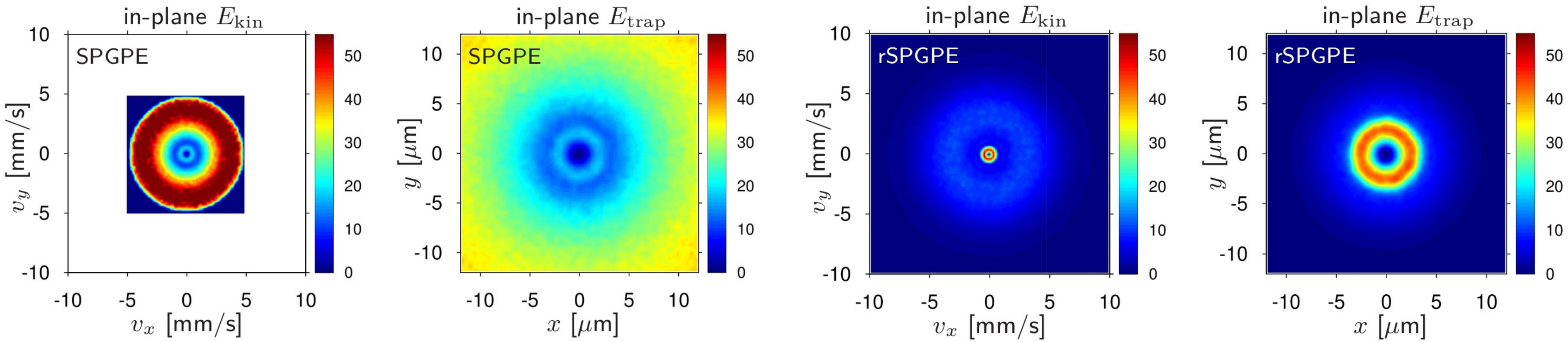


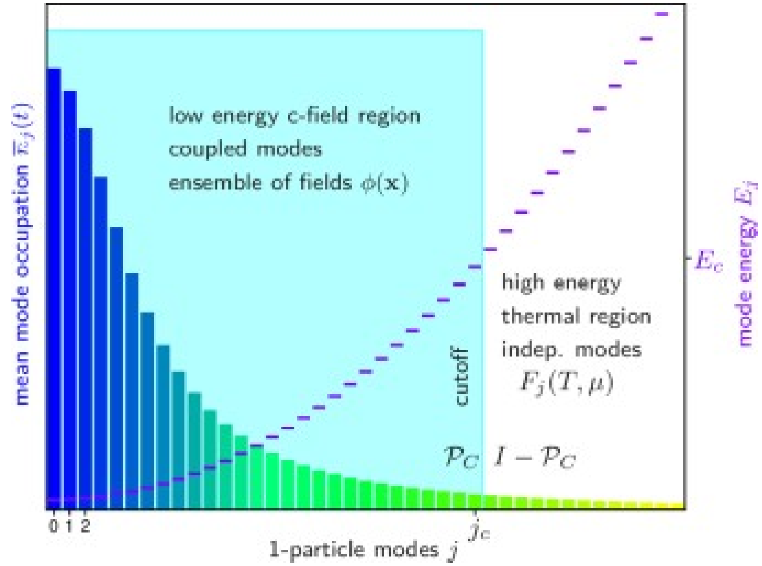
Outline

- Background: semiclassical wave fields for ultracold atoms
- The UV divergence and cutoff issues
- Fixing it
- Simple example: 1d

  (~1000 modes)
- Serious example: 3d collective breathing mode near Tc

  (1000000s of modes)

Simplest semiclassical method = "classical" wave fields



Assuming high occupation:

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk}$$

$$\hat{a}_j \gg 1 \;\rightarrow \hat{a}_j \approx \alpha_j$$

Bose field — semiclassical replacement

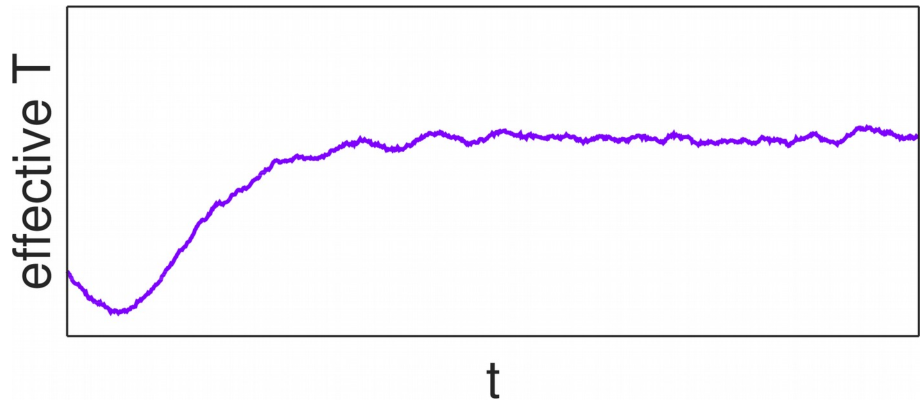
$$\hat{\Psi}(\mathbf{x}) = \sum_j \hat{a}_j \psi_j(\mathbf{x}) \longrightarrow \phi(\mathbf{x}) = \left\{ \sum_{j\in\mathcal{C}} \alpha_j \psi_j(\mathbf{x}) \right\}$$

Evolution: GPE

$$\hbar \frac{d\phi(\mathbf{x})}{dt} = -i\mathcal{E}(\mathbf{x})\phi(\mathbf{x})$$

$$\mathcal{E}\phi(\mathbf{x}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g\,|\phi(\mathbf{x})|^2 \right] \phi(\mathbf{x})$$

Thermalised ensemble at long time



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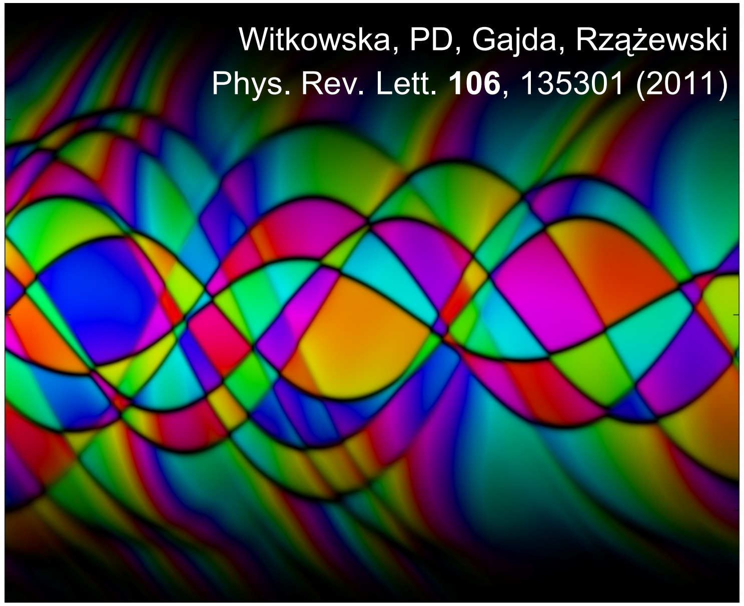
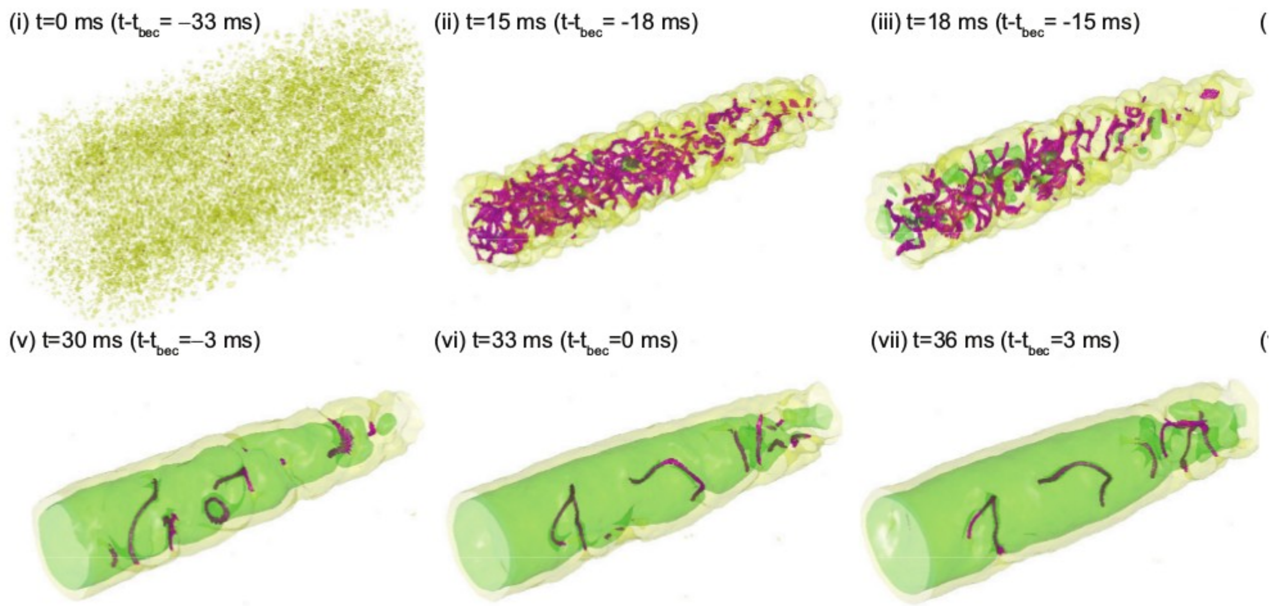
Treatment of truly large systems

- Good scaling: makes 10^7 modes tractable
- Nonperturbative
- Makes $T \gg 0$ possible
- Single shots = single realizations



1d gas – phase domains after cooling



Liu, Donadello, Lamporesi, Ferrari, Gou, Dalfovo, Proukakis, Commun. Phys. **1**, 24 (2018)

1d gas – thermal equilibrium

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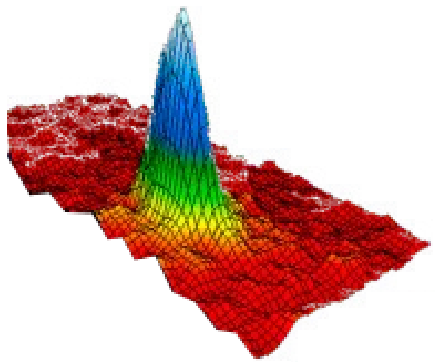
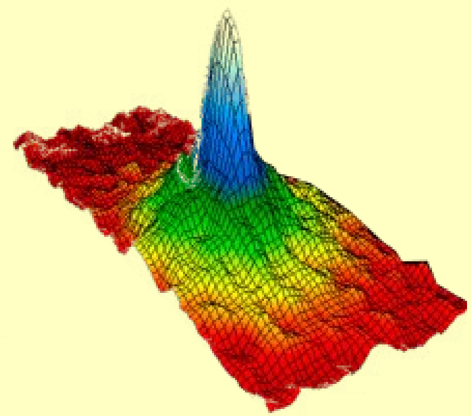
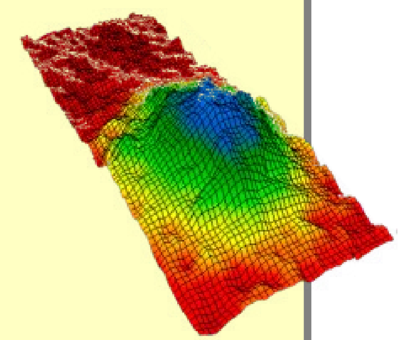
Thermal clouds - Classical fields essential

T=0
1 mode
GPE

Bogoliubov
few modes
perturbative

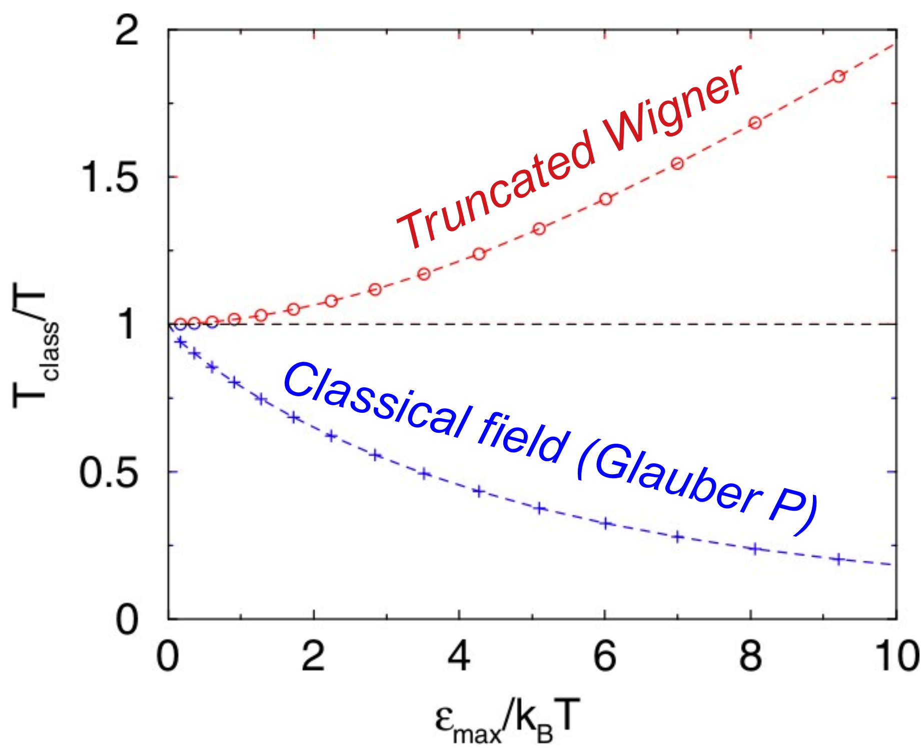
Classical fields, ZNG
zillions of modes
non-perturbative

$T=T_c$

However, problem 1: tainted thermalisation

Annoyingly:

Long time equilibrium temperature depends on cutoff

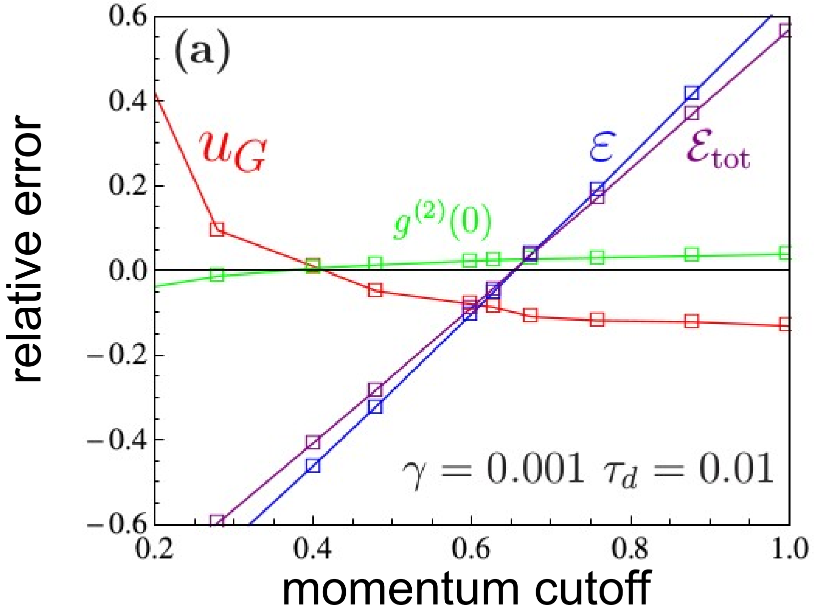


Sinatra, Lobo, Castin, J Phys B **35**, 3599 (2002)

- Stationary ensemble reached at long times can be bogus
- Will CERTAINLY be bogus if lattice is too fine (momentum cutoff too high)

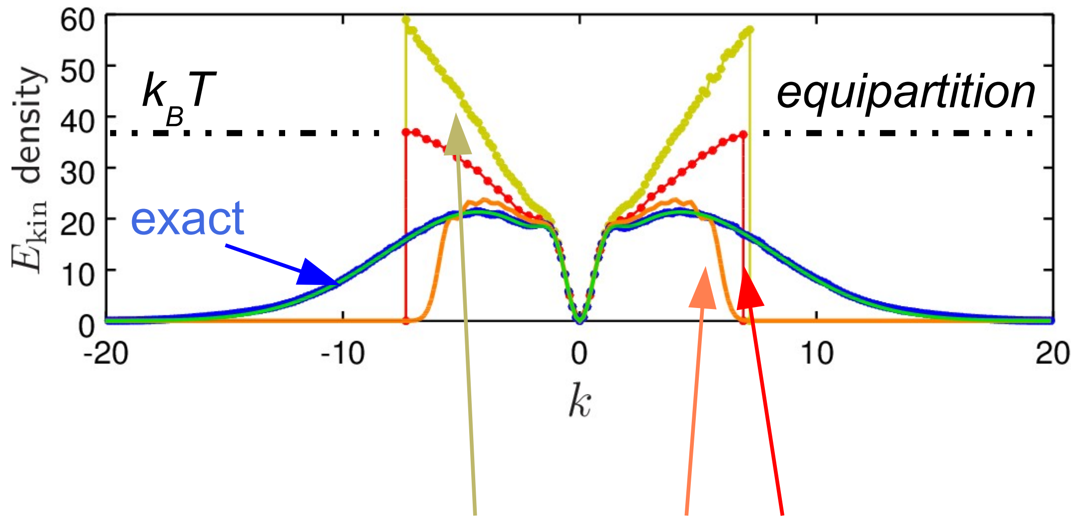
Problem 2: Cutoff dependence

Different observables depend differently on cutoff



PRA **92**, 063620 (2015)
Pietraszewicz, PD, PRA **97**, 053607 (2018)
PRA **98**, 023622 (2018)

Kinetic energy distribution, ideal gas in trap



Various bases with “optimal” cutoff

The end result: results tend to be qualitative only :-(

Studied by many:

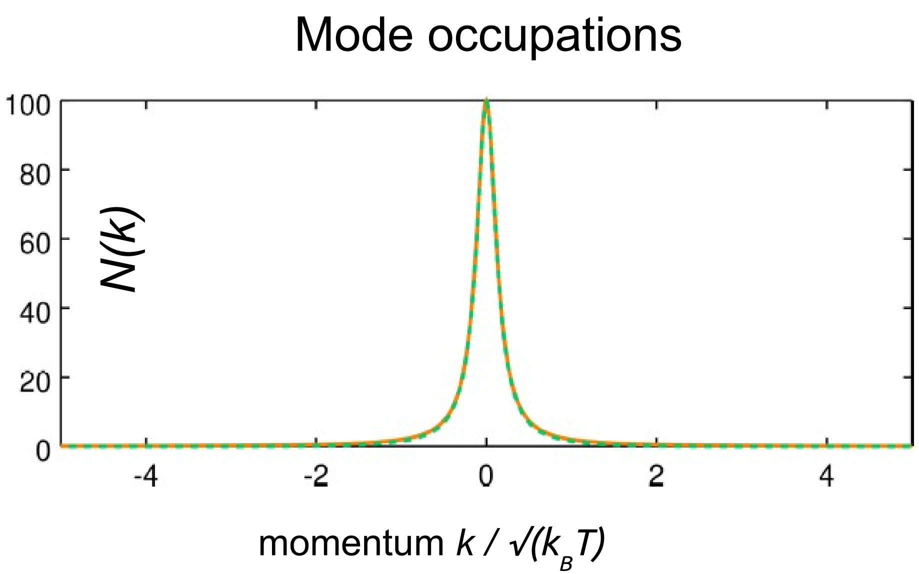
Witkowska, Gajda, Rzążewski, PRA **79**, 033631 (2009)
Karpiuk, Brewczyk, Gajda, Rzążewski, PRA **81**, 013629 (2010)
Zawitkowski, Brewczyk, Gajda, Rzążewski, PRA **70**, 033614 (2004)
Bradley, Blakie, Gardiner, J Phys B **38**, 4259 (2005)
Cockburn, Proukakis, PRA **86**, 033610 (2010)
and the list goes on ...

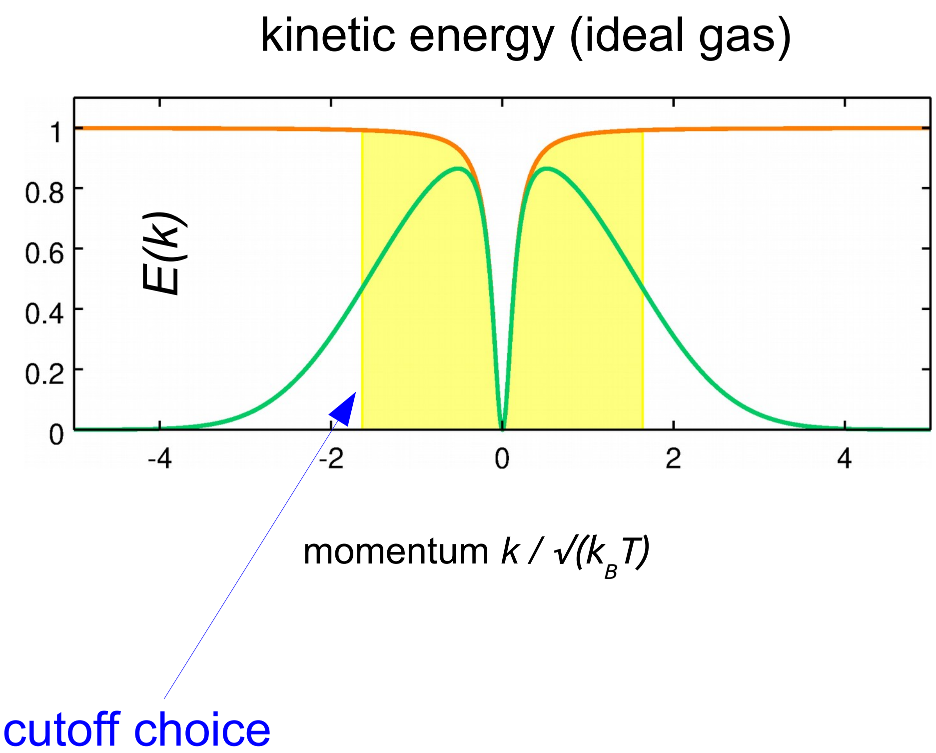
Cause: wrong degrees of freedom = UV catastrophe

Quantum Bose-Einstein distribution $\frac{1}{2}k_B T$ energy per *PARTICLE*

Classical field Rayleigh-Jeans distribution $k_B T$ energy per *MODE*

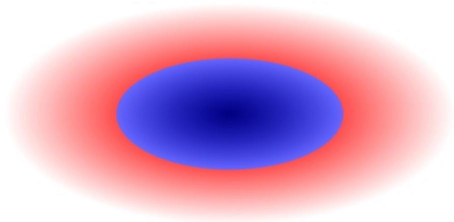
→ *too many particles at high energy*



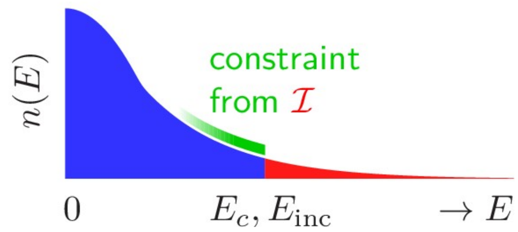


Start of fix: stochastic Gross-Pitaevskii equation (SGPE)

real space



occupations



Commonly used alternative to plain GPE

complex noise

$$\hbar \frac{\partial \phi(\mathbf{x})}{\partial t} = -i\mathcal{E}(\mathbf{x})\phi(\mathbf{x}) - \gamma\left[\mathcal{E}(\mathbf{x}) - \mu\right]\phi(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\,\eta(\mathbf{x},t)$$

Loss rate to “incoherent” tails

Incoherent growth from tails

GPE
Hamiltonian evolution of
“coherent” field

$$\mathcal{E}\phi(\mathbf{x}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g\,|\phi(\mathbf{x})|^2\right]\phi(\mathbf{x})$$

Temperature is tamed, but occupation of tails is still UV divergent.

Invokes a “classical field” linearisation of occupation in tails (correct when $\overline{N}(\omega) \gg 1$)

$$\overline{N}(\omega) \rightarrow \frac{k_B T}{\omega(\mathbf{x}) - \mu}$$

$\rightarrow$ equipartition znowu :(

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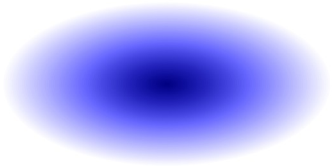
A regularized SGPE

Preserve proper quantum occupations in master equation

$$\overline{N}(\omega) = N_{BE} = \left[e^{\frac{\omega(\mathbf{x})-\mu}{k_B T}} - 1\right]^{-1}$$

real space



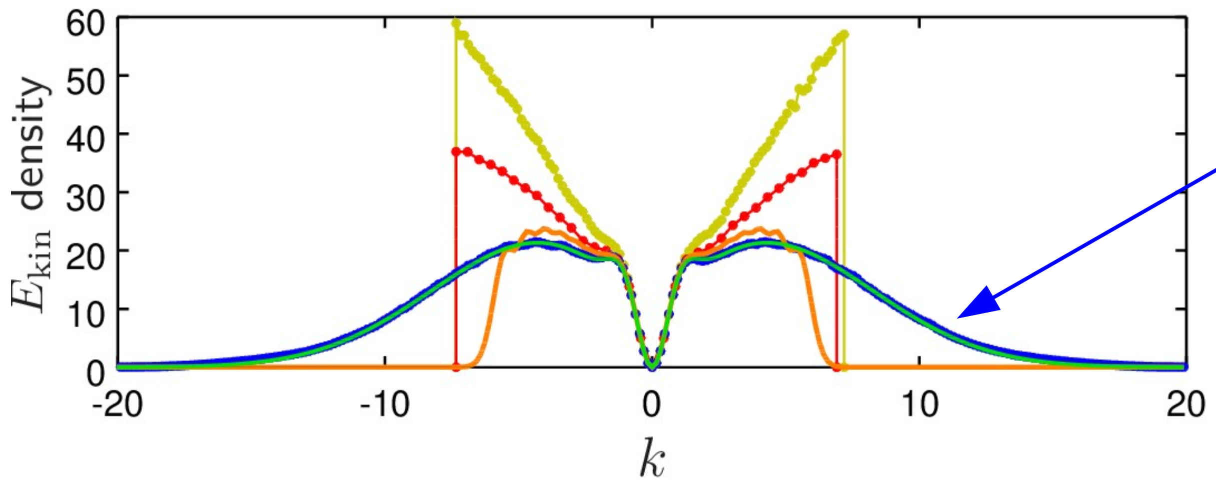
occupations

Obtain *regularised* SGPE

Details: arXiv:1904.06266

$$\hbar\frac{\partial\phi(\mathbf{x})}{\partial t} = -i\mathcal{E}(\mathbf{x})\phi(\mathbf{x}) - \gamma k_B T\left[e^{\frac{\mathcal{E}(\mathbf{x})-\mu}{k_B T}} - 1\right]\phi(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\,\eta(\mathbf{x},t)$$

Crucial development: an algorithm that keeps tractable MlogM scaling



UV divergence is gone

Matches Bose-Einstein

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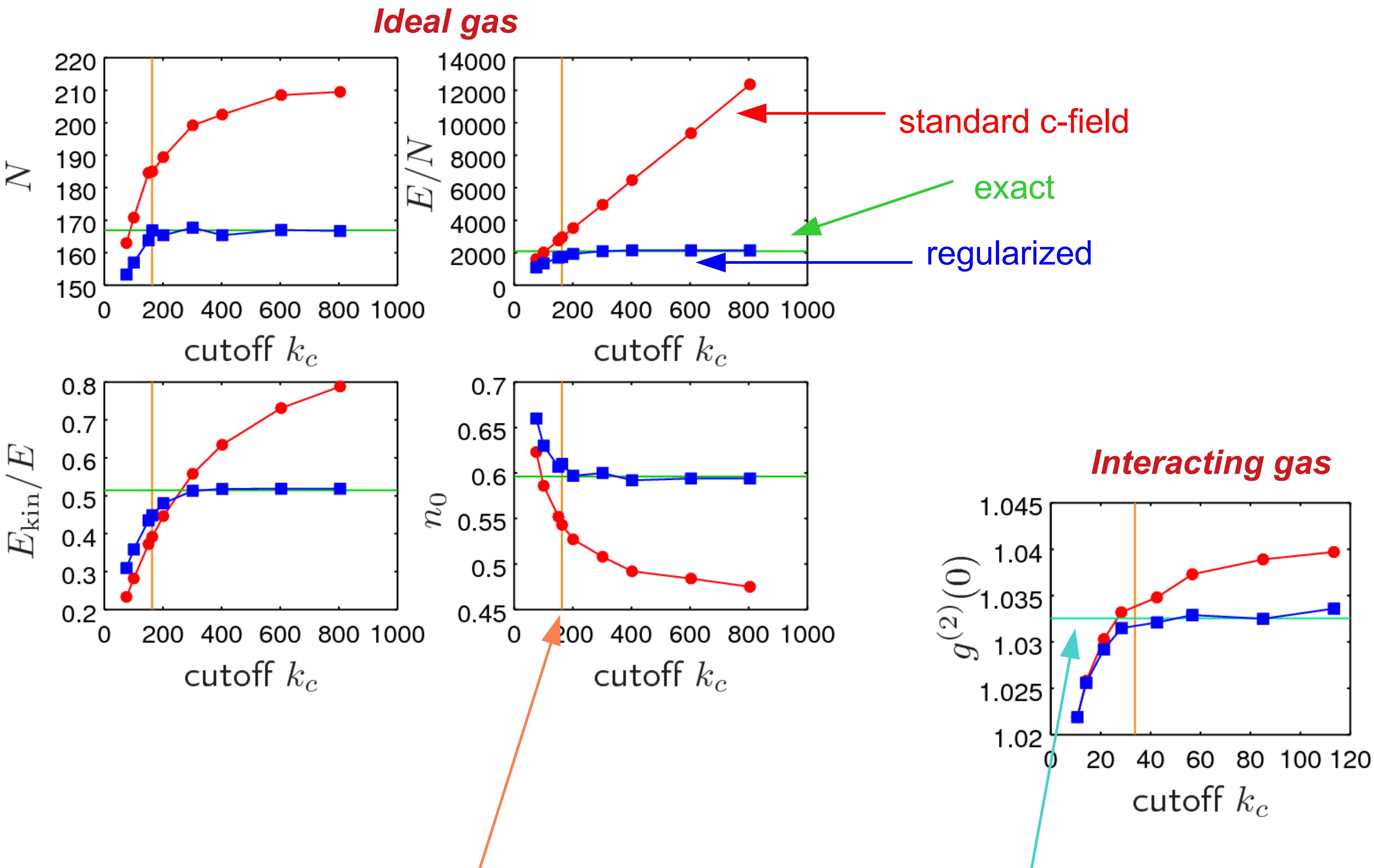
Testing: 1d trapped gas (cutoff dependence)



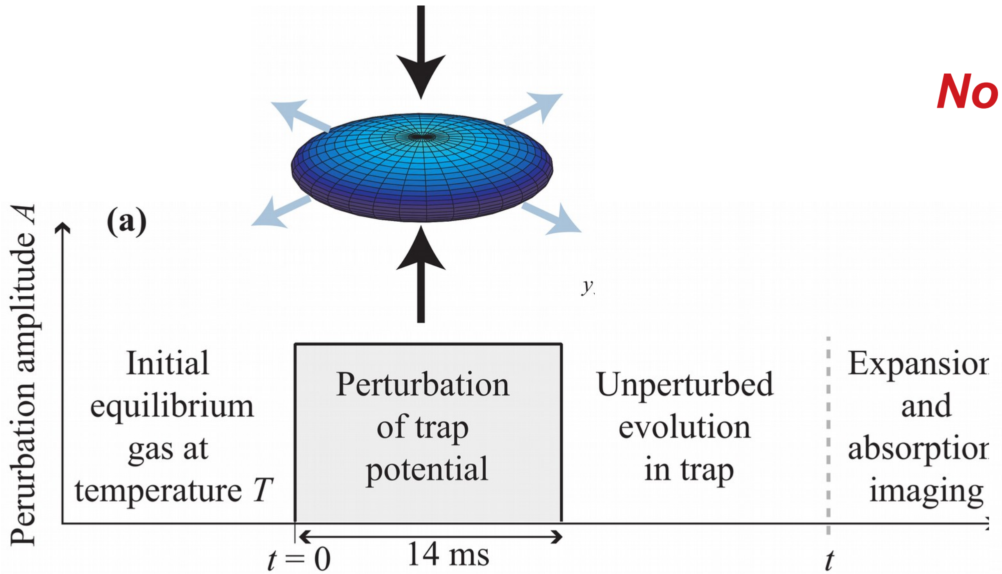
cutoff from PRA 97, 053607 (2018)

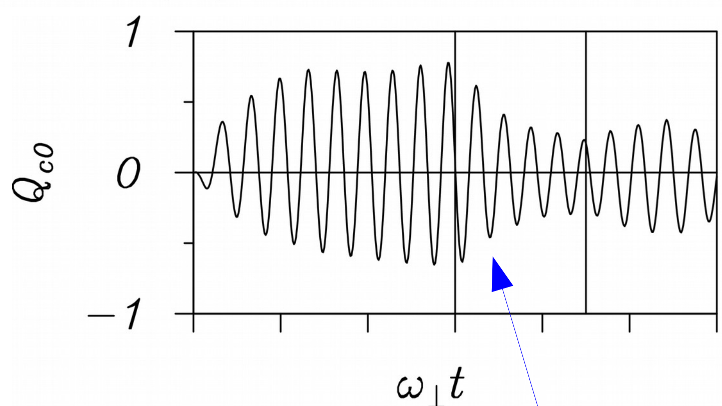
extended Bogoliubov as per Mora+Castin PRA 67, 053615 (2003)

Litmus test for thermal cloud: *m=0* collective mode

Famous experiment: Jin, Matthews, Ensher, Wieman, Cornell, PRL 78, 764 (1997)

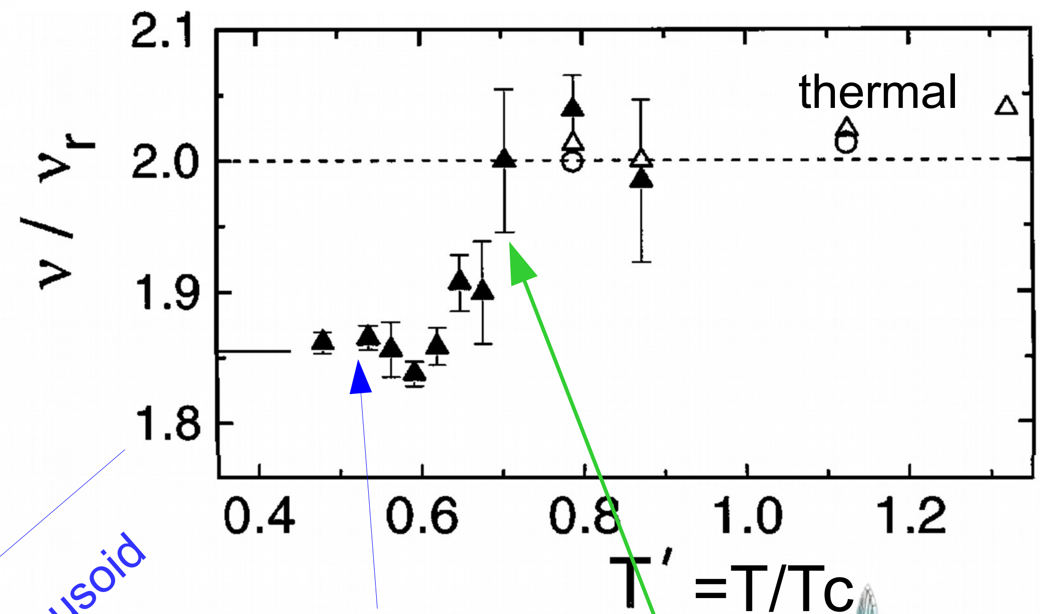
No dynamical theory has been successful in replicating this – over 20 years



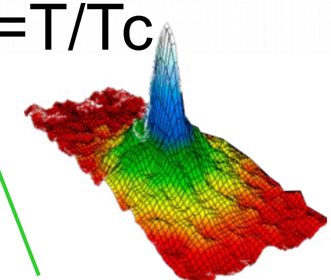


fit decaying sinusoid

adapted from: Jin *et al*, PRL 78, 764 (1997)



condensate oscillations



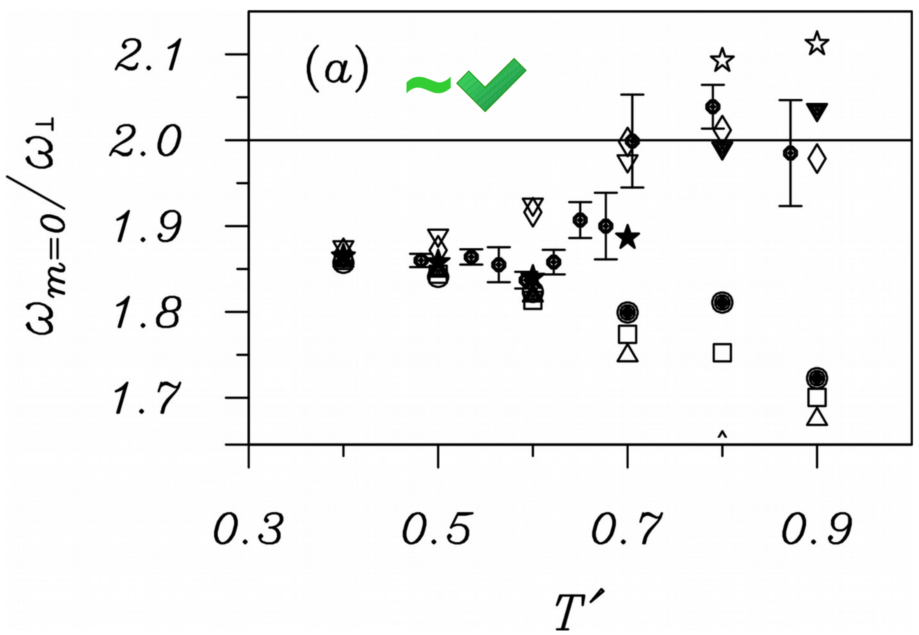
shift due to drag of thermal cloud on condensate

The above figures from:
Jackson, Zaremba, PRL 88, 180402 (2002)
Bezett, Blakie, PRA 79, 023602 (2009)

Targeted approaches

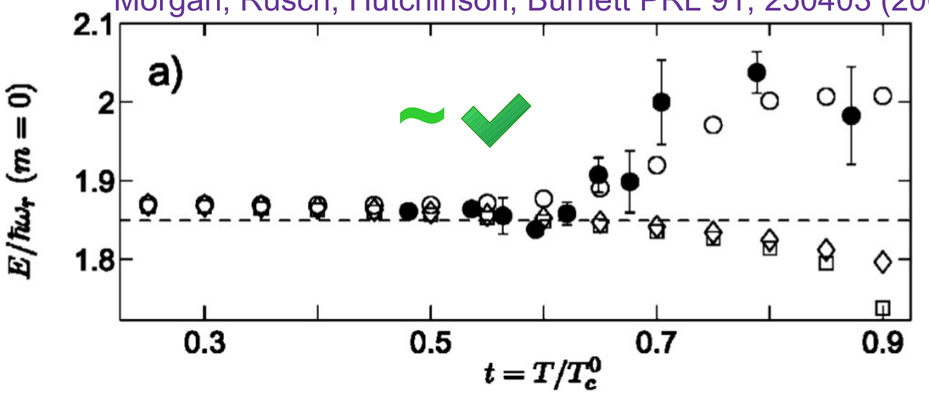
ZNG

Jackson, Zaremba, PRL 88, 180402 (2002)



Frequencies from "2[nd] order Bogoliubov"

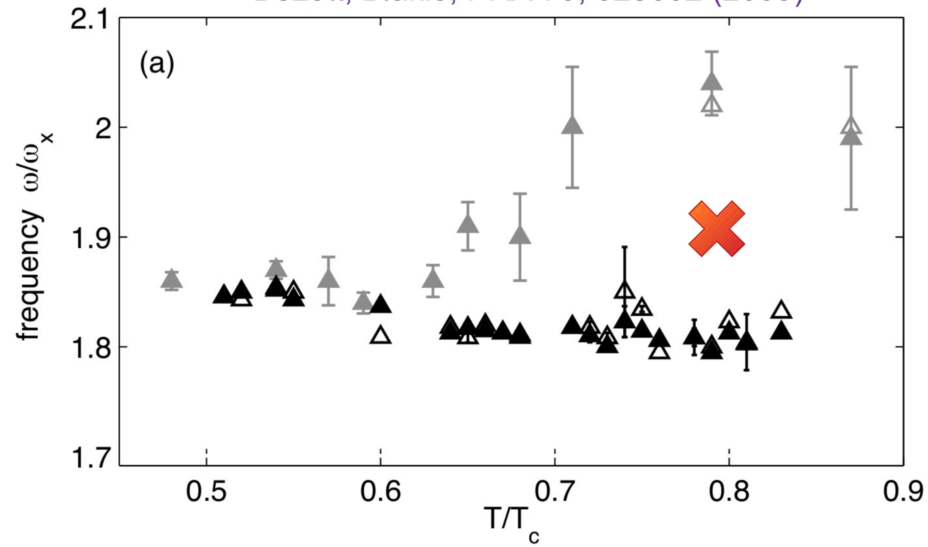
Morgan, Rusch, Hutchinson, Burnett PRL 91, 250403 (2003)



Flexible simulations

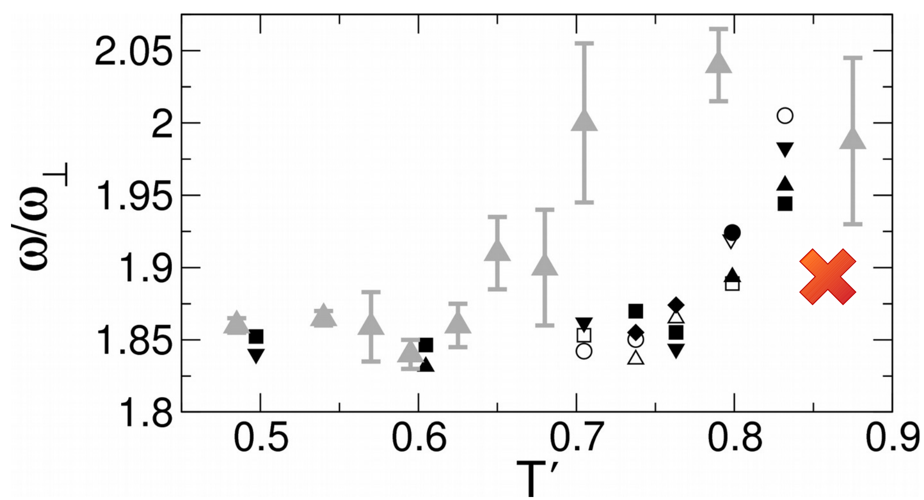
Classical fields + HF

Bezett, Blakie, PRA 79, 023602 (2009)



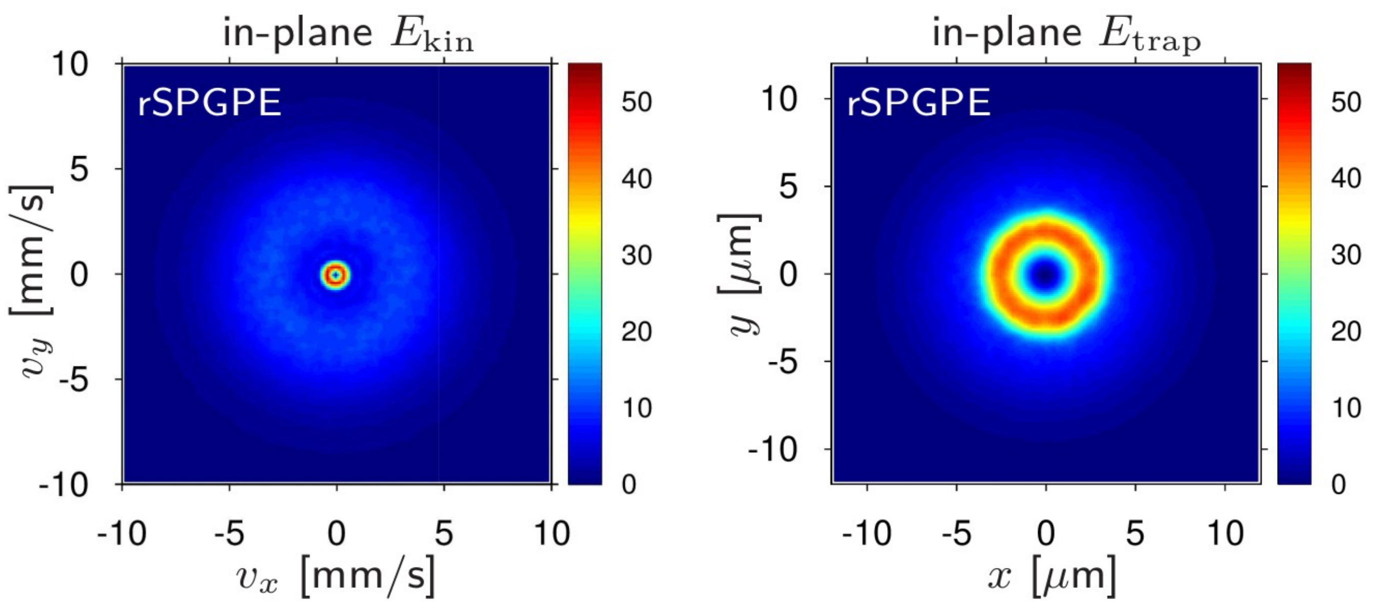
Classical fields (higher cutoff)

Karpiuk, Brewczyk, Gajda, Rzążewski, PRA 81, 013629 (2010)

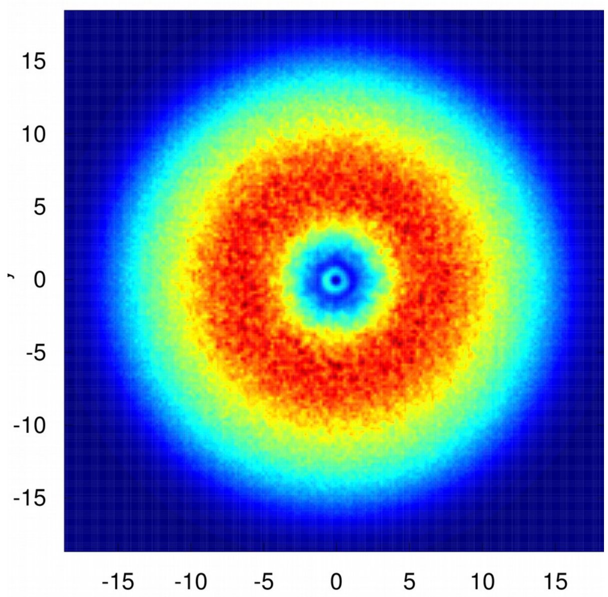


Interacting 3d trapped gas: equilibrium ensemble

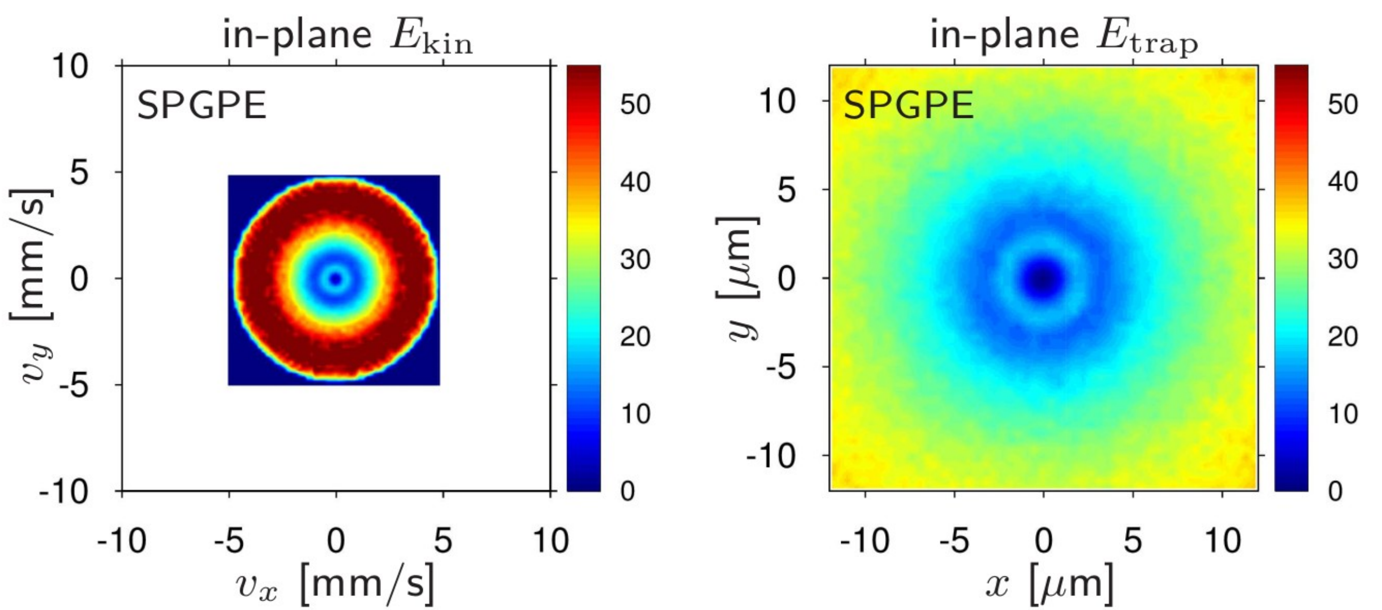
ENERGY DENSITIES



higher temperature

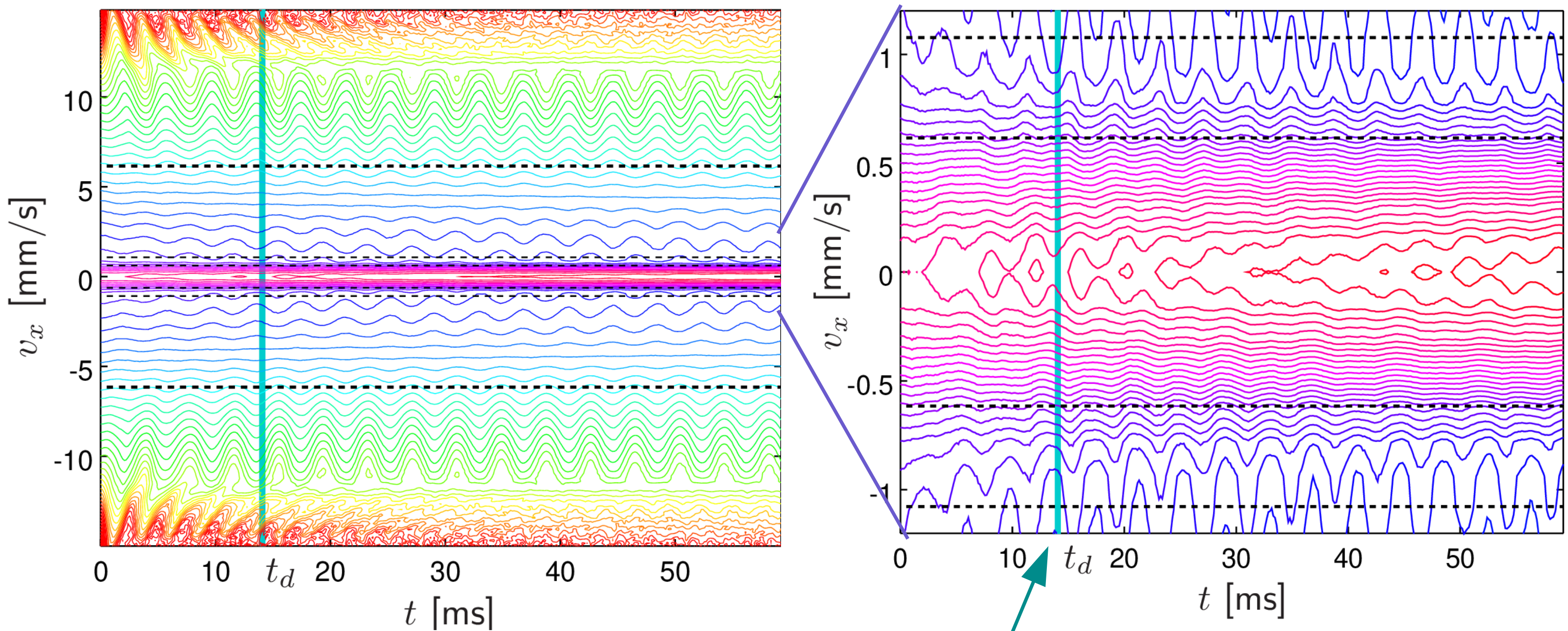


plain classical field (to scale):



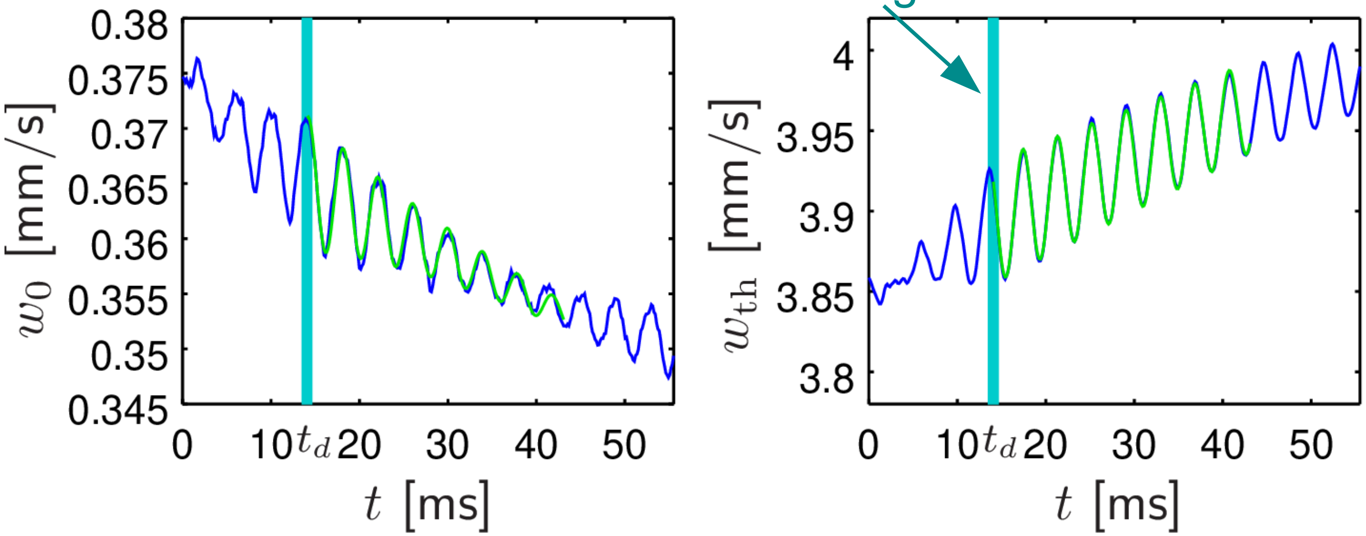
Collective oscillations

Contours of log density during evolution:

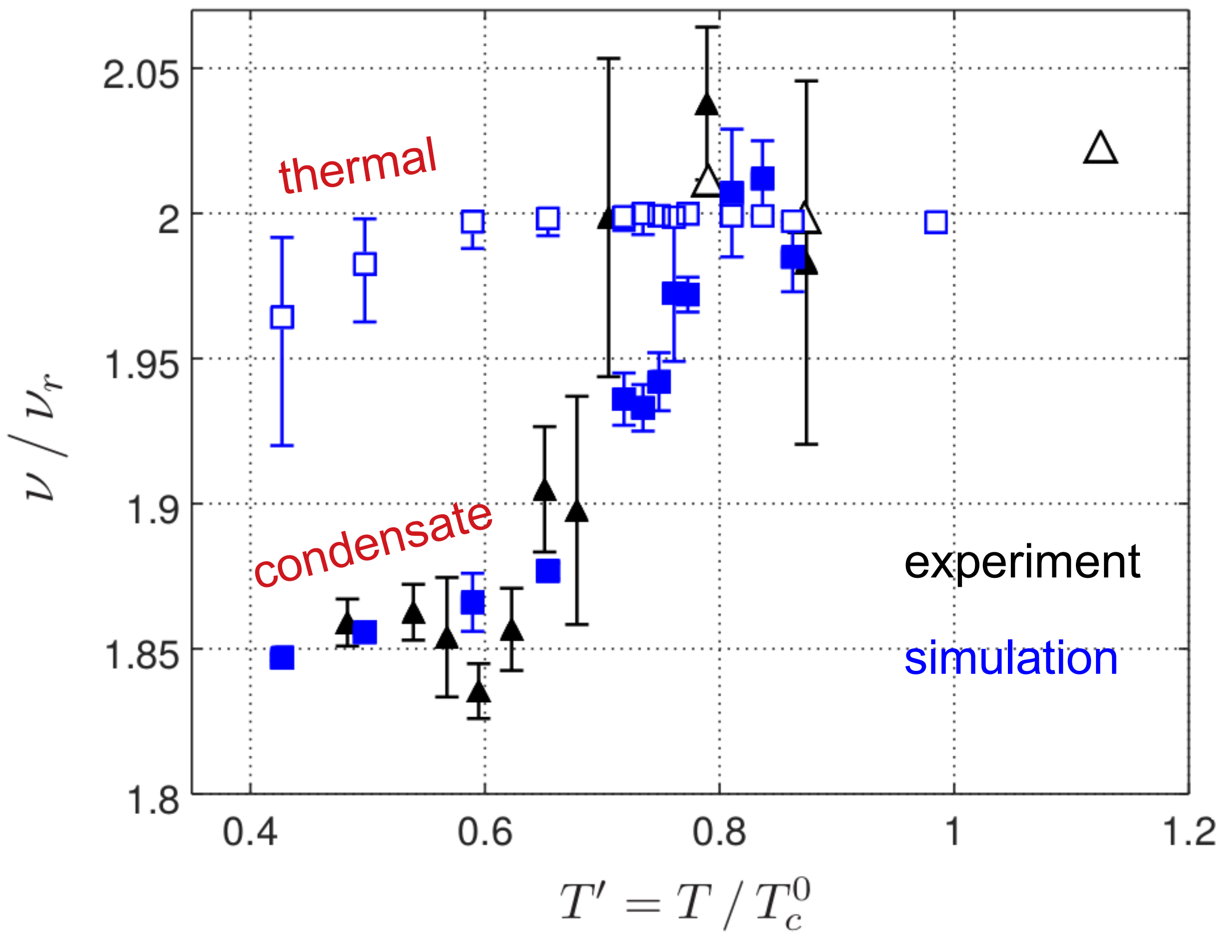


End of driving:

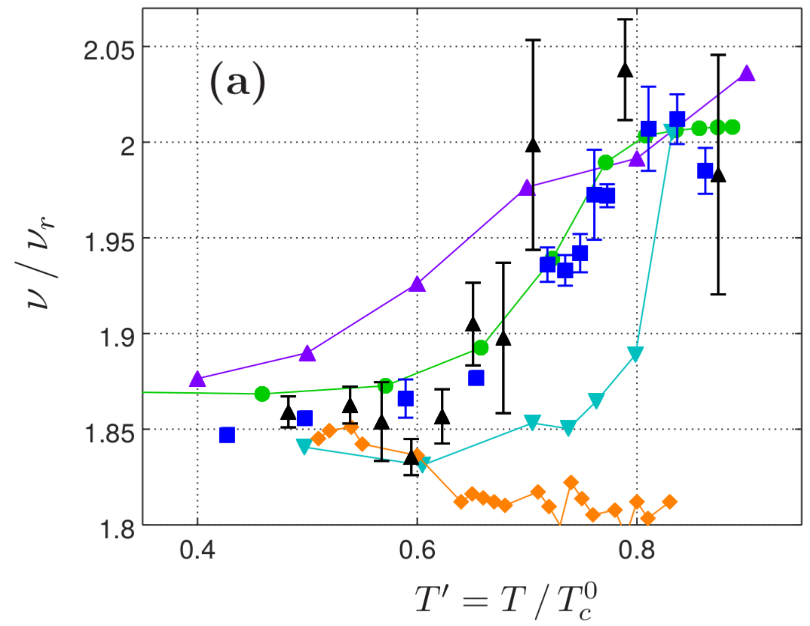
Fitting of width:



Results: *m=0* mode frequency



Compared to past results:



Experiment: Jin, Matthews, Ensher, Wieman, Cornell, PRL 78, 764 (1997)

This work (2018)

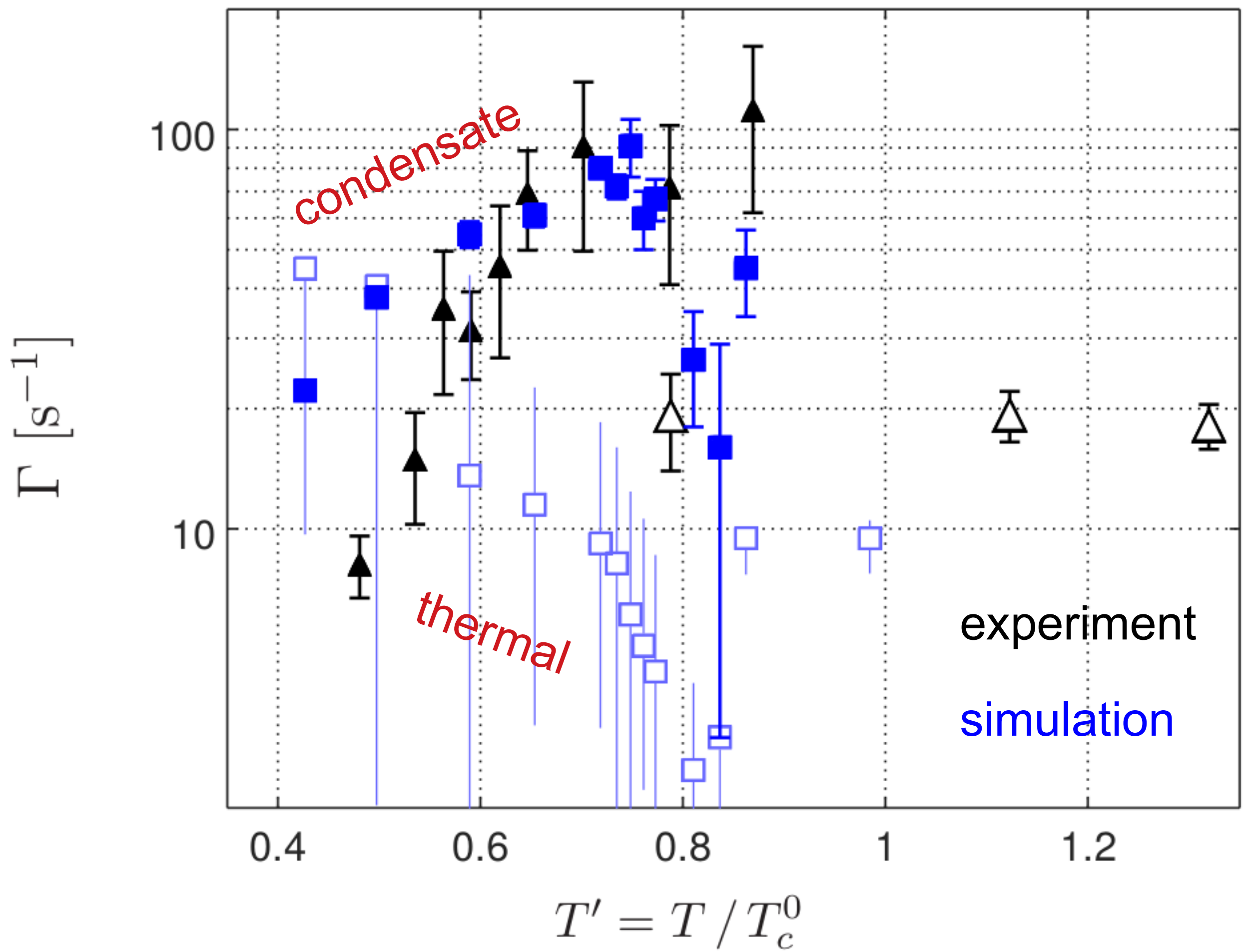
ZNG: Jackson, Zaremba, PRL 88, 180402 (2002)

2nd order Bogoliubov: Morgan, Rusch, Hutchinson, Burnett PRL 91, 250403 (2003) ***[not a simulation]***

PGPE+HF: Bezettt, Blakie, PRA 79, 023602 (2009)

PGPE: Karpiuk, Brewczyk, Gajda, Rzążewski, PRA 81, 013629 (2010)

m=0 mode damping



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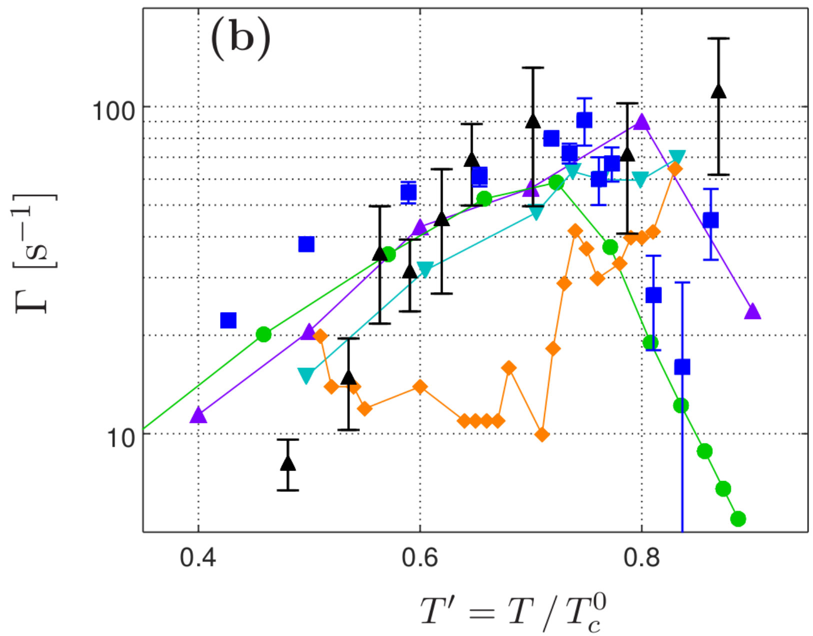

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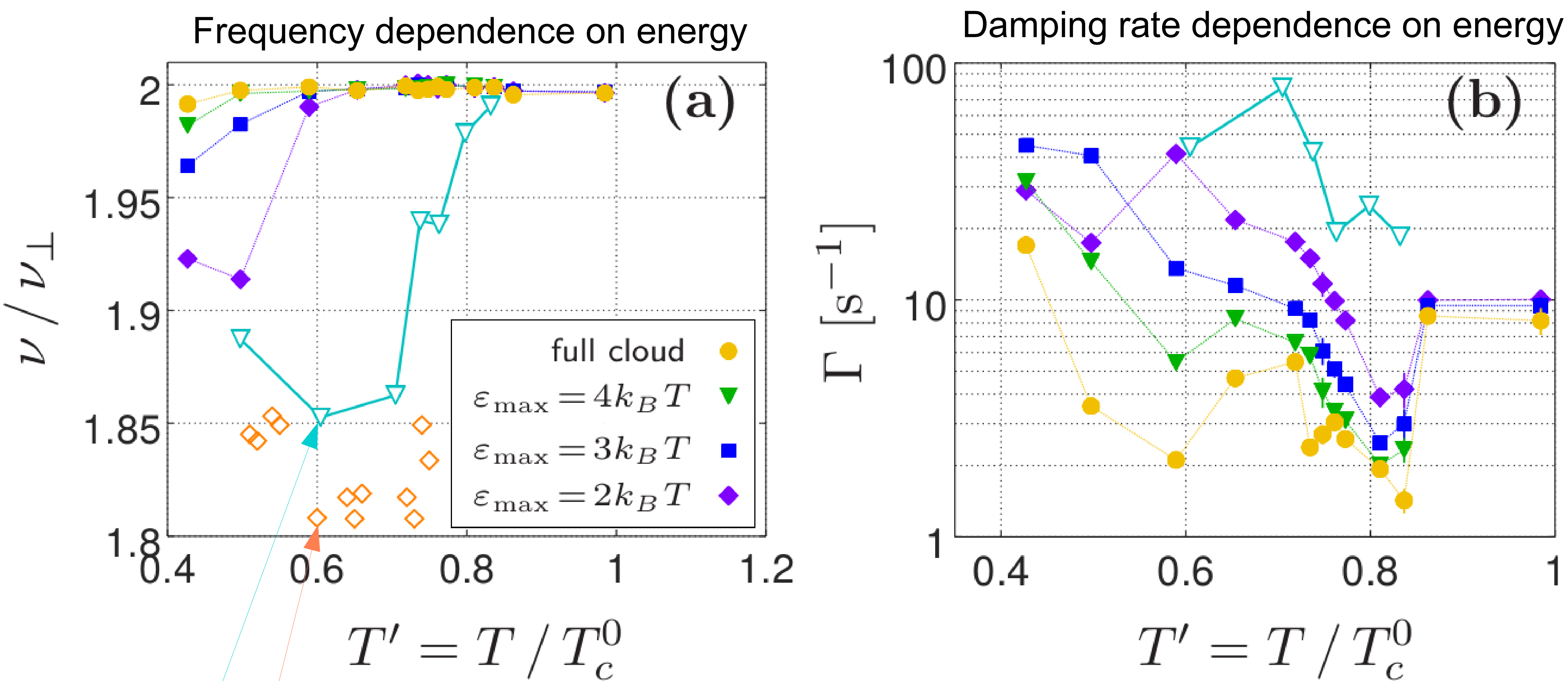
PGPE+HF: Bezettt, Blakie, PRA 79, 023602 (2009)

PGPE: Karpiuk, Brewczyk, Gajda, Rzążewski, PRA 81, 013629 (2010)

Thermal cloud

- Since there are no artificial cutoffs, energies of $k_B T$ and more can be investigated

e.g. it can be seen that the thermal cloud does not move as a coherent body
Frequency and damping depend on distance from the condensate



Karpiuk, Brewczyk, Gajda, Rzążewski 2010 cutoff $2\ k_B T$

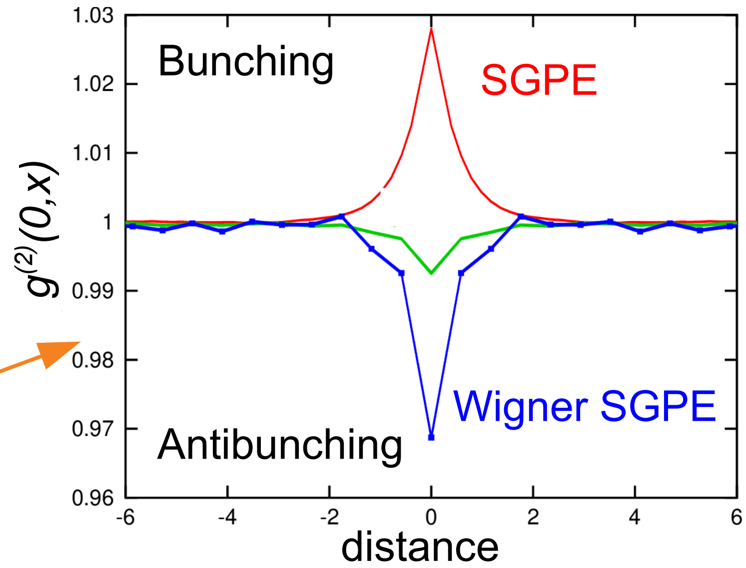
Bezett + Blakie 2009 cutoff $1\ k_B T$

Outlook

- Accurate results with classical fields, also above $k_B T$
  no UV divergence, no cutoff dependence

arXiv:1904.06266

- Numerical effort comparable to SGPE
  though, lattice may need to be larger
- Still lacks wave-particle duality
  but this is harder to spot than expected
- NEXT: Wigner representation version
  to include quantum fluctuations



Thanks to many people for discussions over the years:

Nick Proukakis	Simon Gardiner	Krzysztof Gawryluk
Mariusz Gajda	Crispin Gardiner	Tomasz Karpiuk
Mirosław Brewczyk	Matt Davis	Thomas Gasenzer
Kazimierz Rzążewski	Ashton Bradley	Andrew Daley
Emilia Witkowska	Blair Blakie	Igor Nowicki