The superfluidity of dipolar Fermi gases

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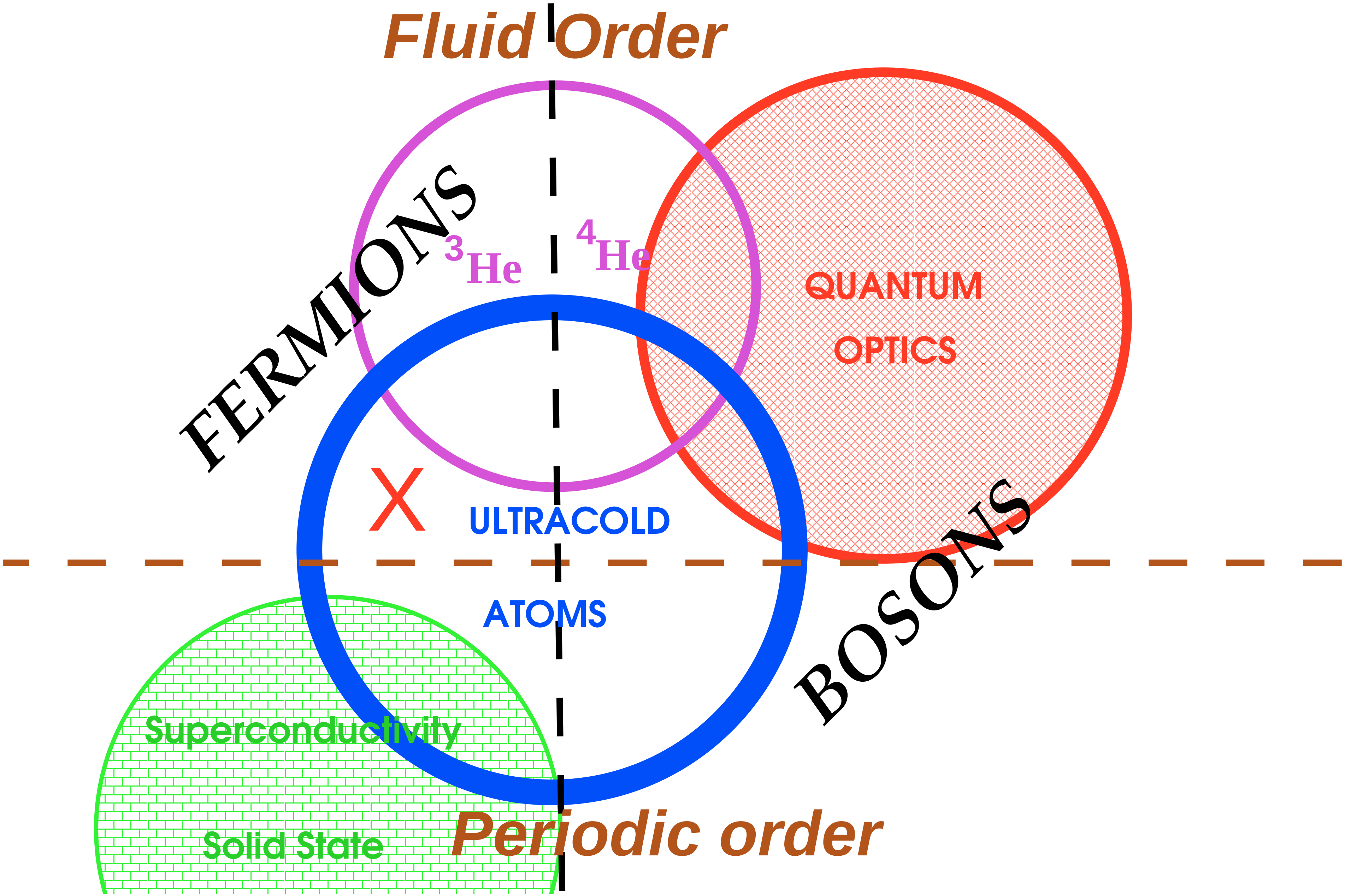
Orientation mudmap



Single-species gas of dipolar fermions

WHY?

- Cooling of heteronuclear molecules to rovibrational ground state with large dipole moment 0.5–5 Debye

  K.-K. Ni et al., arXiv:0808.2963

  J. Deiglmayr et al., arXiv:0812.1002

- Superfluidity is predicted for single-species Fermi gas of dipoles

  (Long range interaction avoids Pauli blocking)

  Baranov et al. PRA **66**, 013606 (2002)

- Order parameter structure similar to long elusive phases of matter

  – Polar phase of $^3\mathrm{He}$ (never experimentally realised)

  – Exotic superconductors (Heavy fermion, p-wave)

- Experimental realisation could be near

  With parameters from K.-K. Ni et al., get $T_c^{\mathrm{BCS}} \approx 1.6\mathrm{nK}$ *(Well, low)*

  However, **with** $10\times$ **more density** (The "theorist's fallacy"!)

  $T_c^{\mathrm{BCS}} \approx 40\mathrm{nK}$ *(plausible?)*

link BCS — bosons — quantum optics

$$|\Psi\rangle = \left(1 + \frac{2}{N}\sum_k \Delta_k \widehat{\Psi}^\dagger_{-k}\widehat{\Psi}^\dagger_k\right)^{N/2} |B\rangle$$

BCS state

$\widehat{\Psi}_k$ fermions

$|B\rangle$ = Bogoliubov vacuum = Fermi sphere

Δ_k = energy gap for pair with momentum k

$$|\Psi\rangle = \left(1 + \frac{2}{N}\sum_k \Delta_k \widehat{B}^\dagger_k\right)^{N/2} |B\rangle$$

$\widehat{B}_k = \widehat{\Psi}_k\widehat{\Psi}_{-k}$ composite bosons

$$|\Psi\rangle \sim \bigotimes_k \exp\left[\Delta_k \widehat{B}^\dagger_k\right] |B\rangle$$

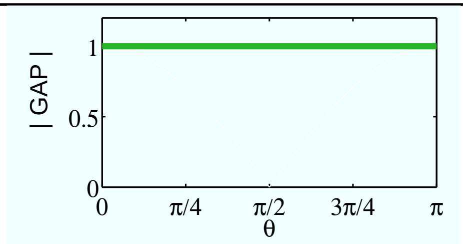
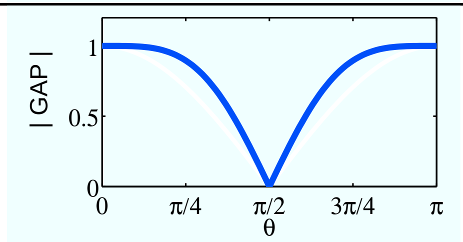
$\sim$ coherent state

$$|\Psi\rangle \sim \bigotimes_k |\Delta_k\rangle$$

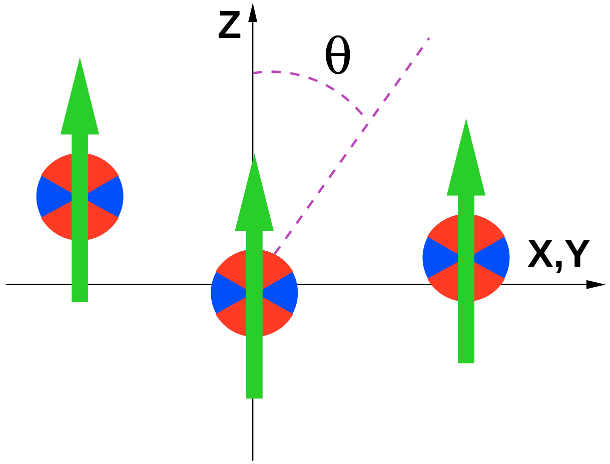
$\sim$ condensate of pairs

order parameter Δ_k

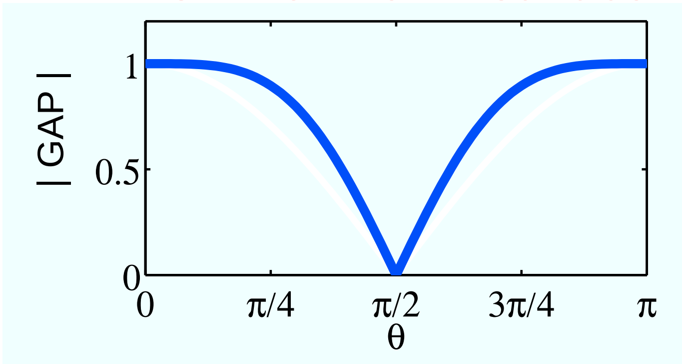
Some superfluids

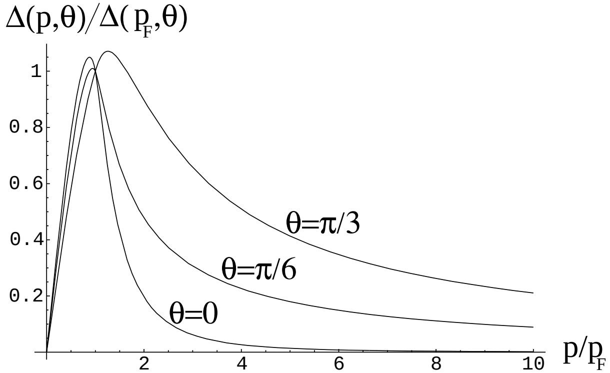
	Gap structure	mean-field energy	sound ω	Γ(θ)
BEC		gn	$\sqrt{\frac{gn}{m}}\,k$	constant
s-wave BCS gas		$2\lvert\Delta\rvert$	$\frac{v_F}{\sqrt{3}}\,k$	constant
fermi dipolar gas		$2\lvert\Delta(k,\theta)\rvert$	$\frac{v_F}{\sqrt{3}}\,k$	direction dependent

Simplest model : Uniform 3D dipolar Fermi gas



GAP on Fermi surface





Baranov et al. PRA **66**, 013606 (2002)

- Uniform & 3D
- Cold: $T < T_c^{BCS}$
- static external field (E or B)

  $\Longrightarrow$ full polarisation in Z direction
- single-species (spin polarised)
- dilute

  $\Longrightarrow$ Energy dominated by Fermi sea

  $\Longrightarrow$ BCS-like model
- Cooper pairs are cheap near gap zero.

comparison to standard BCS gas

dipole potential

$$V(r,\theta) = \frac{d^2}{r^3}(1-3\cos^2\theta)$$

- long range interaction
  $\longrightarrow$ 1 spin component suffices
- always partly attractive
  BCS pairing if *polarised*
- Energy gap has nodes
  $\longrightarrow$ damped at $T=0$?
- *Anisotropic*
- Bogoliubov spectrum

contact s-wave potential

$$V(r) = g\,\delta(r)$$

- short range interaction
  $\longrightarrow$ Needs 2 spin components
  (Pauli blocking)
- arttractive or repulsive
  BCS pairing only if $a_s < 0$
- Energy gap always > 0
  $\longrightarrow$ perfect superfluid at $T=0$
- *Isotropic*
- Bogoliubov spectrum

Hamiltonian

$$\widehat{H} = \text{K.E.} + \frac{1}{2}\int d^3x\, d^3y \left\{ \widehat{\Psi}_x^\dagger \widehat{\Psi}_x \, V_D(x-y)\, \widehat{\Psi}_y^\dagger \widehat{\Psi}_y \right\}$$

Resulting effective BCS mean-field Hamiltonian

$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3x\, d^3y \left\{ \begin{array}{ll} \frac{\hbar^2}{m}\, \widehat{\Psi}_x^\dagger\, \nabla^2 \widehat{\Psi}_x\, \delta(x-y) & \textit{Kinetic} \\ \Delta^*(x-y)\, \widehat{\Psi}_x \widehat{\Psi}_y - \Delta(x-y)\, \widehat{\Psi}_x^\dagger \widehat{\Psi}_y^\dagger & \textit{BCS} \\ W(x-y) \widehat{\Psi}_x^\dagger \widehat{\Psi}_y & \textit{Hartree} \end{array} \right\}$$

Gap *field* consistency equation

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

Exchange mean field

$$W(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y^\dagger \right\rangle_{\text{eff}}$$

Low energy superfluidity

Phase perturbations of the ground state order parameter Δ_0

$$\boxed{\Delta_0(x-y) \quad \longrightarrow \quad \Delta(x,y) = \Delta_0(x-y)\; e^{2i\phi(x,t)}}$$

Goldstone mode

Assumptions:

- Low energy ($\hbar\omega \ll \Delta_0^{\max} \sim T_c$)
- Low ω $\Longrightarrow$ long wavelength ($k \ll k_F$)
  $\Longrightarrow$ insensitive to small-scale of $|x-y|$ $\Longrightarrow$ $\phi \approx \phi(x \text{ only })$
- Weak perturbation $\Longrightarrow$ lowest order in ϕ

$$\omega(k) = \frac{v_F}{\sqrt{3}} k \quad \boxed{-i\,\Gamma(k,T,\theta)}$$

Effective dispersion

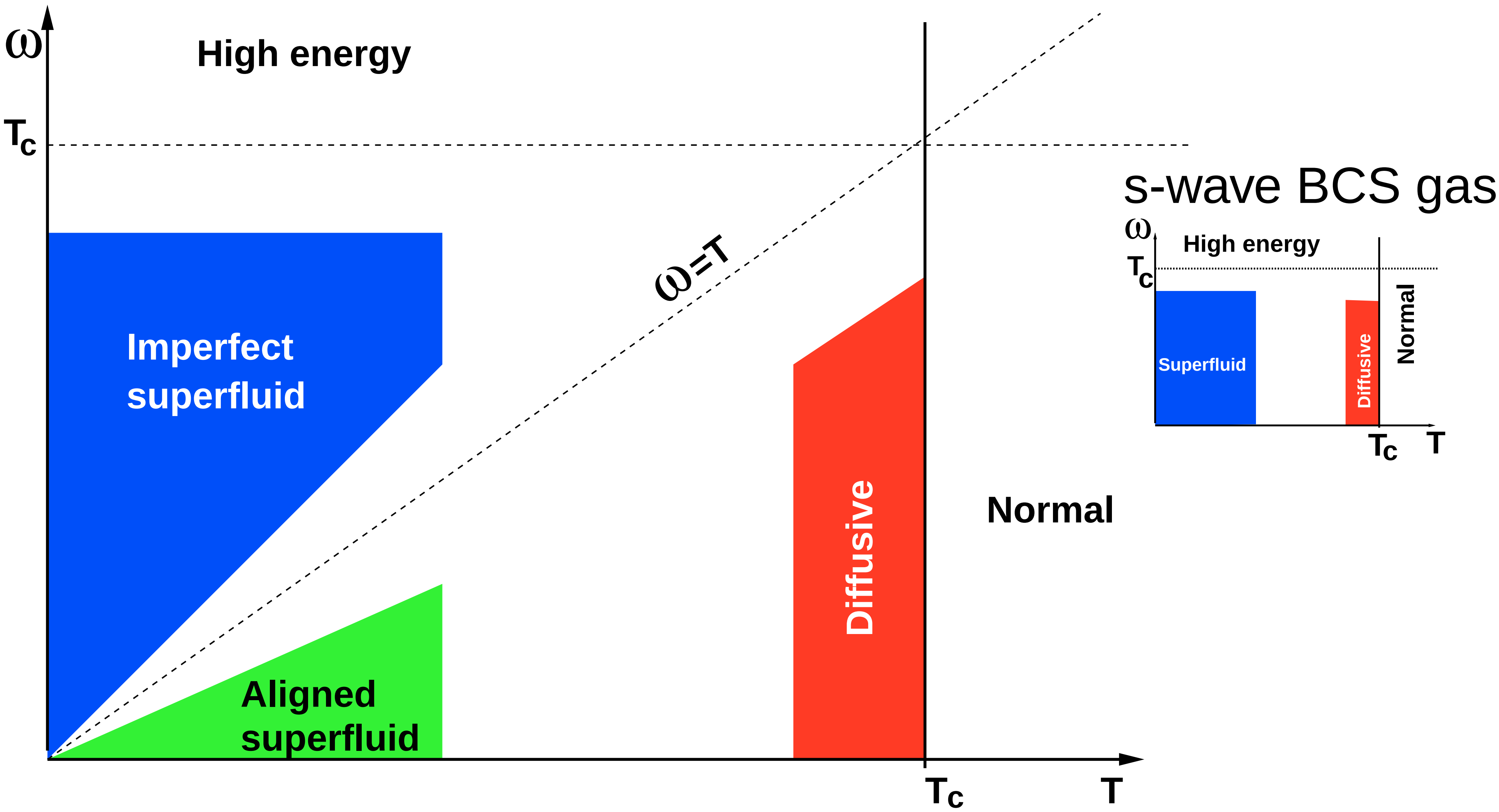
- Bogoliubov diagonalisation $\widehat{\Psi}(\mathbf{r}) = \sum_\nu \left[U_\nu(\mathbf{r})\widehat{b}_\nu + V_\nu(\mathbf{r})^* \,\widehat{b}_\nu^\dagger\right]$
- Solve mean-field theory self-consistently for small perturbation ϕ
- Obtain $\boxed{\omega(k, k_{\text{shortrange}})}$

  LONG wavelength $\mathbf{k}$, SHORT wavelength $k_{\text{shortrange}} \sim 1/k_F$.
- $k_{\text{shortrange}}$ not accessible experimentally :-(
- Integrate out short-wavelength degrees of freedom in a Lagrangian formulation
- Obtain macroscopic effectve dispersion

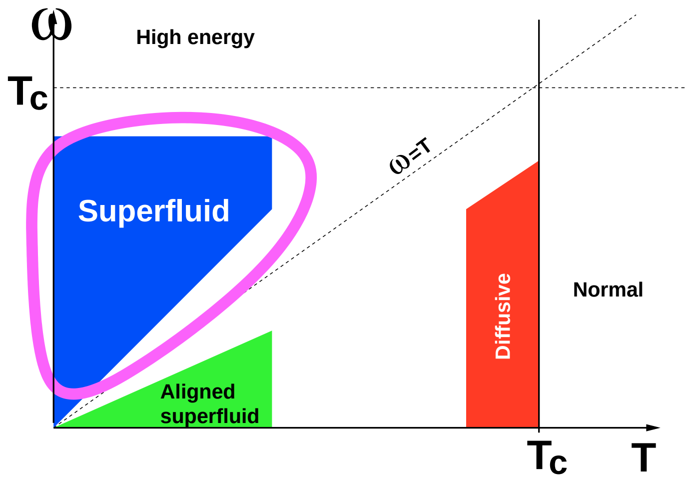
$$\omega(k) = \frac{v_F}{\sqrt{3}}k - i\,\Gamma(k, \theta)$$

- Γ determines quality of superfluidity after a stimulus

Collective excitation regimes



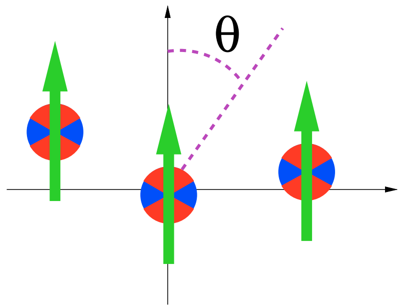
Regime 1: Imperfect BCS superfluid

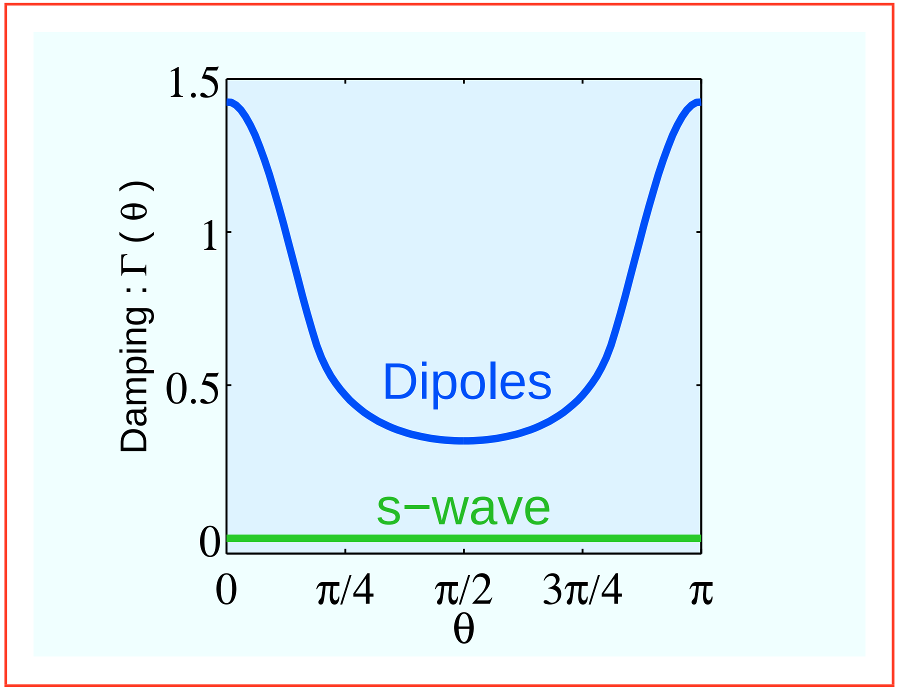


$$\omega = \left(\frac{v_F}{\sqrt{3}}\right) k \left\{1 - i \left(\frac{\hbar\omega_{\rm Bog}}{\Delta_{\max}}\right) \Gamma(\theta)\right\}$$

Beliaev process:

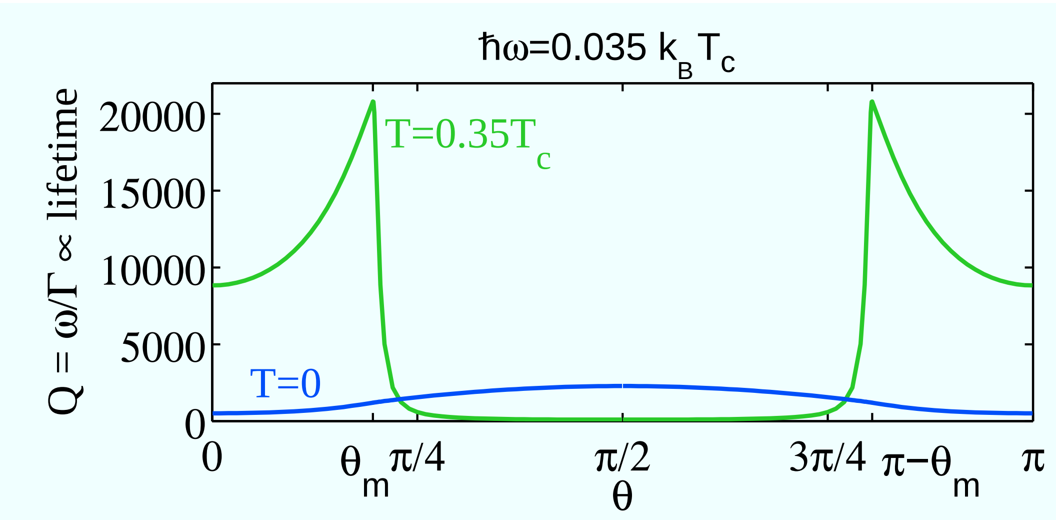
collective $\Longrightarrow$ QP pair

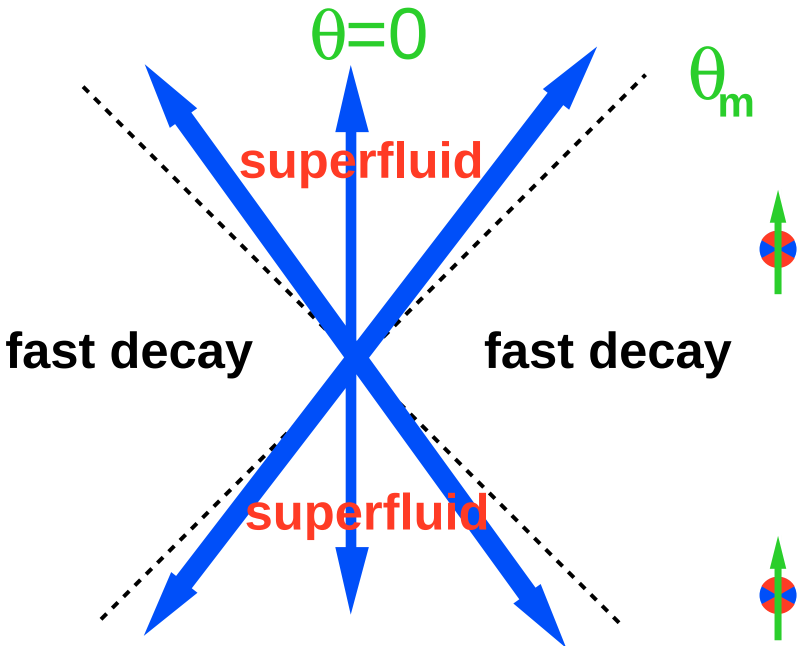




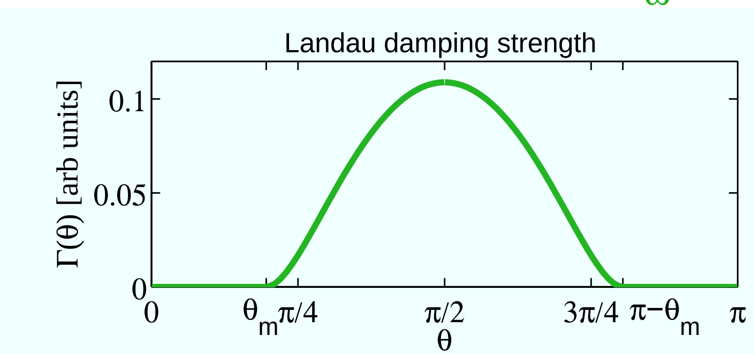
Regime 2: “Aligned superfluid”

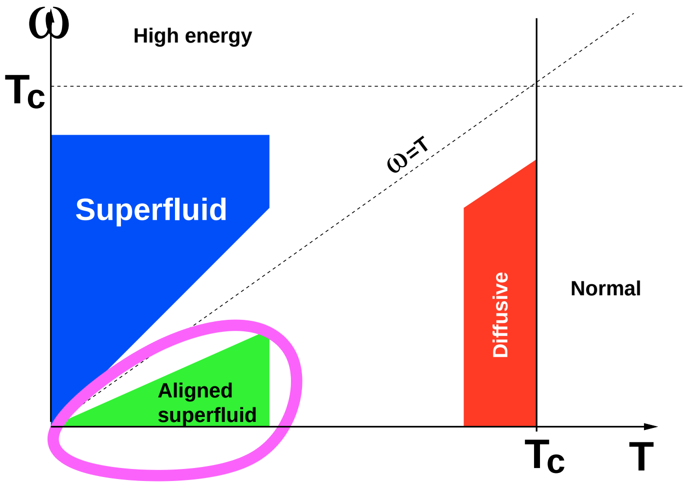
(No s-wave BCS analogue)



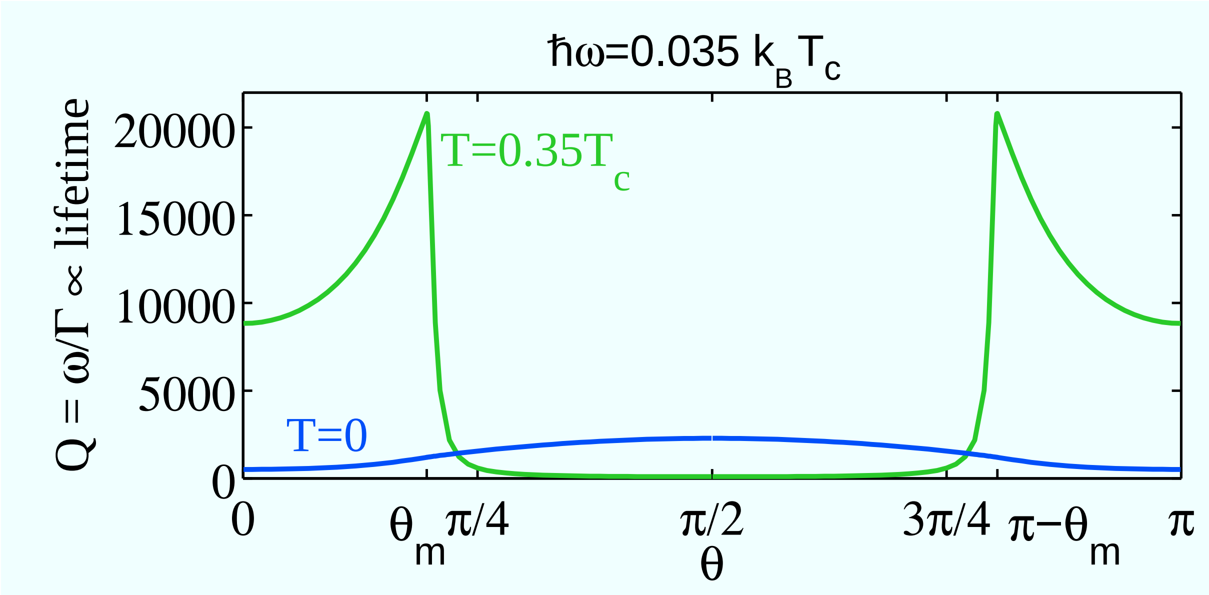


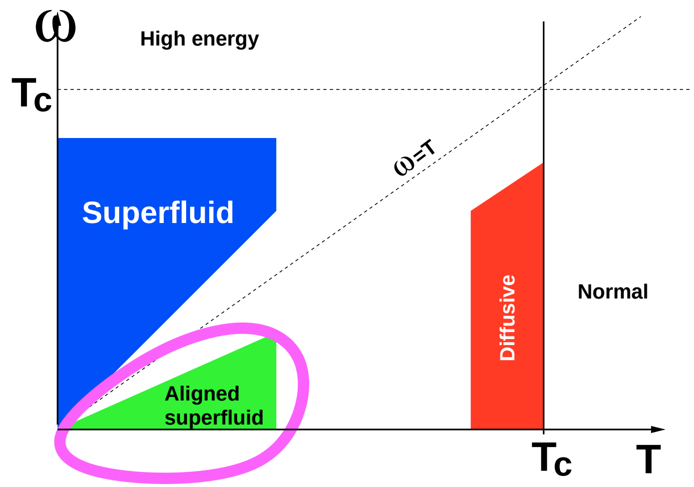
Strong Landau damping $\Gamma \propto \frac{1}{\omega}$





Thermally assisted superfluidity





- MYSTERY: How come the superfluid is better at higher T?
- Occurs when $k_B T \gg \hbar\omega$
- Quasiparticles are fermionic, and low energy pairs are already filled.
- $\implies$ Beliaev decay into quasiparticle pairs is blocked, unlike $T = 0$

Conclusions

- Alignes SF at $\omega \ll T \ll T_c$
- Superfluidity improves with *rising* T
- Should be easily discernible in experiments
  - Anisotropic propagation of disturbance
  - Drop in damping with increasing T