

Thermal properties of quantum droplets

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- ultracold quantum gas

Introduction

- ultracold quantum gas
- behave like an incompressible liquid (self stabilization in the bulk at a well-defined density)

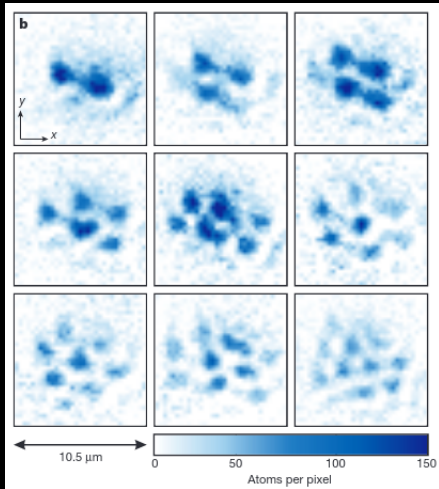
Introduction

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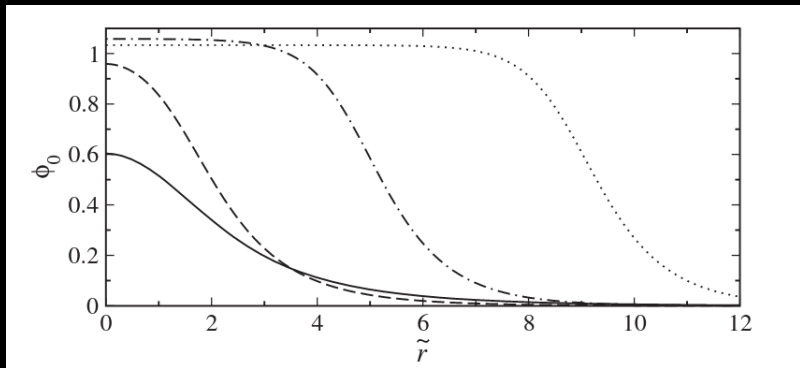
- ultracold quantum gas
- behave like an incompressible liquid (self stabilization in the bulk at a well-defined density)
- self-bound without external trapping
- stabilized and dominated by quantum fluctuations (negligible in all other known dilute quantum matter)

Unexpected discovery



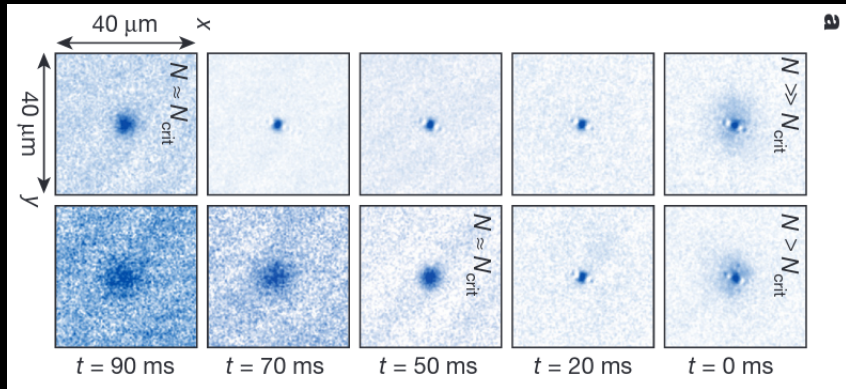
H. Kadau et al., Nature 530, 194 (2016)

Theoretical prediction



D. S. Petrov, Phys. Rev. Lett. 115, 155302 (2015)

Confirmation



M. Schmitt et al., Nature 539, 259 (2016)

2 component Bose-Bose mixture

$$E_{MF} = E_K + E_V + \frac{1}{2} \sum_{i,j} g_{ij} n_i n_j$$

2 component Bose-Bose mixture

$$E_{MF} = E_K + E_V + \frac{1}{2} \sum_{i,j} g_{ij} n_i n_j$$

$$g_{ii} > 0 \wedge g_{12}^2 < g_{11}g_{22} \implies \text{stable (gas)}$$

2 components

Beyond the MF: Bogoliubov spectrum in the LDA

$$E_{k,\pm} = \sqrt{\frac{\omega_1^2 + \omega_2^2}{2} \pm \sqrt{\left(\frac{\omega_1^2 - \omega_2^2}{2}\right)^2 + \frac{g_{12}^2 n_1 n_2 k^4}{m_1 m_2}}}$$

$$\omega_i = \sqrt{\frac{k^2}{2m_i} \left(\frac{k^2}{2m_i} + 2g_{ii} n_i \right)}$$

$$E_{LHY} = \int \frac{d^3 k}{2(2\pi)^3} \left(E_{k,+} + E_{k,-} - \frac{k^2}{2m_r} - g_{11} n_1 - g_{22} n_2 + \frac{m_1 g_{11}^2 n_1^2 m_2 g_{22}^2 n_2^2 + 4m_r g_{12}^2 n_1 n_2}{k^2} \right)$$

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LHY significant when $g_{12} \approx -\sqrt{g_{11}g_{22}} \rightarrow$ protects the mixture from collapse

2 components

$$E_I = \frac{1}{2} \sum_{ij} g_{ij} n_i n_j = \sum_{\pm} \lambda_{\pm} n_{\pm}^2$$

$$\delta g = g_{12} + \sqrt{g_{11} g_{22}}$$

$$\lambda_+ \approx \frac{g_{11} + g_{22}}{2} \quad \lambda_- \approx \frac{\delta g \sqrt{g_{11} g_{22}}}{g_{11} + g_{22}}$$
$$n_+ = \frac{n_1 \sqrt{g_{11}} - n_2 \sqrt{g_{22}}}{\sqrt{g_{11} + g_{22}}} \quad n_- = \frac{n_1 \sqrt{g_{22}} + n_2 \sqrt{g_{11}}}{\sqrt{g_{11} + g_{22}}}$$

+ branch locks the equilibrium densities: $\frac{\bar{n}_1}{\bar{n}_2} = \frac{\sqrt{g_{22}}}{\sqrt{g_{11}}}$

LHY "beats" the - branch

2 components

$$\delta g = g_{12} + \sqrt{g_{11}g_{22}}$$

$$\delta g < 0 \wedge g_{ii} > 0 \wedge |\delta g| \ll g_{ii}$$



Droplets!

Single component

$$i\partial_t\phi = \left(-\frac{\nabla^2}{2} - 3|\phi|^2 + \frac{5}{2}|\phi|^3 - \mu\right)\phi$$
$$\phi = \phi(x/\xi, t/\tau)$$

For ^{39}K atoms:

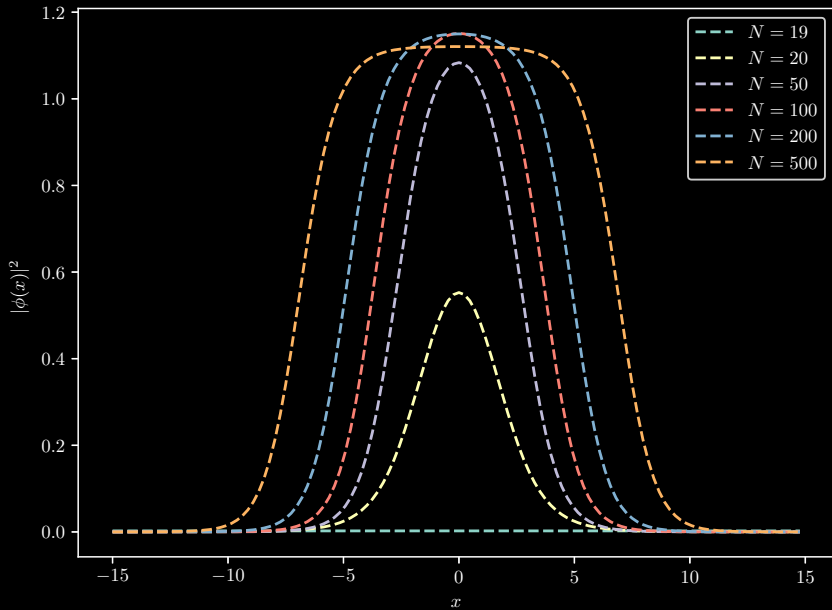
coordinate units:

$$\xi \approx 2\mu\text{m}$$

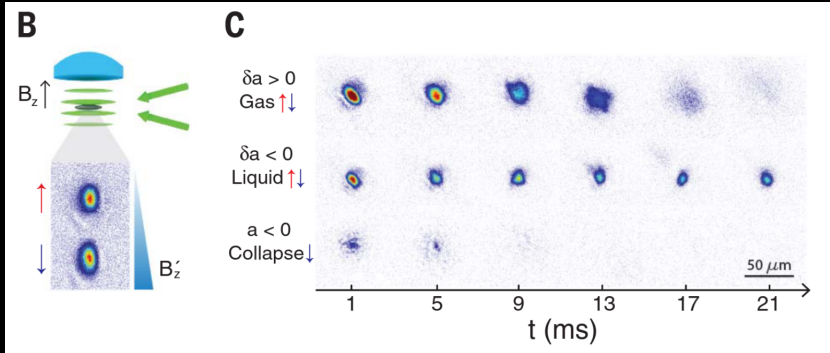
time units:

$$\tau \approx 2.5\text{ms}$$

3D droplets

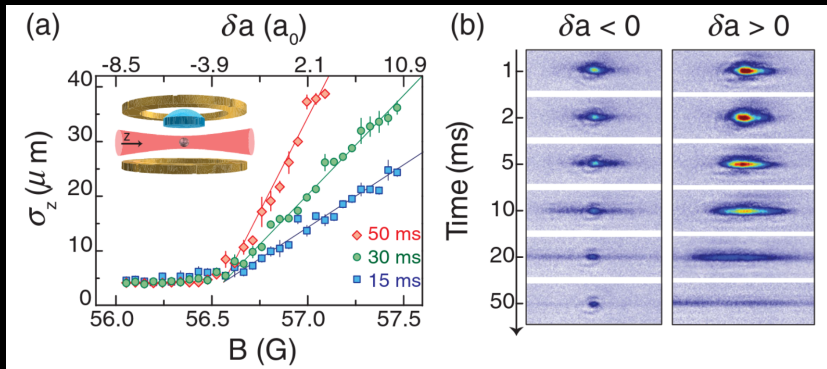


Sausages and pancakes: an effective low dimensional description



C. R. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, *Science* 359, 301 (2018)

Sausages and pancakes: an effective low dimensional description



P. Cheiney et al., Phys. Rev. Lett. 120, 135301 (2018)

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Most strict to least strict (quasi) low-D regimes:

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Strict 1D, 2D

D. S. Petrov and G. E. Astrakharchik, Phys. Rev. Lett. 117, 100401 (2016)

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Dimensional crossover

P. Zin, M. Pylak, T. Wasak, M. Gajda, and Z. Idziaszek, Phys. Rev. A 98, 051603 (2018)

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Flattened/elongated 3D

this presentation

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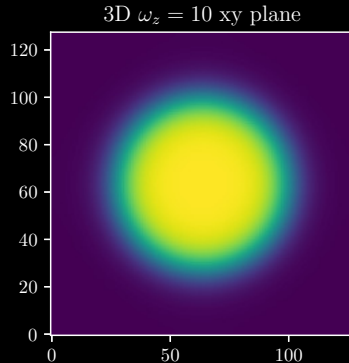
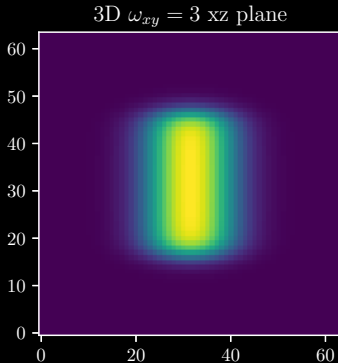
this presentation



Full 3D

D. S. Petrov, Phys. Rev. Lett. 115, 155302 (2015)

Sausages and pancakes: an effective low dimensional description



Sausages and pancakes: an effective low dimensional description

2D

$$\phi(x, y, z) = \phi_{2D}(x, y)\phi_{\text{gaussian}}(z)$$

1D

$$\phi(x, y, z) = \phi_{\text{gaussian}}(x, y)\phi_{1D}(z)$$

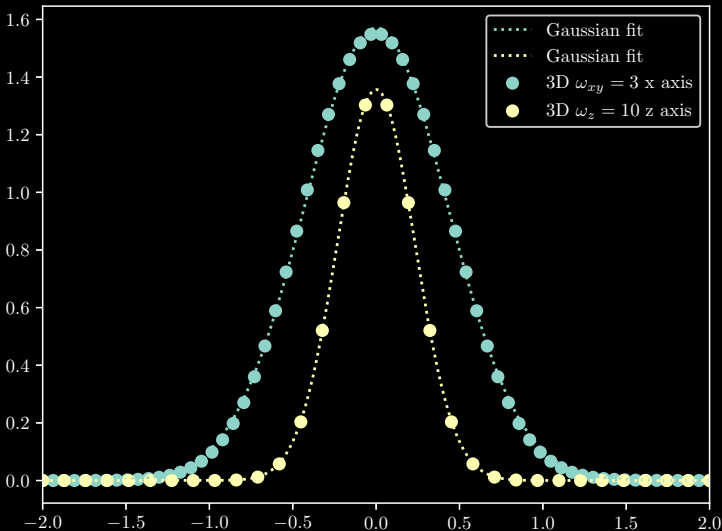
Sausages and pancakes: an effective low dimensional description

$$i\partial_t\phi = \left(-\frac{\nabla^2}{2} - 3|\phi|^2 + \frac{5}{2}|\phi|^3 - \mu\right)\phi$$

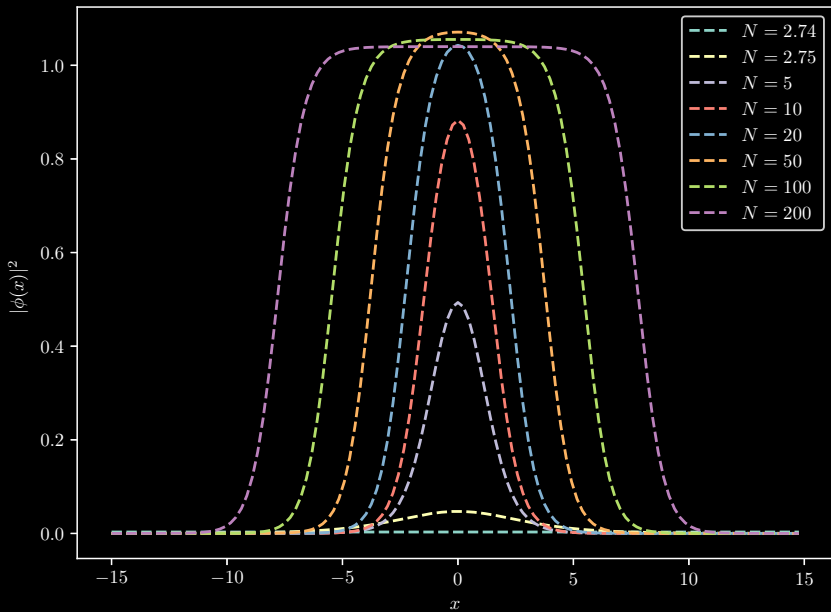
$$i\partial_t\phi_{2D} = \left(-\frac{\nabla^2}{2} - g_2(\omega)|\phi_{2D}|^2 + c_2(\omega)|\phi_{2D}|^3 - \mu\right)\phi_{2D}$$

$$i\partial_t\phi_{1D} = \left(-\frac{\nabla^2}{2} - g_1(\omega)|\phi_{1D}|^2 + c_1(\omega)|\phi_{1D}|^3 - \mu\right)\phi_{1D}$$

Sausages and pancakes: an effective low dimensional description



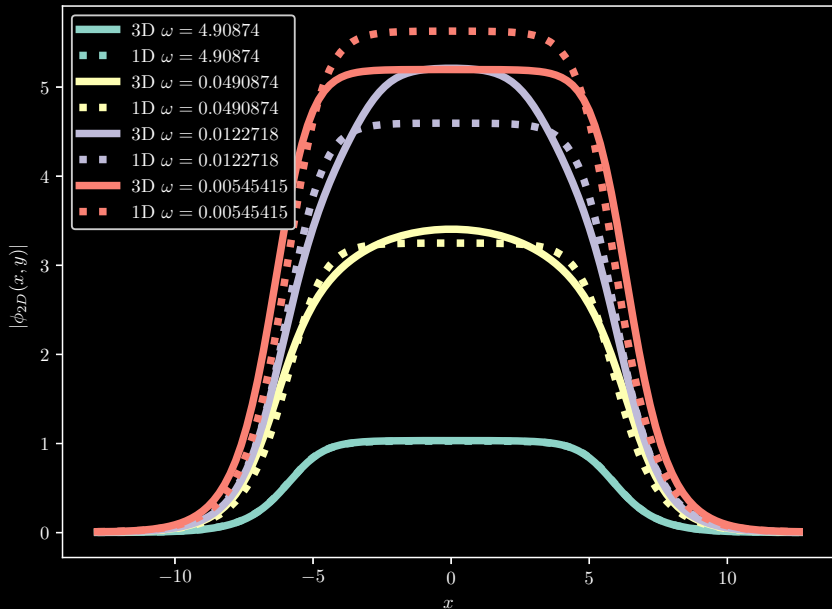
Effective 2D (flattened 3D)



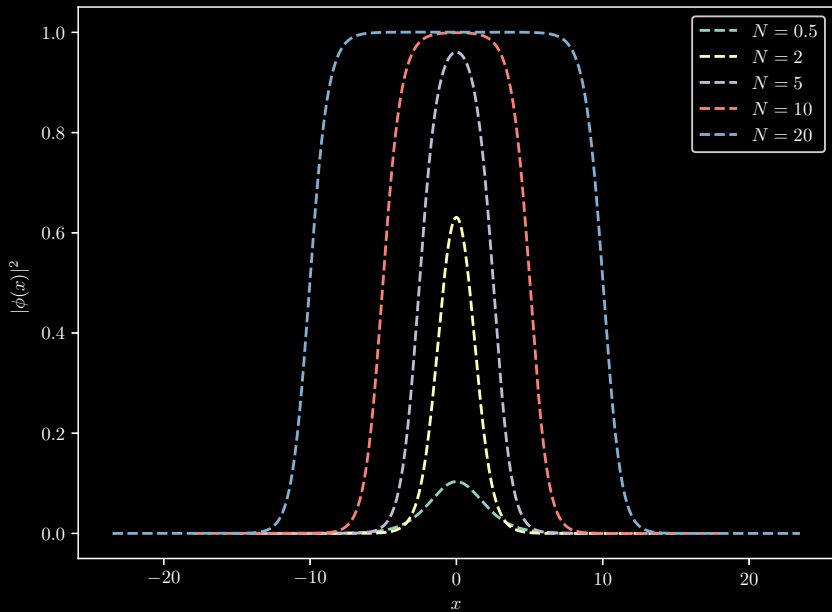
Effective 2D (flattened 3D)

$$2.74 < N_c < 2.75$$

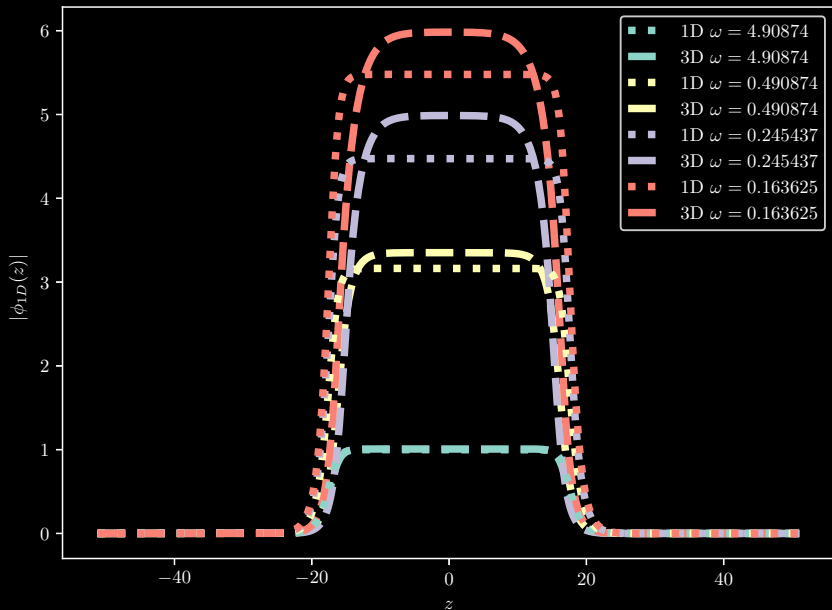
Effective 2D (flattened 3D)



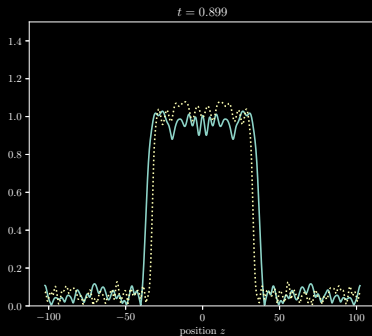
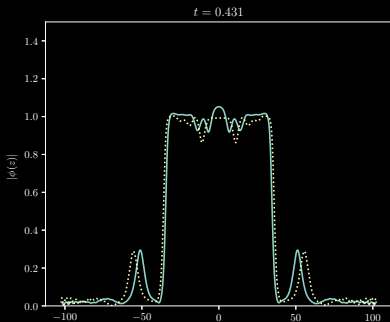
Effective 1D (elongated 3D)



Effective 1D (elongated 3D)

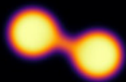


Effective 1D dynamics

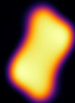
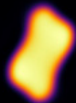
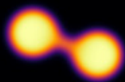


Effective 2D dynamics

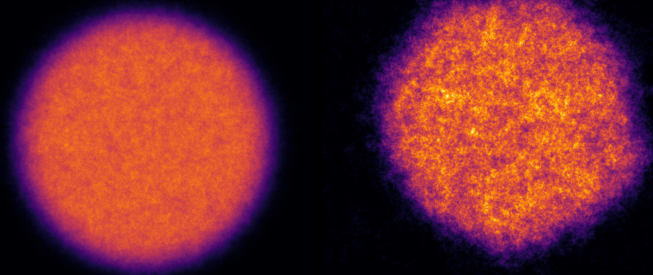
quasi-2D



3D



Thermal properties



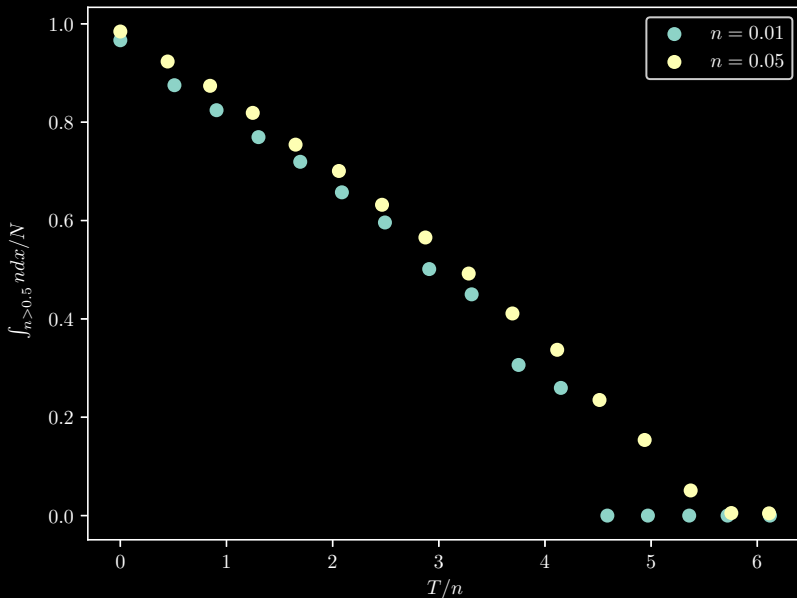
Modified Projected Stochastic GPE:

$$\begin{aligned}i\partial_t\phi &= \mathcal{P}_C\left((1 - i\gamma)\left(-\frac{\nabla^2}{2} - 3|\phi|^2 + \frac{5}{2}|\phi|^3 - \mu\right)\phi\right. \\ &\quad \left.- \frac{\gamma T}{\sigma^2}(N(\phi) - \bar{N})\phi\right. \\ &\quad \left.+ \sqrt{2\gamma T}\eta(x, t)\right)\end{aligned}$$

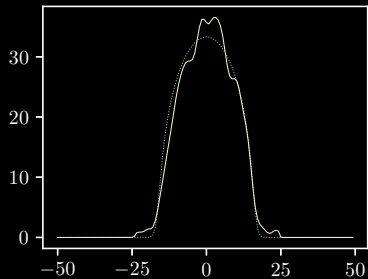
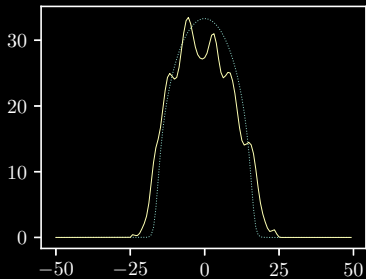
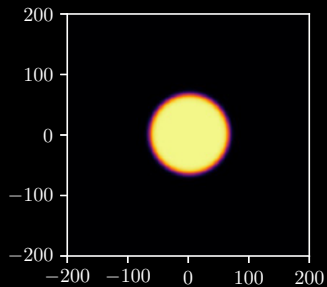
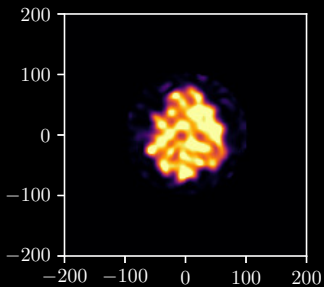
$$C = |\mu| + (\log 2)T$$

complex Gaussian noise: $\eta, \langle \eta^*(x, t)\eta(x', t') \rangle = \delta(x - x')\delta(t - t')$

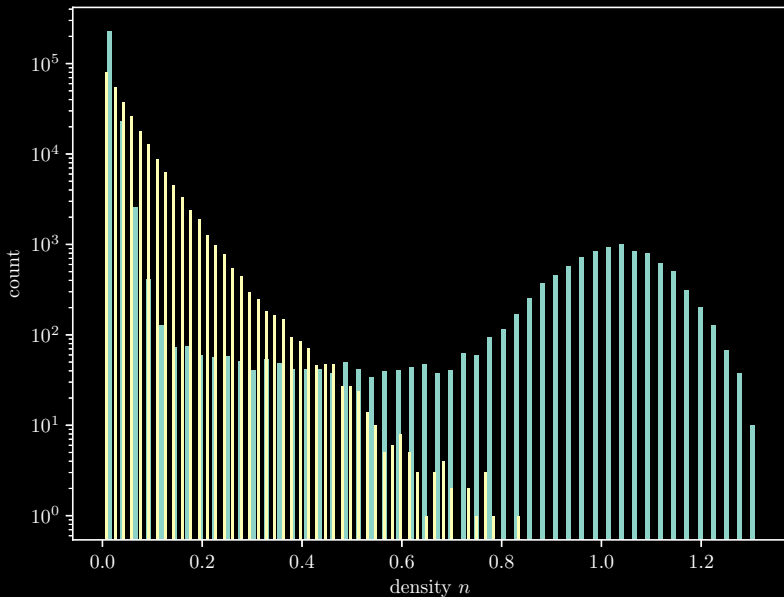
Thermal properties



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