Spontaneous defects in 1D Bose gases: Simulating the early universe

King Lun Ng [1,2]

Peter D. Drummond [2], Margaret Reid [2], Bogdan Opanchuk [2], Manushan Thenabadu [2]

1. *IFPAN Quantum Noise Group, Institute of Physics, Polish Academy of Sciences, Warszawa, Poland*
2. *Centre for Quantum Technology Theory, Swinburne University of Technology, Melbourne, Australia*

Contents

1. The proposed experiment:
How to simulate the early universe in BEC?

Fate of the false vacuum

- Coleman [1]: decay of relativistic scalar field; from metastable false vacuum to stable true vacuum

$$\partial_t^2\psi - c\nabla^2\psi = -\partial_\psi V(\psi)$$

- Bubble nucleation at speed c

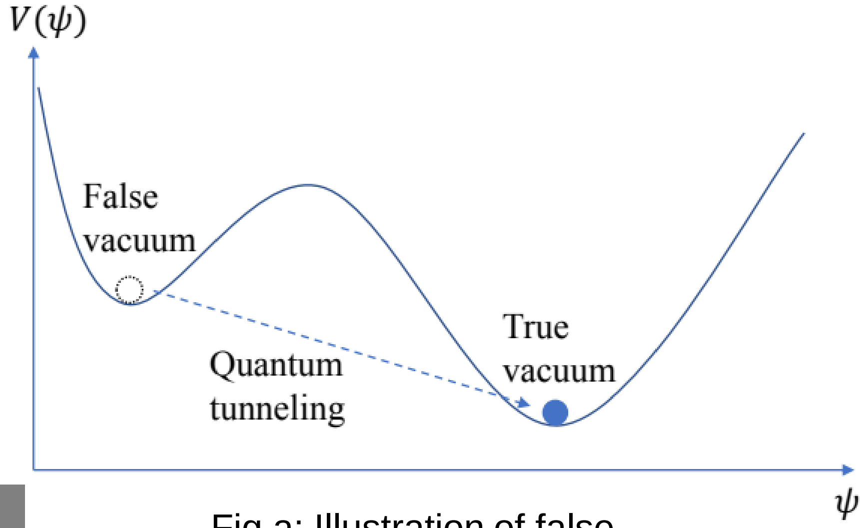

Fig a: Illustration of false vacuum tunneling

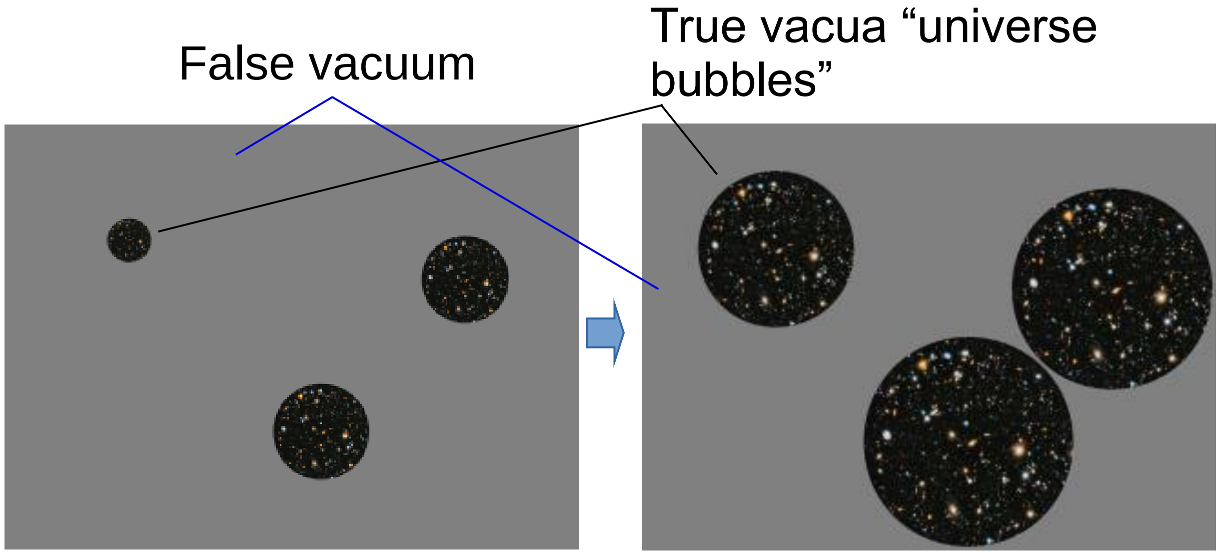

Fig b: Illustration of bubble nucleation of inflationary universes (Figures edited from https://www.nasa.gov/)

1. S. Coleman, *Phys. Rev. D* 15, 2929 (1977).

Proposed experiment of the false vacuum

- BEC with two occupied hyperfine levels, well mixed with minimized interspecies interaction $g_{12} \approx 0$, and a phase difference π
- For simplicity, we assume intraspecies interaction $g_{11} \approx g_{22} = g$
- This is possible for $^{41}\mathrm{K}$ ($|1\rangle = |F=1, m_F = 1\rangle$ and $|2\rangle = |F=1, m_F = 0\rangle$)
- Fialko *et al 2015 Europhys.Lett. 110 56001*

$$g_{11} \approx g$$

$$g_{22} \approx g$$

$$g_{12} \approx 0$$

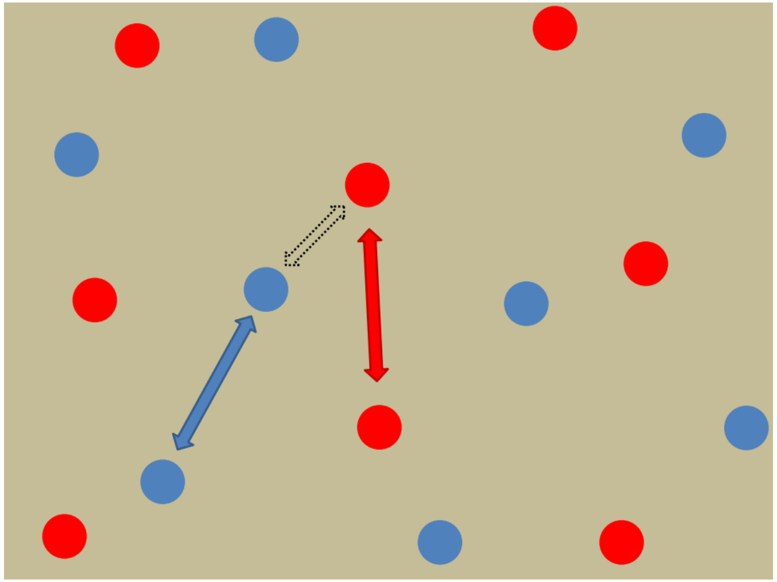

Fig: Illustration of well mixed BEC components

Proposed experiment of the false vacuum

- The Hamiltonian

$$\hat{H} = \sum_{j=1}^{2} \int dx \left\{ \left(-\hat{\Psi}_j^\dagger \frac{\hbar^2 \nabla^2}{2m} \hat{\Psi}_j + \frac{g}{2} \hat{\Psi}_j^{\dagger 2} \hat{\Psi}_j^2 \right) - \nu(t) \left(\hat{\Psi}_2^\dagger \hat{\Psi}_1 + \hat{\Psi}_1^\dagger \hat{\Psi}_2 \right) \right\}$$

- Modulated time-dependent sinusoidal coupling

$$\nu(t) = \nu + \delta \hbar \omega \cos \omega t$$

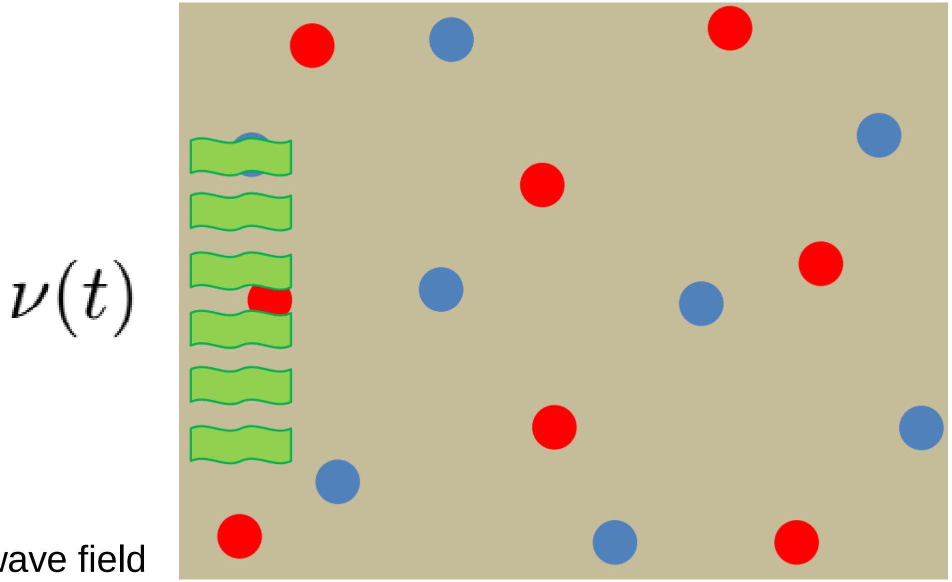

Fig: Modulated coupling by external cw microwave field

Kapitza pendulum: Phase potential creation

- Create metastable+stable potential:
  $\nu \quad \rightarrow \quad \nu(t)$
- Applying high driving frequency at the pivot point of a rigid pendulum
- metastable false vacuum -> small perturbation angle at lower position
- stable true vacuum -> upper vertical position

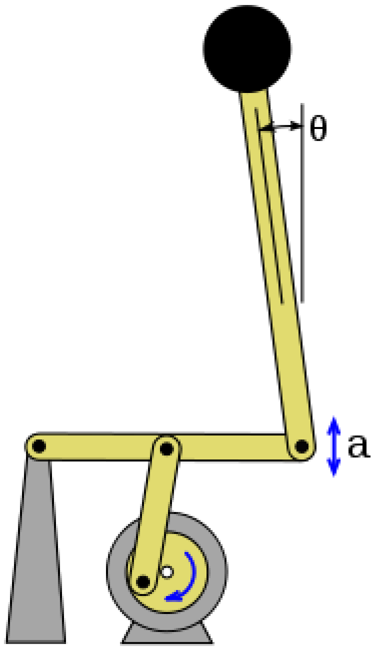

Fig: Illustration of Kapitza pendulum (Figure source:https://en.wikipedia.org/wiki/Kapitza%27s_pendulum)

Relative phase of the BEC

- Two component BEC with relative phase: $\phi_a = \phi_1 - \phi_2 - \pi$
- Phase potential in the condition of "Kapitza pendulum":

$$U(\phi_a) = \omega_0^2 \left[\cos(\phi_a) + \frac{\lambda^2}{2} \sin^2(\phi_a) \right]$$

- Characteristic frequency due to the coupling amplitude:

$$\omega_0 = 2\sqrt{\nu g \rho_c}/\hbar$$

- Fast oscillation amplitude:

$$\lambda = \delta\sqrt{2 g \rho_c/\nu}$$

- BEC in false vacuum: $\phi_a = 0$
- BEC in true vacuum: $\phi_a = \pm\pi$

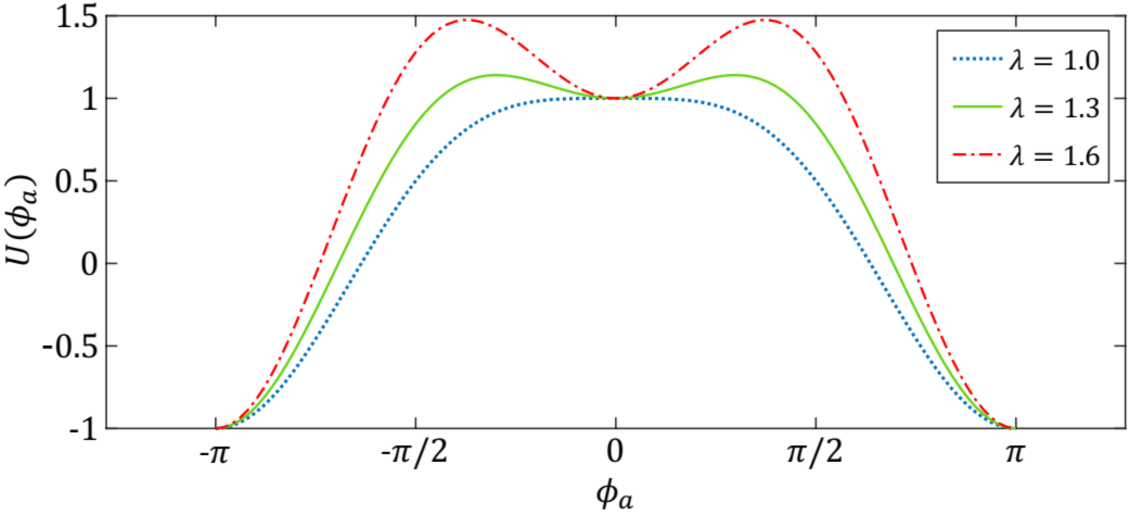

Fig: Phase potential vs relative phase of BEC

2. Theoretical model of the experiment

Initial state (part1): Bogoliubov method

- Assuming component $j = 2$ is in a vacuum state; component $j = 1$ is in thermal equilibrium at temperature T : $\hat{\Psi}_1(x,0) = \psi_c + \delta\hat{\Psi}_1$

- Fluctuations: $\delta\hat{\Psi}_1 = \frac{1}{\sqrt{L}}\sum_k \left[u_k \hat{b}_k e^{ikx} - v_k \hat{b}_k^\dagger e^{-ikx}\right]$

- Bogoliubov coefficients for $k \neq 0$: $u_k = \frac{\epsilon_k + E_k}{2\sqrt{\epsilon_k E_k}}$, $v_k = \frac{\epsilon_k - E_k}{2\sqrt{\epsilon_k E_k}}$

- Free particle energy $E_k = \hbar^2 k^2 / (2m)$ and excitation energy $\epsilon_k = \sqrt{E_k(E_k + 2g\rho_c)}$

- Phonon distribution: $\langle \hat{n}_k \rangle = \langle \hat{b}_k^\dagger \hat{b}_k \rangle \equiv n_k = \frac{1}{\exp(\beta\epsilon_k) - 1}$, $\beta = 1/k_B T$

Initial state (part 2):
truncated Wigner Approximation (TWA)

- Long simulation time
- Include thermal and vacuum fluctuations
- Correction of order $1/N^2$ for N particles
- Taking $\hat{b}_k \sim \beta_k \to \beta_{\widetilde{k}}$, the corresponding Wigner representation for the BEC fields are (in dimensionless):

$$\hat{\Psi}_1 \to \widetilde{\psi}_1 = \widetilde{\psi}_c + \frac{1}{\sqrt{\widetilde{L}}} \sum_{\widetilde{k}} (u_{\widetilde{k}} \beta_{\widetilde{k}} e^{i\widetilde{k}\widetilde{x}} - v_{\widetilde{k}} \beta^*_{\widetilde{k}} e^{-i\widetilde{k}\widetilde{x}})$$

$$\hat{\Psi}_2 \to \widetilde{\psi}_2 = \frac{1}{\sqrt{\widetilde{L}}} \sum_{\widetilde{k}} \alpha_{\widetilde{k}} e^{i\widetilde{k}\widetilde{x}}$$

Initial state (part 2): truncated Wigner Approximation (TWA)

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$$\hat{\Psi}_2 \to \widetilde{\psi}_2 = \frac{1}{\sqrt{\widetilde{L}}} \sum_{\widetilde{k}} \alpha_{\widetilde{k}} e^{i\widetilde{k}\widetilde{x}}$$

- Complex Gaussian random variables $\beta_{\widetilde{k}} = \dfrac{\eta_{1,\widetilde{k}}}{\sqrt{2\tanh(\widetilde{\epsilon}_{\widetilde{k}}/8\sqrt{\widetilde{\nu}}\widetilde{\rho}_0^2\tau)}}$ and $\alpha_{\widetilde{k}} = \dfrac{\eta_{2,\widetilde{k}}}{\sqrt{2}}$
- Expectation values of noises: $\langle|\beta_{\widetilde{k}}|^2\rangle = n_{\widetilde{k}} + 1/2$ and $\langle|\alpha_{\widetilde{k}}|^2\rangle = 1/2$, $n_{\widetilde{k}} = (\exp(\beta\epsilon_{\widetilde{k}}) - 1)^{-1}$

Initial state (part 3)

- The BEC is rabi rotated by a microwave pulse to give equal occupation for both spin species with initial relative phase $\phi_1 - \phi_2 = \pi$
- In simulation, this is equivalent to a rotation matrix acting on the BEC fields:

$$\begin{pmatrix} \widetilde{\psi}_1' \\ \widetilde{\psi}_2' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -ie^{-i\phi}\sin\frac{\theta}{2} \\ -ie^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \widetilde{\psi}_1 \\ \widetilde{\psi}_2 \end{pmatrix}$$

where $\theta = \pi/2$ and $\phi = -\pi/2$

- Initial conditions $\langle|\widetilde{\psi}_1'|^2\rangle = \langle|\widetilde{\psi}_2'|^2\rangle$

3.Decay of false vacuum

Some parameters

Proposed experimental parameters[3,4]	
Trap circumference L	254μm
Number of atoms N_c	4×10^4
Condensate density ρ_c	$\approx 1.58 \times 10^6 \text{cm}^{-1}$
Degeneracy temperature	$\approx 147\mu$K
$T_d = (\hbar^2 \rho_c^2 / 2mk_B)$	
BEC termperature T	$\approx 1.47 \sim 147n$K
Characteristic frequency ω_0	$2\pi \times 191.26$Hz
Oscillator frequency ω	$2\pi \times 9.56$kHz
Speed of sound c	3.05mms^{-1}
Observation time t_f	49.9ms

Dimensionless simulation parameters	
Circumference $\widetilde{L}$	100
Atom density $\widetilde{\rho}_0$	200
Reduced temperature τ	$10^{-5} \sim 10^{-3}$
Oscillator frequency $\widetilde{\omega}$	$50 \sim 200$
Modulation amplitude λ	$1.2 \sim 1.4$
Coupling $\widetilde{\nu}$	$0.004 \sim 0.01$
Number of mode M	256

3. A. Kumar et al., *Phys. Rev. A* 95, 021602(R) (2017).

4. M. Kunimi and I. Danshita, *Phys. Rev. A* 99, 043613 (2019).

The decay of false vacuum and the bubble nucleation of true vacua

- The Wigner field trajectory in real time

$$\frac{d\widetilde{\psi}_j}{d\widetilde{t}} = -i\left[-\sqrt{\widetilde{\nu}}\widetilde{\nabla}^2\widetilde{\psi}_j + \tilde{g}\widetilde{\psi}_j|\widetilde{\psi}_j|^2\right] + i\frac{\sqrt{\widetilde{\nu}}}{2}\left[1+\sqrt{2}\lambda\widetilde{\omega}\cos(\widetilde{\omega}\widetilde{t})\right]\widetilde{\psi}_{3-j}$$

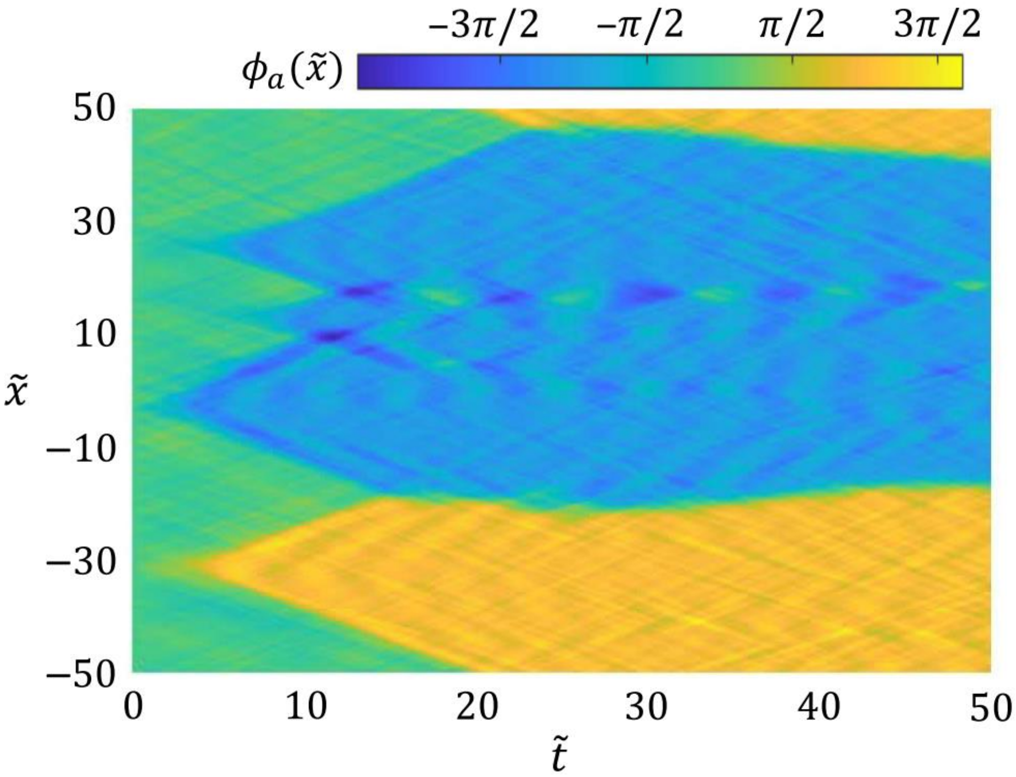
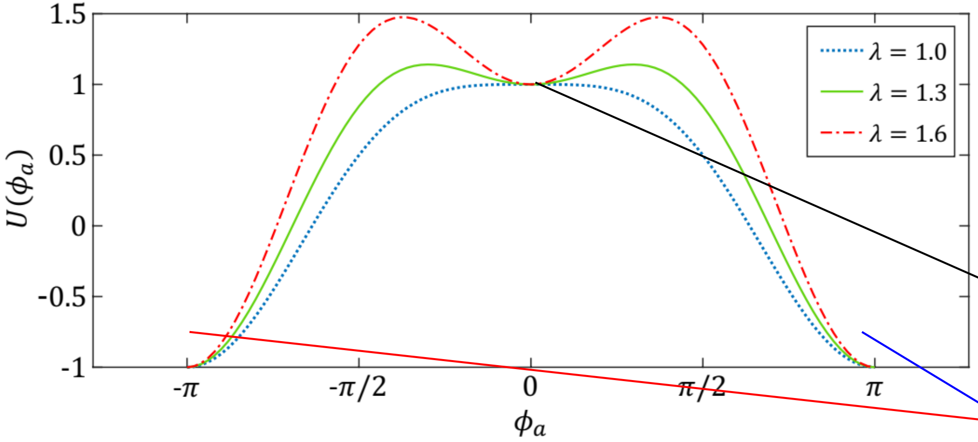


Fig: Decay of 1D false vacuum from a single trajectory simulation with reduced temperature $\tau = 10^{-5}$, corresponds to $T \sim 1.5\mathrm{nK}$.

The decay of false vacuum and the bubble nucleation of true vacua



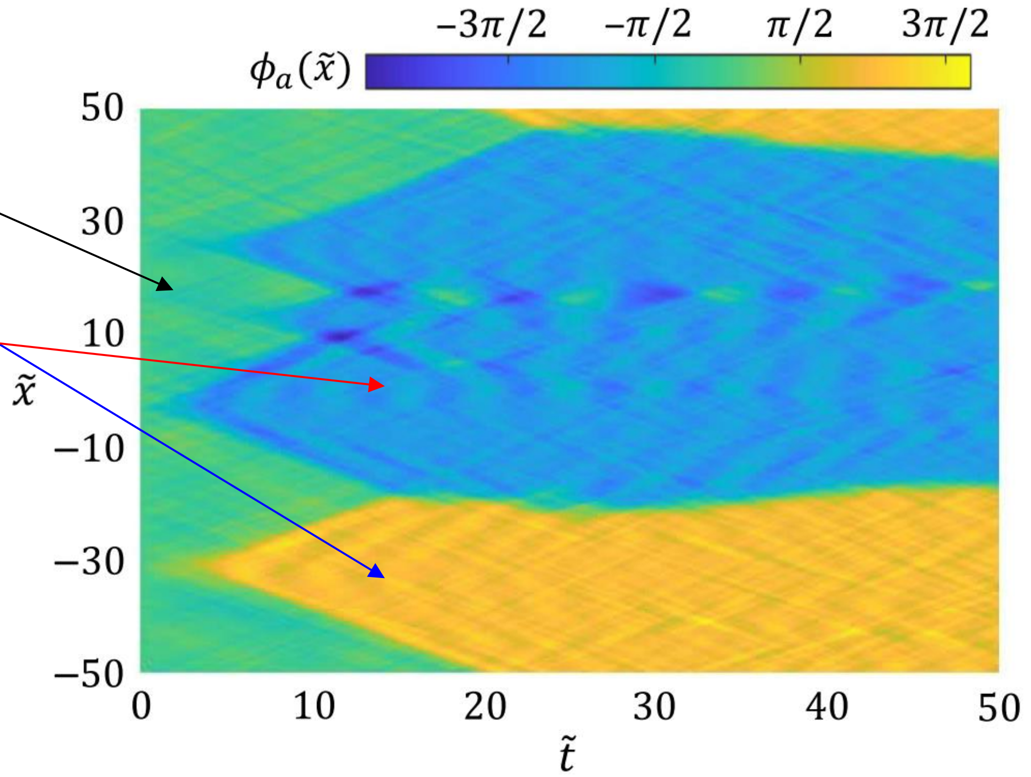


Fig: Decay of 1D false vacuum from a single trajectory simulation with reduced temperature $\tau = 10^{-5}$, corresponds to $T \sim 1.5\text{nK}$.

1D false vacuum at finite temperatures

- Relative number density distribution: $p_z(\widetilde{x}) = \frac{I_2(\widetilde{x})-I_1(\widetilde{x})}{I_2(\widetilde{x})+I_1(\widetilde{x})}$,

where $I_1(\widetilde{x}) = |\frac{\widetilde{\psi}_1+\widetilde{\psi}_2}{\sqrt{2}}|^2 - \frac{1}{2\Delta\widetilde{x}}$ and $I_2(\widetilde{x}) = |\frac{\widetilde{\psi}_1-\widetilde{\psi}_2}{\sqrt{2}}|^2 - \frac{1}{2\Delta\widetilde{x}}$

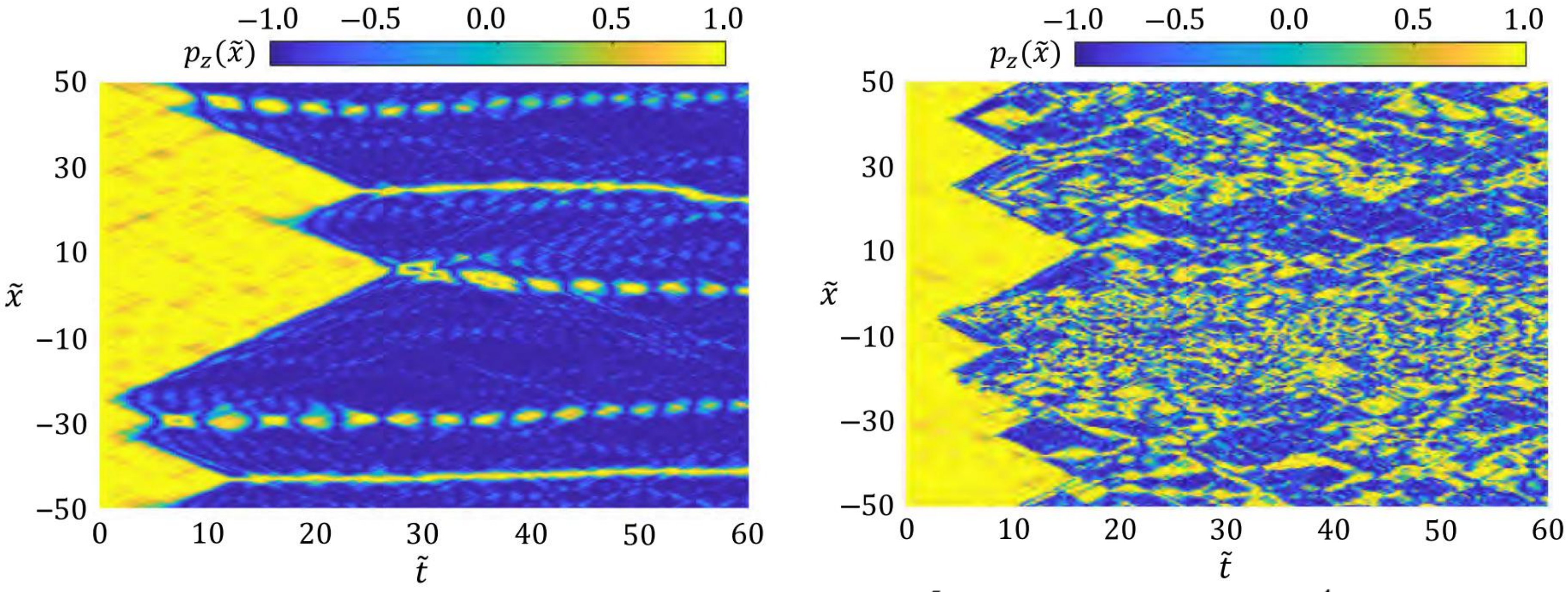
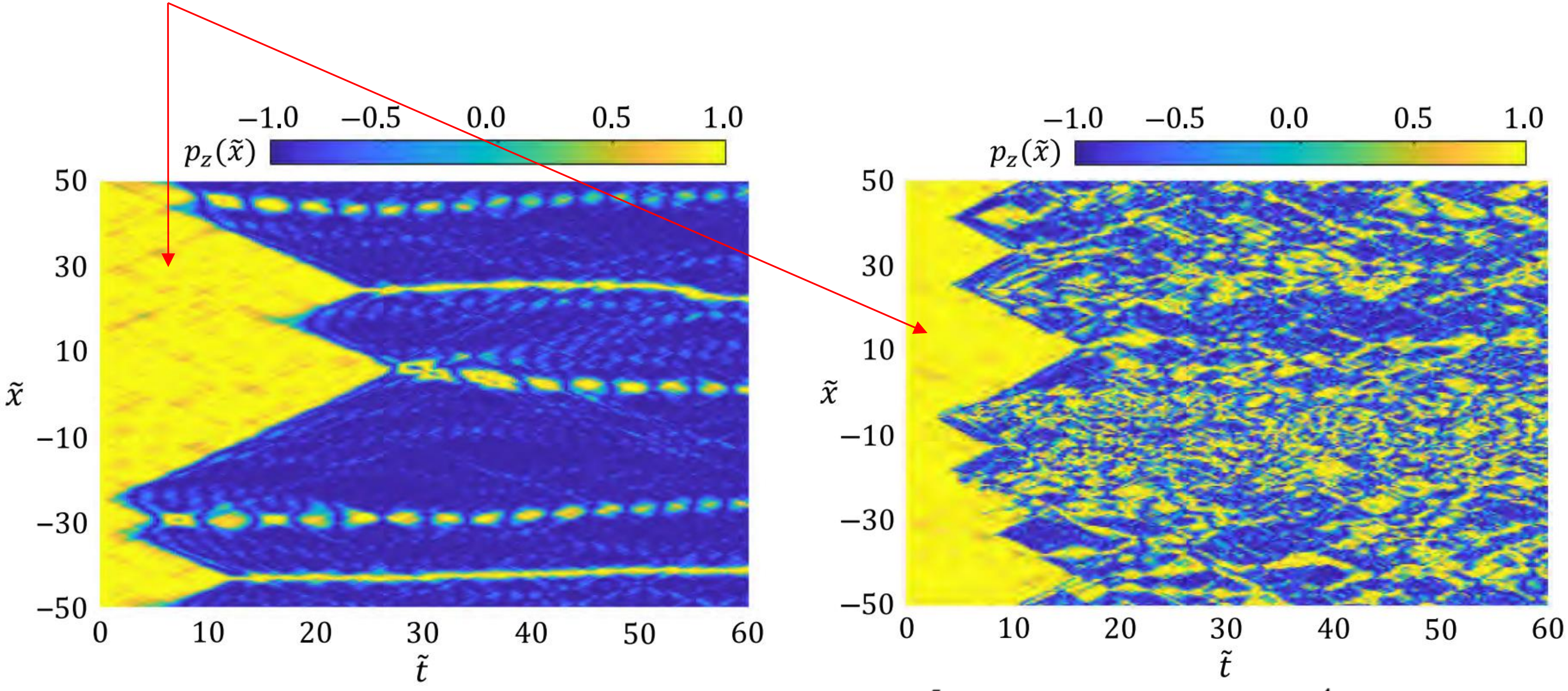


Fig: (left) reduced temperature $\tau = 1 \times 10^{-5}$; (right) $\tau = 3 \times 10^{-4}$

1D false vacuum at finite temperatures

- False vacuum and true vacua (bubble universes)



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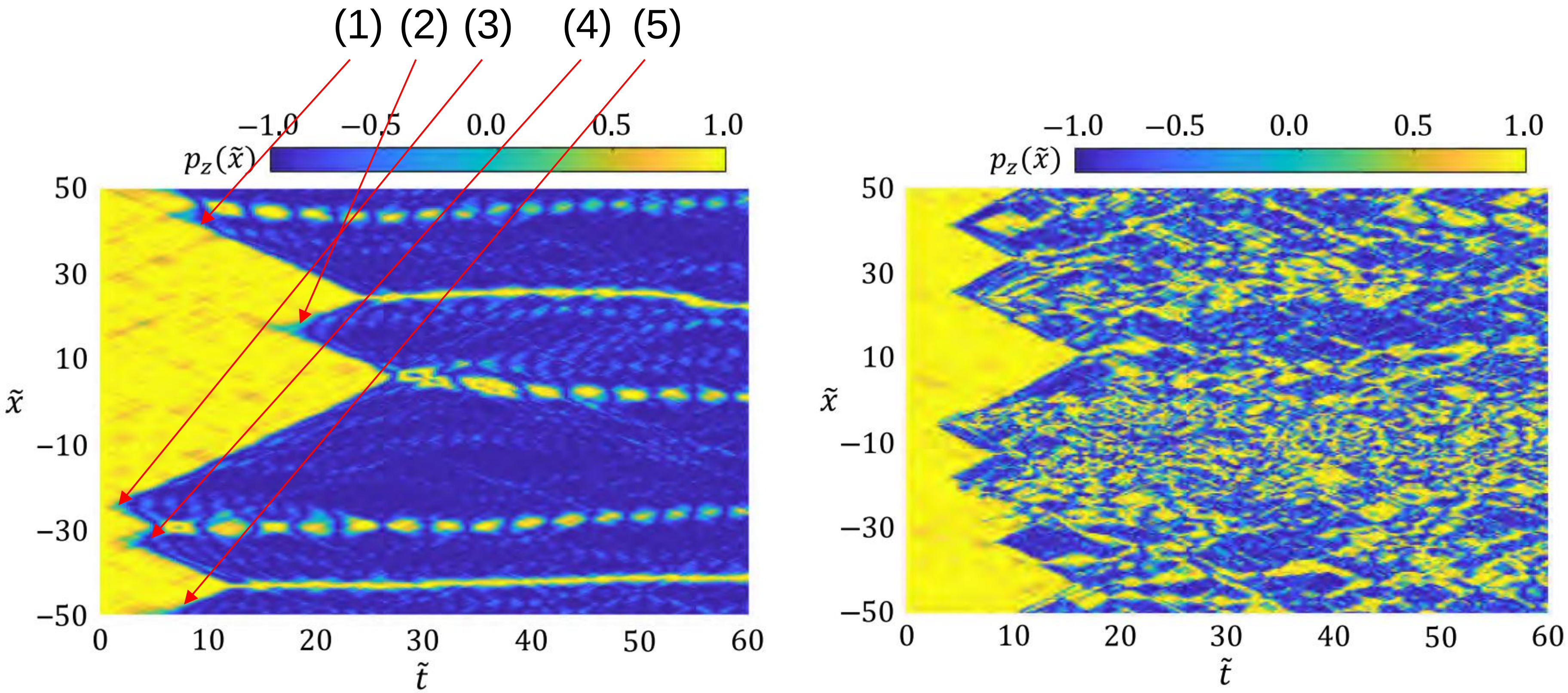


Fig: (left) reduced temperature $\tau = 1 \times 10^{-5}$; (right) $\tau = 3 \times 10^{-4}$

1D false vacuum at finite temperatures

- Domain walls and oscillons

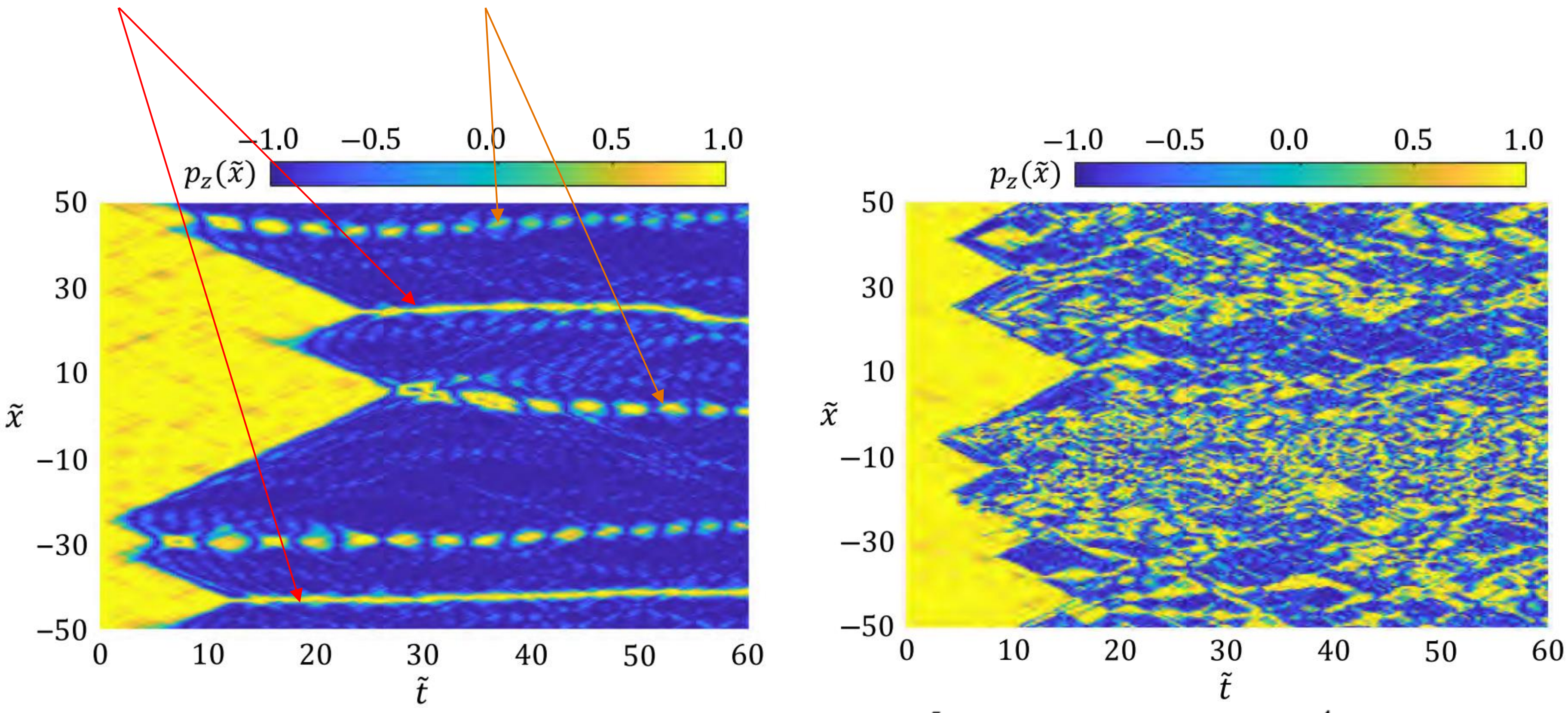


Fig: (left) reduced temperature $\tau = 1 \times 10^{-5}$; (right) $\tau = 3 \times 10^{-4}$

2D false vacuum at finite temperatures

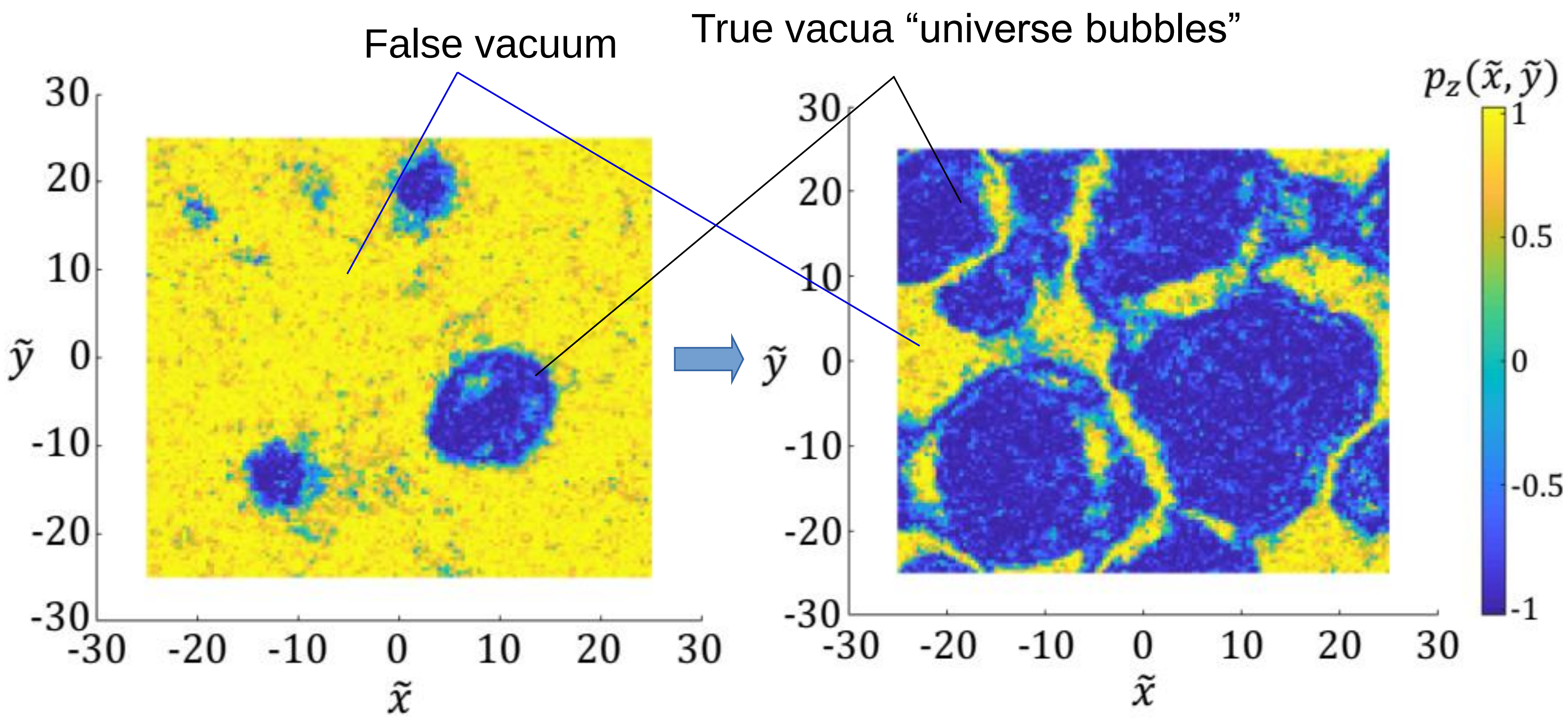


Fig: Simulation of bubble nucleation in 2D BEC

Tunneling rate: quantify bubble nucleation

- Average cosine of the relative phase:

$$\langle \cos\phi_a \rangle = \frac{1}{\widetilde{L}} \int^{\widetilde{L}} \cos\phi_a(\widetilde{x})\,d\widetilde{x}$$

- Threshold value for bubble nucleation
- Survival probability and tunneling rate[5]

$$\mathcal{F} = \exp(-\Gamma\widetilde{t})$$

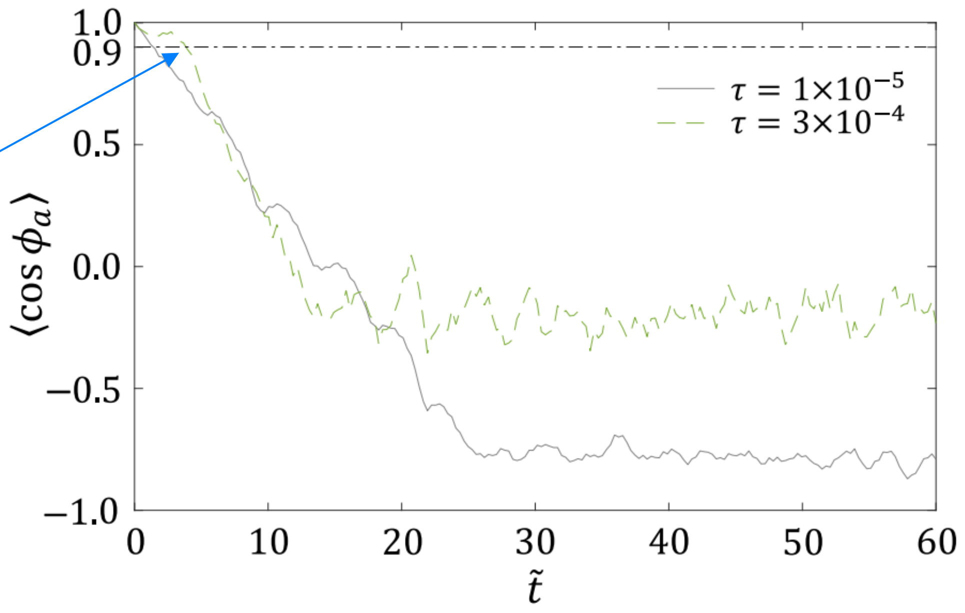
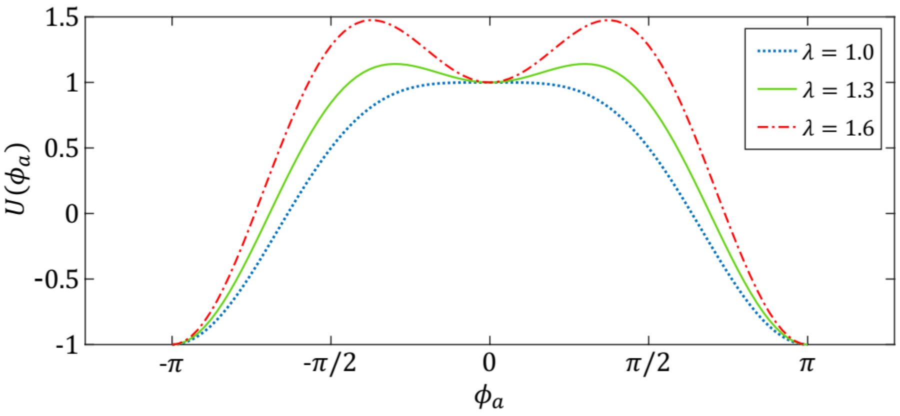


Fig: Example of average cosine of the BEC relative phase at two temperatures

5. S. Takagi, *Macroscopic Quantum Tunneling* (Cambridge University Press 2002).

Tunneling rate

- Statistical results from 80000 Wigner trajectories
- Coherent state with no thermal effect included
- Various external coupling $\tilde{\nu}$
- Various oscillation amplitude λ



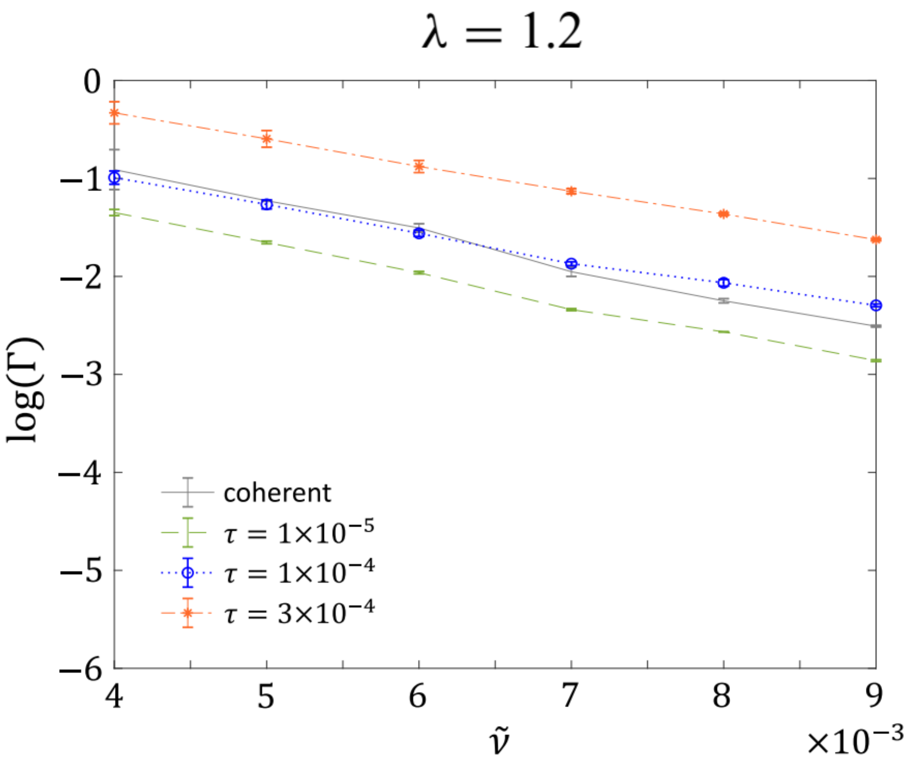
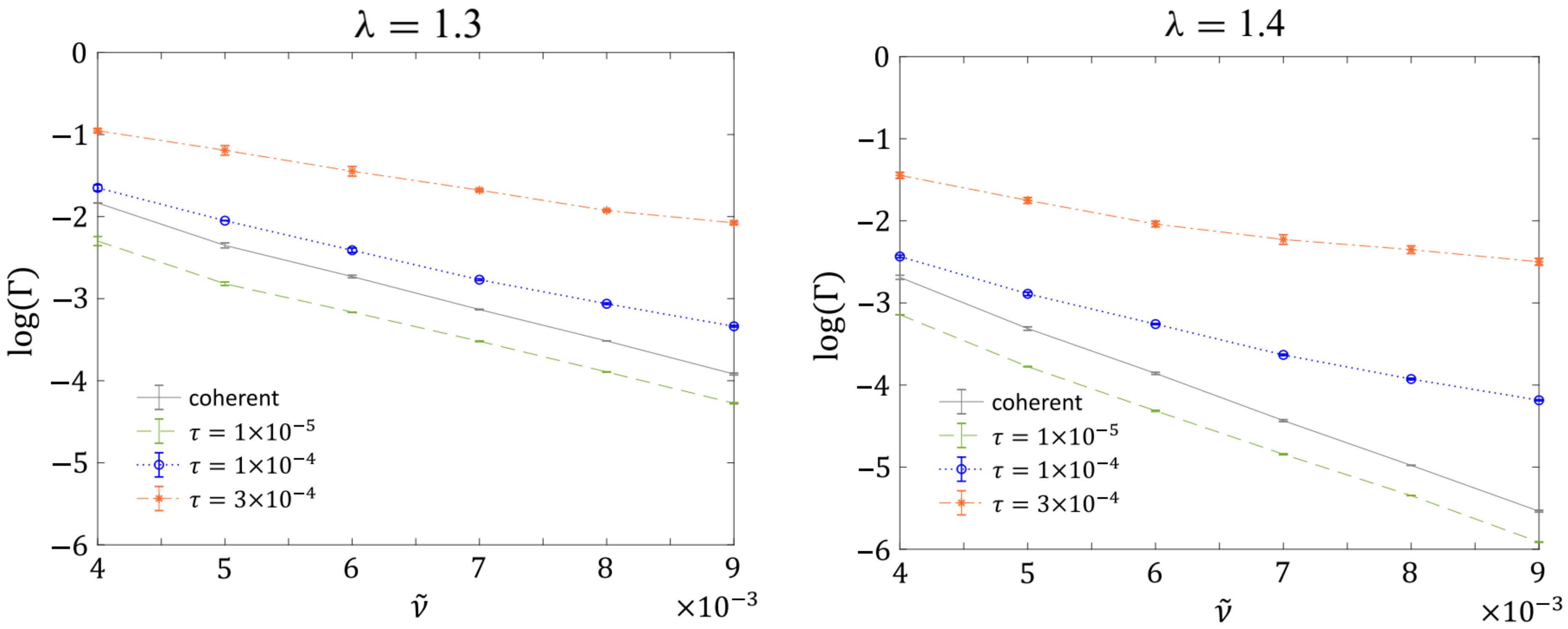


Fig: Tunneling rate at various temperatures

Tunneling rate

- High oscillation amplitude (deeper phase potential “depth”) reduces tunneling rate
- Strong external coupling reduces tunneling rate
- Tunneling rate is dominated by the thermal fluctuations at high temperature



4.Floquet instability

Floquet instability

- If modulation frequency $\widetilde{\omega}$ too low: unstable Floquet modes occur [6]

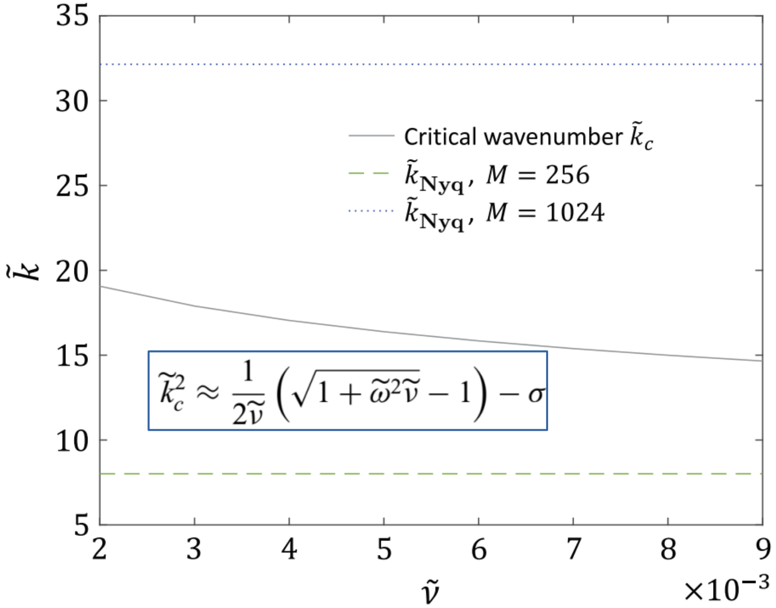
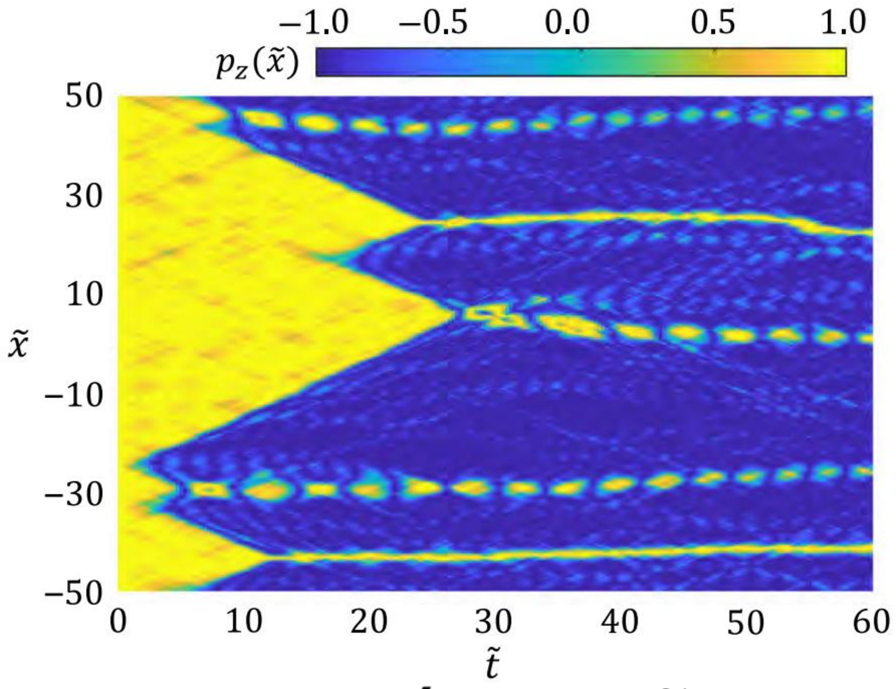


Fig. Critical wavenumber at various coupling

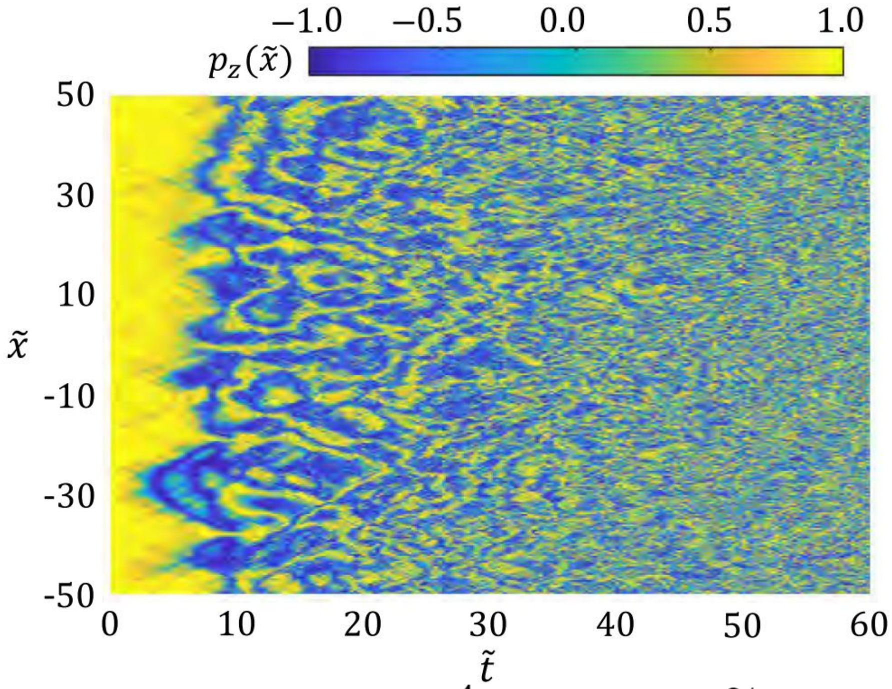


$\tau = 1 \times 10^{-5}; M = 256\,; \widetilde{\omega} = 50$

6. J. Braden et al., *JHEP* 2019, 174 (2019).

Floquet instability

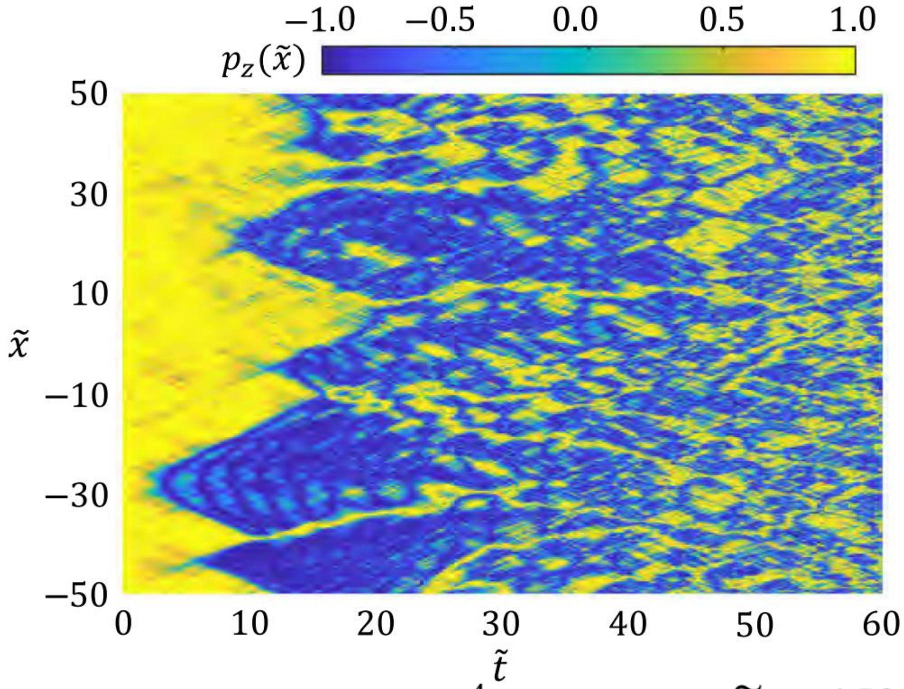
- Increase the momentum cutoff to include the unstable Floquet modes
- True vacua gradually destroyed
- Chaotic fluctuations
- Short lived vacua



$\tau = 1 \times 10^{-4}; M = 1024; \widetilde{\omega} = 50$

Floquet instability

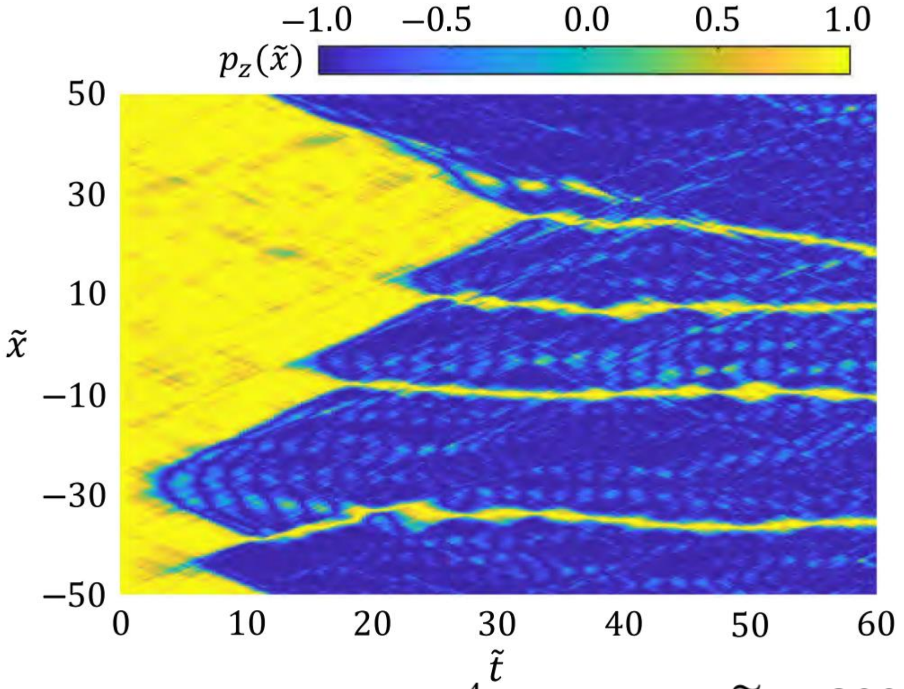
- Increase modulation frequency
- Partially stabilize the true vacua



$\tau = 1 \times 10^{-4}; M = 1024; \widetilde{\omega} = 150$

Floquet instability

- Stable true vacua in the simulation time



$\tau = 1 \times 10^{-4}; M = 1024; \widetilde{\omega} = 200$

Floquet instability

- Statistical results from 8000 trajectories
- True vacua stabilization at large modulation frequency (Fig.a)
- Initiation of bubble nucleation is delayed by modulation amplitude (Fig.b)

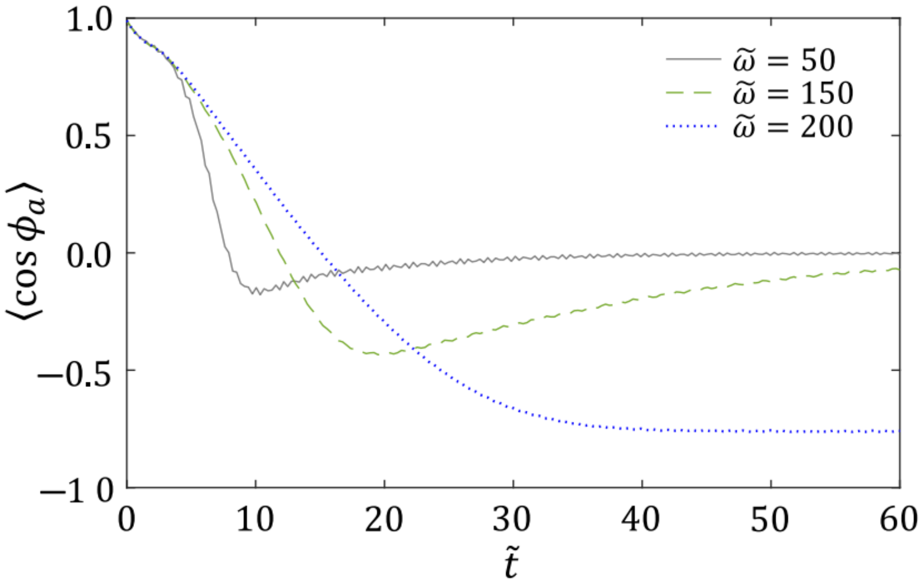


Fig.a: $\tau = 1 \times 10^{-4}; M = 1024; \lambda = 1.2$

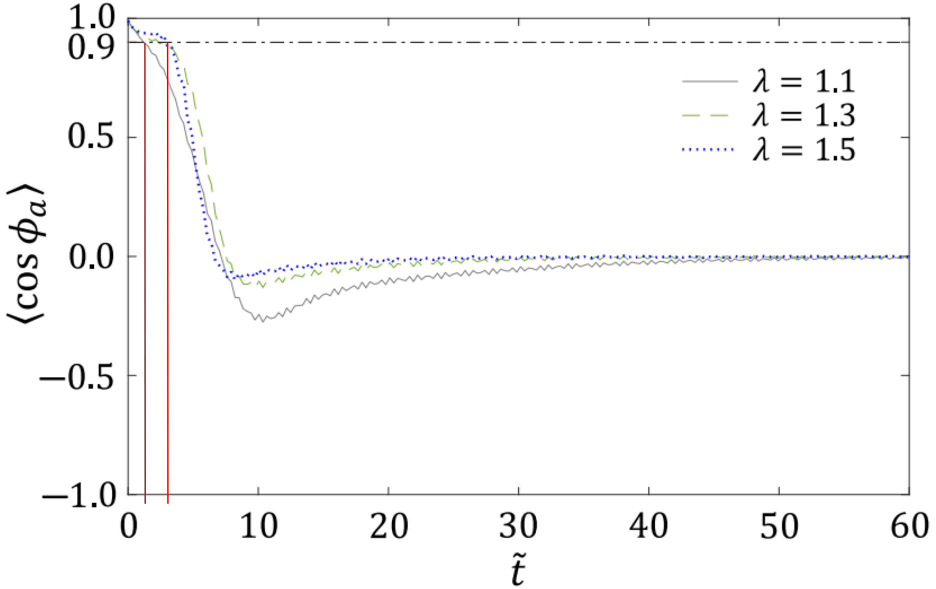


Fig.b: $\tau = 1 \times 10^{-4}; M = 1024; \widetilde{\omega} = 50$

5.Summary

Conclusion

- BEC with two spin components as the analogous relativistic quantum field
- Relative phase corresponds to the false/true vacuum
- Components are coupled via modulation microwave
- Thermal fluctuations coexist with true vacua
- Bubble nucleation is accelerated at finite temperature
- True vacua may be stabilized by high modulation frequency

Reference

- O. Fialko *et al*, *EPL* 110, 56001 (2015); O. Fialko *et al*, *J. Phys. B: At. Mol. Opt. Phys.* 50, 024003 (2017); K.L. Ng *et al, PRX Quantum 2*, 010350 (2021)
- T. P. Billam *et al*, *Phys. Rev. D* 100, 065016(2019) ; T.P. Billam *et al*, *Phys. Rev. A* 104 053309 (2021)
- J. Braden *et al*, *Phys. Rev. Lett.* 123 031601 (2019); J. Braden *et al*, *JHEP* 2019, 174 (2019)

Thank You

Other works

- Simulated universe in BEC (theory – modulation coupling):

Alexander Dellios, Andrei Sidorov, Peter Drummond (Swinburne University, Australia)

- Simulated universe in BEC experiment

(*JHEP* 2018, 014 (2018); *Phys.Rev.Lett.* 123, 031601 (2019); *JHEP* 2019, 174 (2019)):

Jonathan Braden (University of Toronto, Perimeter Institute for Theoretical Physics, Canada)

Matthew C. Johnson (York University, Perimeter Institute for Theoretical Physics, Canada)

Hiranya V. Peiris (University College London, U.K.)

Andrew Pontzen (University College London, U.K.)

Silke Weinfurtner (University of Nottingham, U.K.)