Superfluid dipolar Fermi gases and their excitations

Piotr Deuar, Misha Baranov, Gora Shlyapnikov

LPTMS

Université Paris-sud XI

Universiteit van Amsterdam

LPTMS, 19 Mars 2008

Overview

1. Motivation

   Comparison with standard BCS gas;

   A “clean” realisation of solid-state phases

2. Experimental prospects

   possible realisations; Behaviour of critical temperature T_c

3. Model for the uniform 3D gas

   $\widehat{H}$, mean-field theory , assumptions

4. Quasiparticle (pair) excitations

   Anisotropic energy gap for pair breaking, gap nodes

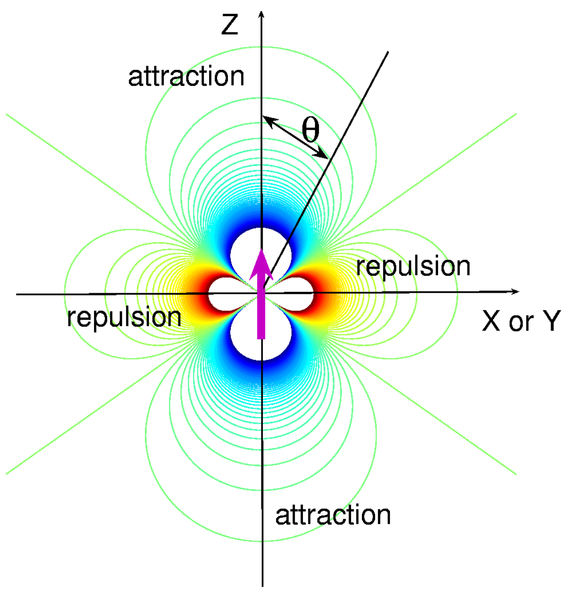
5. Collective excitations & superfluid component

   Hydrodynamics, anisotropic damping,

   unusual superfluid current response

Physical system

$$V_D(R,\theta) = \frac{d^2}{R^3}\left(1 - 3\cos^2\theta\right)$$



- uniform 3D gas
- Cold: $T < T_c$
- static external field (E or B) $\Longrightarrow$ full polarisation
- single-species (spin polarised)
- dilute $\Longrightarrow$ Energy dominated by Fermi sea to leading order
- short-range interaction assumed negligible (Fermi exclusion, no p-wave resonances)

(1) Motivation

BCS superfluidity

dipole–dipole potential

- *LONG* range interaction
- *Anisotropic*
- always partly attractive
  BCS pairing if *polarised*
- Needs 1 spin component
- Energy gap has nodes
- Stability conditions nontrivial

standard s-wave $\uparrow\downarrow$ potential

- *SHORT* range interaction
- *Isotropic*
- arttractive or repulsive
  BCS pairing only if $a_s < 0$
- Needs 2 spin components
- Energy gap always $\neq 0$

Góral, Brewczyk, Rzążewski PRA 67,025601 (2003)

Condensed matter analogue

- The node structure of the direction-dependent order parameter is similar to that of solid state and liquid He phases, e.g.:
  - Polar phase of $^3\mathrm{He}$.
    (Never experimentally realized) Aoyama & Ikeda PRB 73, 060504 (2006), Elbs etal. arXiv:0707.3544
  - Heavy-fermion superconductors like UPt_3.
    (Difficult to get pure system, many potential phases)
- Qualitatively similar behaviour expected in some respects.
- Dipole gas is a much “cleaner” system.
  - $\widehat{H}$ well known
  - spin degrees of freedom can be removed.
- It is potentially well controllable.

(2) Prospects for superfluidity

Possible Physical Realisations

1. Heteronuclear polar molecules

   - Several groups actively aiming to cool to ultracold T.
     e.g. Bigelow (Rochester), Grimm (Innsbruck), . . .
   - Method 1: Photoassociaton from cold atomic gases
   - Method 2: Buffer gas cooling

2. Magnetic atomic dipoles

   - e.g. ^{53}Cr (6 parallel spins in valence electron shell)
   - Current experiments: O. Gorceix (Uni Paris-Nord)

3. Induce electric dipoles in atoms with strong E fields

Critical Temperature for BCS

standard ↑↓ gas:

$$T_c = 0.28\, E_F \exp\left(-\frac{\pi}{2|a_s|k_F}\right)$$

Dipole gas:

MB, Mar'enko, Rychkov, GS, PRA **66**, 013606 (2002)

$$T_c = 1.44\, E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right)$$

$\Longrightarrow$ *Effective* scattering length a_D:

$$a_D = -2m\left(\frac{d}{\pi\hbar}\right)^2$$

T_c rises strongly with $a_D \propto md^2$

Candidates for BCS pairing

(large $|a_D|$ desirable)

Short-range interactions

- Two spin components. For example ^{6}Li : $a_s = -114$ nm

Dipoles

- Heteronuclear polar molecules

  ^{15}ND^3 : $a_D = -145$ nm

  HCN : $a_D = -740$ nm

  NaCs : $a_D \gtrsim -500$ nm

- Magnetic atomic dipoles

  ^{52}Cr : $a_D = -0.5$ nm (pretty weak)

- Atoms with induced electric dipole

  $a_D \approx -1$ to -10 nm (need $\approx 10^6$ V/cm)

(3) Model

Hamiltonian

$$\widehat{H} = \text{K.E.} + \frac{1}{2}\int d^3x\, d^3y \left\{ \widehat{\Psi}_x^\dagger \widehat{\Psi}_x V_D(x-y) \widehat{\Psi}_y^\dagger \widehat{\Psi}_y \right\}$$

- $\widehat{\Psi}_x$ is the anihilating Fermi field operator at point x.

BCS Mean field theory: Postulate the quadratic effective Hamiltonian:

$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3x\, d^3y \Big\{ \frac{\hbar^2}{m} \widehat{\Psi}_x^\dagger \nabla^2 \widehat{\Psi}_x \delta(x-y) \quad \textit{Kinetic}$$

$$\Delta^*(x-y)\widehat{\Psi}_x\widehat{\Psi}_y - \Delta(x-y)\widehat{\Psi}_x^\dagger\widehat{\Psi}_y^\dagger \quad \textit{BCS}$$

$$+W(x-y)\widehat{\Psi}_x^\dagger\widehat{\Psi}_y \Big\} \quad \textit{Hartree}$$

- With some “appropriate” $\Delta(x-y)$ and $W(x-y)$

Gap equation

Choose $\Delta(x-y)$ and $W(x-y)$ to minimise the full Free energy

$$F = \langle \widehat{H} \rangle_{\text{eff}} - \mu N - TS$$

when calculated with eigenstates of $\widehat{H}_{\text{eff}}$.

Obtain:

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}} \qquad \text{GAP}$$

$$W(x-y) = -V_D(x-y) \left\langle \widehat{\Psi}^\dagger_x \widehat{\Psi}_y \right\rangle_{\text{eff}} \qquad \text{“Hartree“ field}$$

Δ, W and Ψ must be self-consistent.

Uniform gas

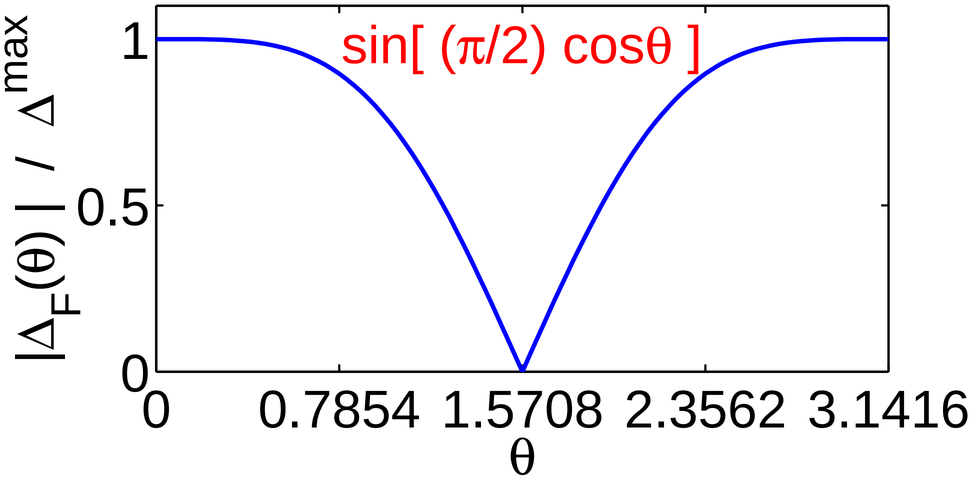
In k–space

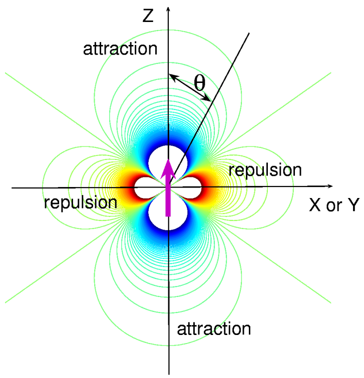
$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3k \left\{ \left(\frac{\hbar^2 k^2}{m} - 2\mu - W(k) \right) \widehat{\Psi}_k^\dagger \widehat{\Psi}_k + \Delta^*(k) \widehat{\Psi}_k \widehat{\Psi}_{-k} - \Delta(k) \widehat{\Psi}_k^\dagger \widehat{\Psi}_{-k}^\dagger \right\}$$

- $W(k)$ is a minor energy shift of Fermi surface
  $\Longrightarrow$ ignore it in leading order
- Order parameter $\Delta(k) \neq 0$ corresponds to BCS pairing of k and $-k$ atoms.
- Important difference to standard $\uparrow\downarrow$gas: $\Delta(k)$ is anisotropic and has nodes on the Fermi surface

(4) Quasiparticle (pair) excitations

BCS gap $\Delta_F(\theta)$ on Fermi surface





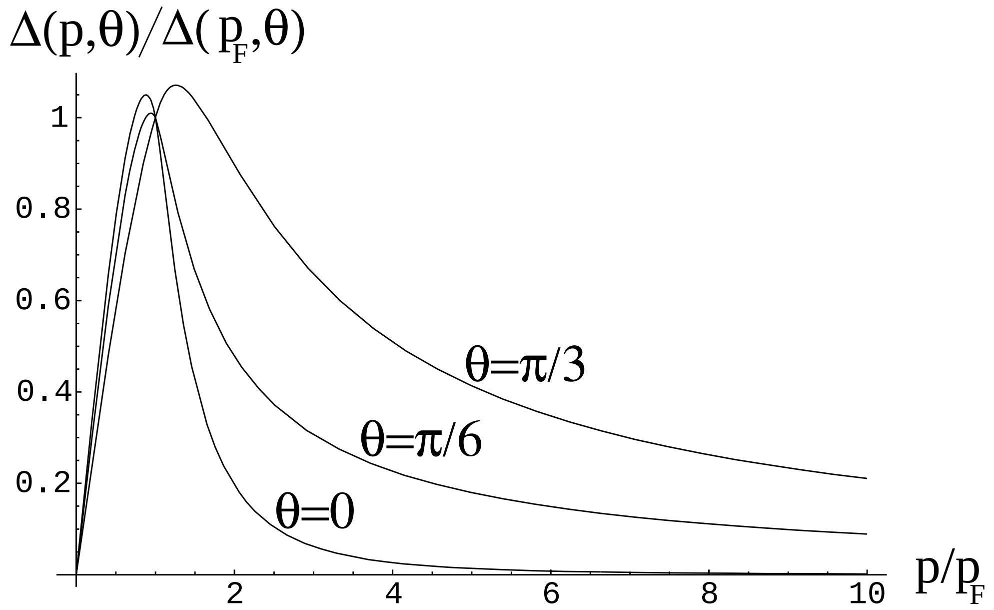
NODE in plane $\perp$ to polarisation

Breaking a pair costs $2 \times E$, where $E(k) = \sqrt{(\text{K.E.} - E_F)^2 + \Delta^2} \geq |\Delta|$.

- Dipoles: Easy to excite a pair in plane $\perp$ to polarisation because energy cost is small.

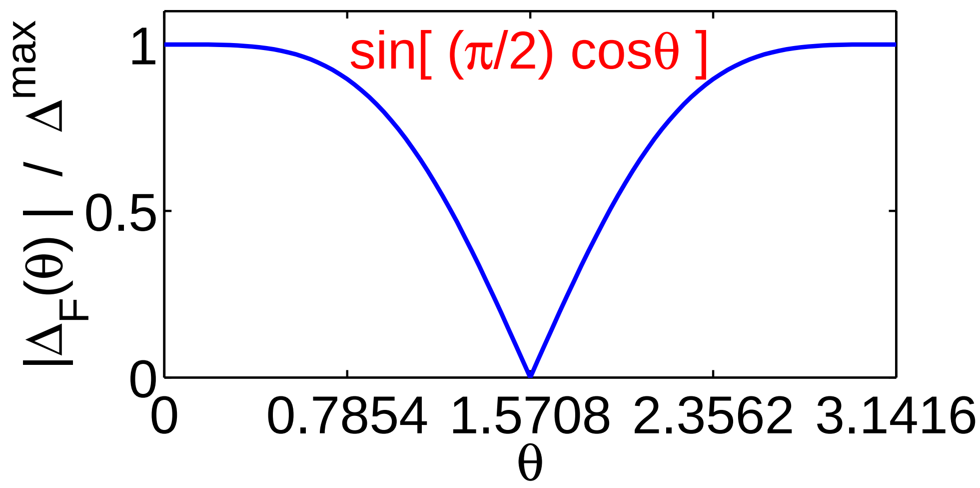
- $\uparrow\downarrow$gas: Appreciable energy cost of excitations always.

BCS gap away from Fermi surface



MB, Mar’enko, Rychkov, GS, PRA **66**, 013606 (2002)

Consequences of pole in Δ



	$\uparrow\downarrow$ gas	dipoles
dispersion	isotropic	anisotropic
damping of sound at $T = 0$	0	nonzero
Specific heat at low T	$\sim \exp(-\Delta/T)$	$\sim T^2$
normal component at low T	$\sim \exp(-\Delta/T)$	polynomial in T

(5A) Collective excitations

Low energy modes

Phase perturbations of the ground state order parameter (Goldstone mode)

$$\boxed{\Delta_0(x-y) \quad \rightarrow \quad \Delta(x,y) = \Delta_0(x-y)\, e^{2i\phi(x,t)}}$$

Assumptions:

- Low energy ($\hbar\omega \ll \Delta_0^{\max}$)
- Phase perturbations only (amplitude perturbations are gapped)
- Low ω $\Longrightarrow$ long wavelength ($k \ll k_F$)
  $\Longrightarrow$ insensitive to small-scale of $|x-y|$ $\Longrightarrow$ $\phi \approx \phi(x$ only $)$
- Weak perturbation $\Longrightarrow$ lowest order in ϕ

Perturbation

Single-particle wavefunctions $U_\nu(\mathbf{r},t)$ and $V_\nu(\mathbf{r},t)$ from a Bogoliubov diagonalization

$$\widehat{\Psi}(\mathbf{r}) = \sum_\nu \left[U_\nu(\mathbf{r})\widehat{b}_\nu + V_\nu(\mathbf{r})^* \, \widehat{b}_\nu^\dagger \right]$$

obey BDG equations

$$i\hbar\frac{\partial}{\partial t}\begin{bmatrix} U_\nu(\mathbf{r}) \\ V_\nu(\mathbf{r}) \end{bmatrix} = H_0(\mathbf{r})\begin{bmatrix} U_\nu(\mathbf{r}) \\ -V_\nu(\mathbf{r}) \end{bmatrix} - \int d^3\mathbf{r}' \begin{bmatrix} \Delta(\mathbf{r},\mathbf{r}')V_\nu(\mathbf{r}') \\ \Delta^*(\mathbf{r},\mathbf{r}')U_\nu(\mathbf{r}') \end{bmatrix}$$

Expand them in terms of the uniform-gas wavefunctions $U^0(\mathbf{r})$ and $V^0(\mathbf{r})$ and coefficients $C^{(\eta)} \sim O(\phi)$

$$\begin{bmatrix} U_\nu(\mathbf{r}) \\ V_\nu(\mathbf{r}) \end{bmatrix} = \sum_j \left\{ (\delta_{j\nu} + C_{j\nu}^{(1)}) \begin{bmatrix} U_\nu^0(\mathbf{r}) \\ V_\nu^0(\mathbf{r}) \end{bmatrix} + C_{j\nu}^{(2)} \begin{bmatrix} V_\nu^0(\mathbf{r})^* \\ -U_\nu^0(\mathbf{r})^* \end{bmatrix} \right\},$$

finf $C^{(\mu)}$ from BDG equation, and substitute it all into Gap equation, which must be satisfied up to $O(\phi)$.

$$\Delta(\mathbf{r},\mathbf{r}') = \frac{V_D(\mathbf{r}-\mathbf{r}')}{2}\sum_\nu \tanh\left(\frac{E_\nu}{2k_BT}\right)\left[U_\nu(\mathbf{r})V_\nu^*(\mathbf{r}') - U_\nu(\mathbf{r}')V_\nu^*(\mathbf{r})\right]$$

Consistency equation in k-space

$$-\frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}\tau^0_{\mathbf{M}}}{2E^0_{\mathbf{M}}} = \frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}}{4E^0_{\mathbf{m}}E^0_{\mathbf{n}}}\left\{\left(\frac{\tau^0_{\mathbf{n}}-\tau^0_{\mathbf{m}}}{2}\right)\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0} - \frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]\right.$$
$$\left. +\tau^0_{\mathbf{n}}\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right] - \tau^0_{\mathbf{m}}\left[\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}\right]\right\}.$$

where $\mathbf{n}=\mathbf{M}+\mathbf{k}/2$, $\mathbf{m}=-\mathbf{M}+\mathbf{k}/2$, $\varepsilon_{\mathbf{k}}=\hbar^2k^2/2m-E_F$, $E^0_{\mathbf{k}}=\sqrt{\varepsilon^2_{\mathbf{k}}+(\Delta^0_{\mathbf{k}})^2}$, and $\tau^0_{\mathbf{k}}=\tanh(E^0_{\mathbf{k}}/2T)$.

- Landau processes ($E+\omega \leftrightarrow E'$ — 1st line) and Beliaev processes ($E+E' \leftrightarrow \omega$ — 2nd line).
- LONG wavelength $\mathbf{k}$, SHORT wavelength $\mathbf{M}$.
- Similar form to $\uparrow\downarrow$gas, but there's a practical PROBLEM . . .

Practical problem

- For any long wavelength $\mathbf{k}$ of $\phi_{\mathbf{k}}$, there are many solutions with different ω, parametrised by the wavenumber $\mathbf{M} \sim k_F$ from $\Delta^0_{\mathbf{M}}$.

- Experiments can control/perturb/see long wavelengths $\mathbf{k}$, but not $\mathbf{M}$, which is an internal microscopic parameter at high energy $\sim \mu$

- Presumably, if you perturb system externally with wavenumber $\mathbf{k}$ the result will be some weighted average over all $\mathbf{M}$ solutions.

- But what are the weights?

The solution — an effective Lagrangian

1. In the action integral formulation of quantum mechanics where $\langle \hat{O} \rangle = \int \mathcal{D}^2\Delta, \mathcal{D}^2\Psi\, e^{iS/\hbar} O[\Delta, \Psi, \mathrm{c.c}]$ etc., write down an action $S(\Delta, \Psi)$ so that its saddle point $\partial S / \partial \{\Delta, \Psi\} = 0$ gives the full BCS theory.

2. Substitute perturbation $\Delta \longrightarrow \Delta_0 e^{2i\phi}$ to give $S(\Delta_0, \phi, \Psi)$.

3. An effective action S_{eff} for the small perturbation ϕ is obtained by integrating over the irrelevant variables Ψ.

4. get $S_{\mathrm{eff}}(\phi, \Delta_0, \Psi_0) = -i\hbar \log \left[\langle e^{i\delta S/\hbar} \rangle \right]$ where Ψ_0 is the unperturbed ground state wavefunction.

5. Consistency equation for ϕ is given by the saddle-point solution $\partial S_{\mathrm{eff}} / \partial \phi = 0$.

6. Weights turn out to be $\Delta^0_{\mathbf{M}}$.

(5B) Predictions

(diagrams - hooray)

$T = 0$ Superfluid

Find Bogoliubov sound, same as for the standard $\uparrow\downarrow$BCS gas

$$\boxed{\omega = \left(\frac{v_F}{\sqrt{3}}\right)\, k}$$

To lowest order in $\omega \ll E_F/\hbar$ and $k \ll k_F$.

Not too surprising from hydrodynamics . . .

$T = 0$ Hydrodynamics

Relies on the hydrodynamic Hamiltonian for superfluid velocity v_s

$$H \approx \int d^3x \left\{ \frac{1}{2} m\rho v_s(x)^2 + U(\rho) \right\}$$

and the continuity and current equations

$$\vec{v}_s = \frac{\vec{J}_s(x)}{\rho} = \frac{\hbar}{m}\,\rho\,\vec{\nabla}\phi(x) \qquad \text{and} \qquad \vec{\nabla}\cdot\vec{J}_s(x) = -\frac{\partial\rho}{\partial t}$$

which are found to be the same for dipoles and short-range gases to order $O(\Delta^{\mathrm{max}}/E_F)$.

Since $U(\rho)$ arises overwhelmingly from the filled Fermi sphere,

$\Longrightarrow$ interaction details have minor effect locally

(But give leading corrections to $\hbar\omega$ by flattening the Fermi sphere)

Beyond hydrodynamics

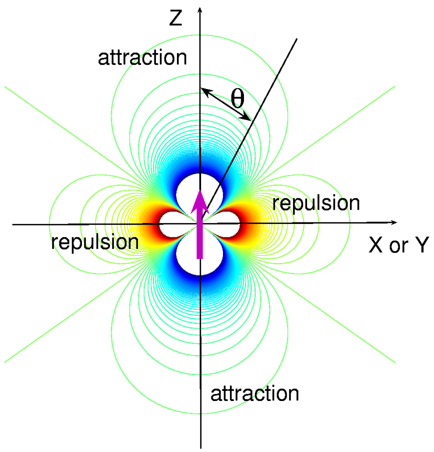
$T=0$ Anisotropic damping of sound

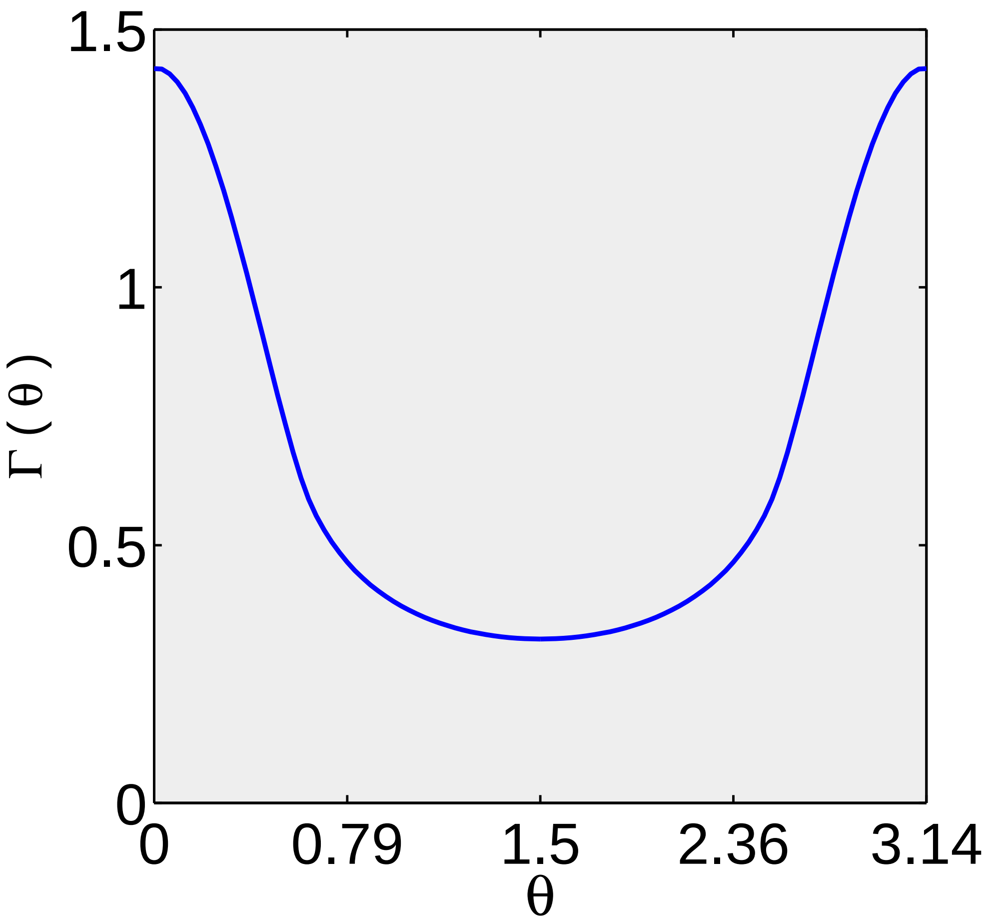
$$\omega = \left(\frac{v_F}{\sqrt{3}}\right) k \left\{1 - i \left(\frac{\hbar\omega_{\text{Bog}}}{\Delta_{\text{max}}}\right) \Gamma(\theta)\right\}$$

absent for standard $\uparrow\downarrow$ gas

Beliaev process:

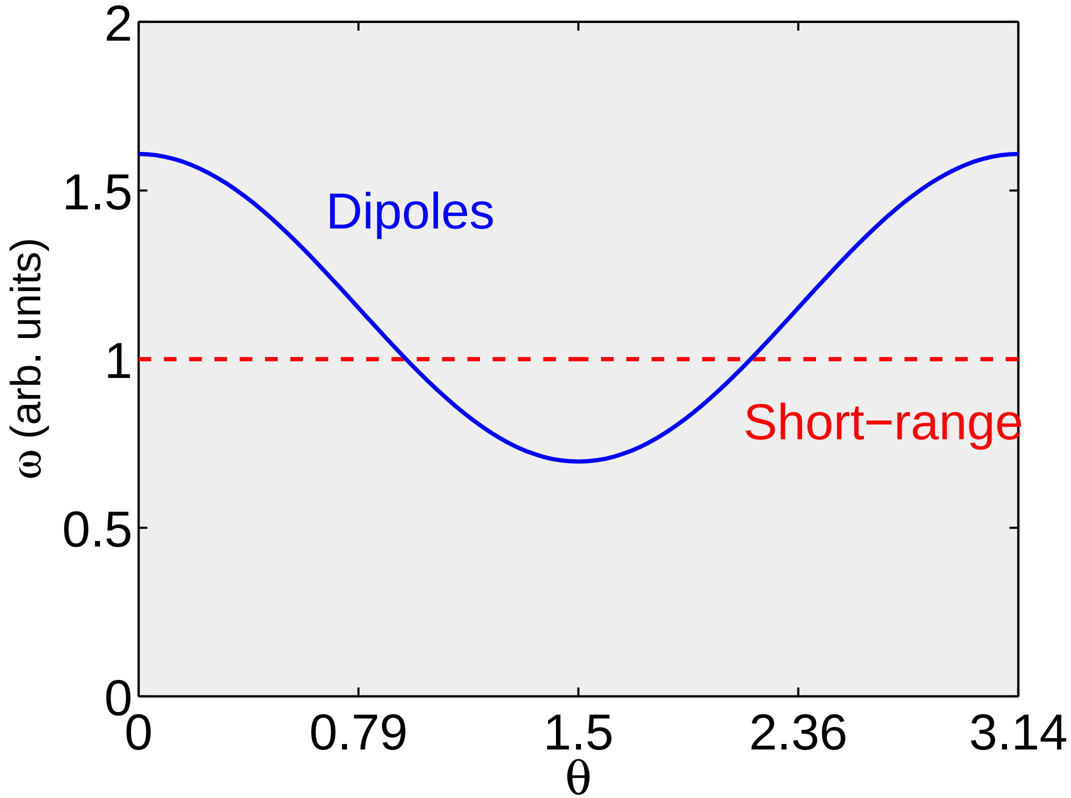
collective $\Longrightarrow$ $2\times$quasipart.

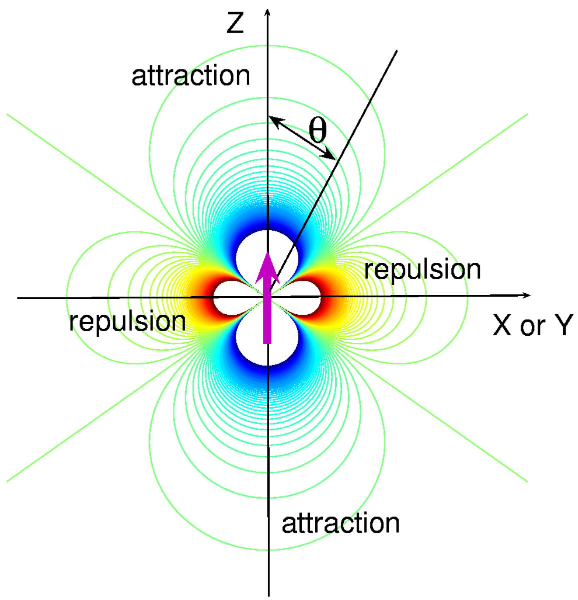




$T \approx T_c$ behaviour

$$\omega = -i \left(\frac{7\zeta(3)}{6\pi^3}\right) \left(\frac{\hbar v_F^2}{T_c}\right) k^2 \left(1 + \frac{3}{2\pi^2}(1 + 3\cos 2\theta)\right)$$





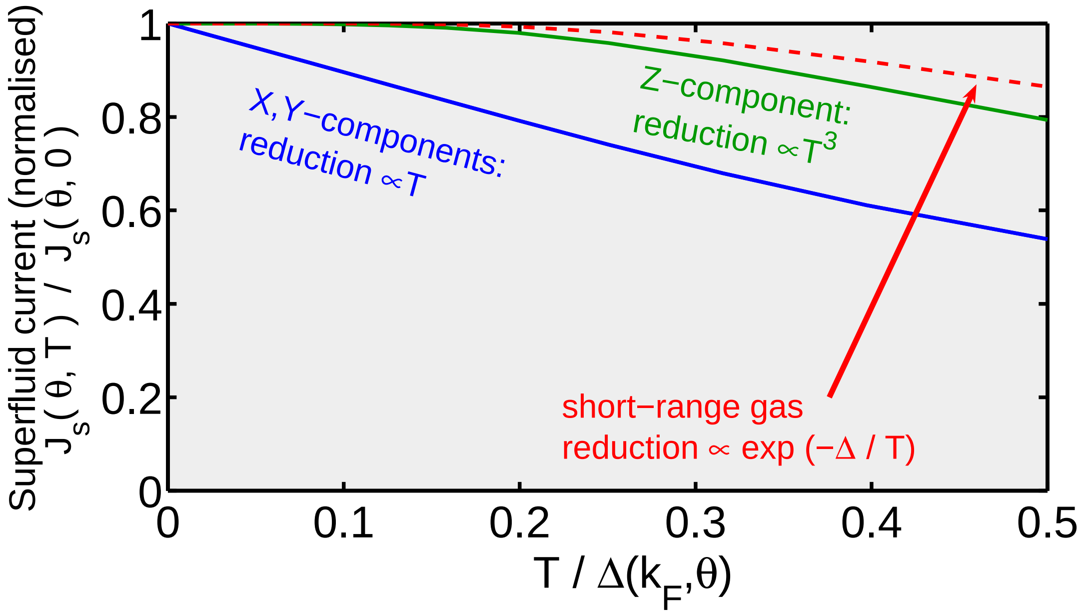
- Purely diffusive (as for standard short-range ↑↓gas)
- Anisotropic (differently to ↑↓gas)

Veering superfluid current $0 < T < T_c$

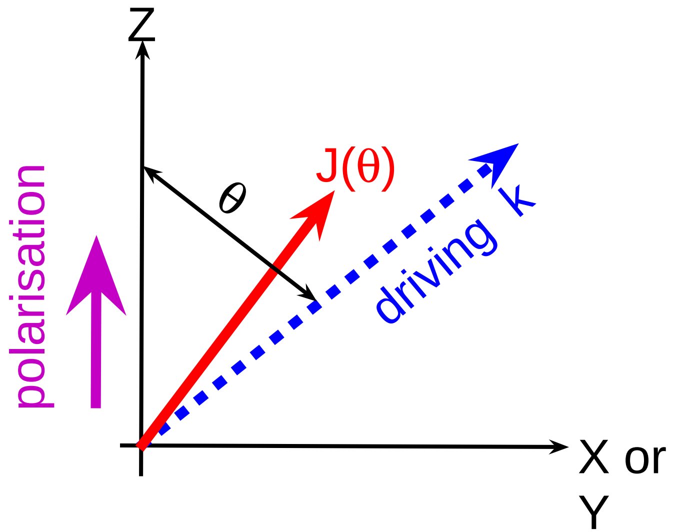
- Current response J_s to an external phase perturbation of the gap

$$\Delta(x,y,t) = \Delta_0(x-y)e^{2i\phi(x,t)}$$

- Strable driving frequency ω, wave-vector k, in direction θ.



Veering current



Direction-dependent superfluid

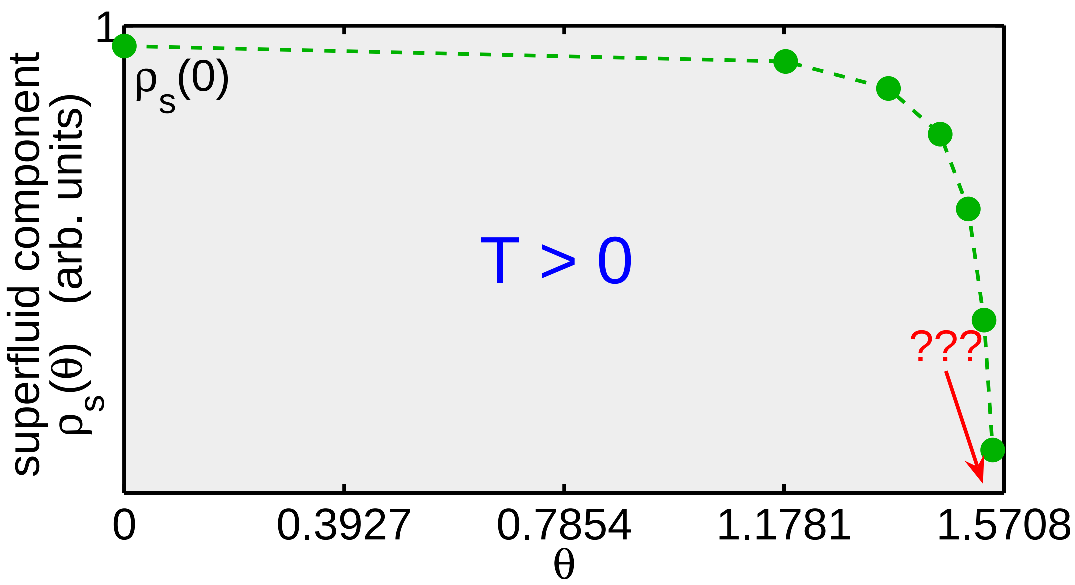
(tentative)

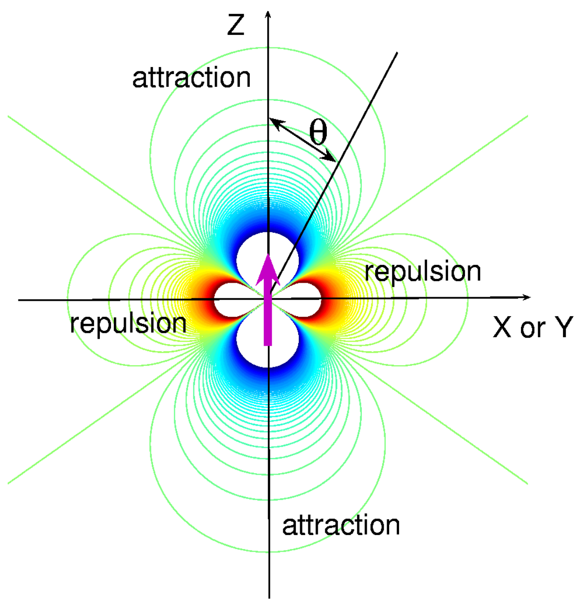
Can define direction-dependent "normal" and "superfluid" components

$$\rho = \rho_n(\theta) + \rho_s(\theta)$$

so that the usual current equation applies to within a modulus:

$$|\vec{J}_s| \;=\; \frac{\hbar}{m}\,\rho_s\,|\vec{\nabla}\phi|$$





Potential related research

- analogues with phases known in Helium
- Are there other low energy modes? - e.g. from perturbation of the polarisation axis.
- What's going on with the current near $\theta = \pi/2$.
- Are the Δ-amplitude modulation modes low-energy near $\theta = \pi/2$?
- Are there interesting low energy perturbations of the discarded Hartree field $W(x,y)$?

Merci!