

# Stationary, dynamic and thermal properties of quantum droplets

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$$\delta g < 0, \alpha > 0$$

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Bogoliubov spectrum of uniform density (local density approximation)

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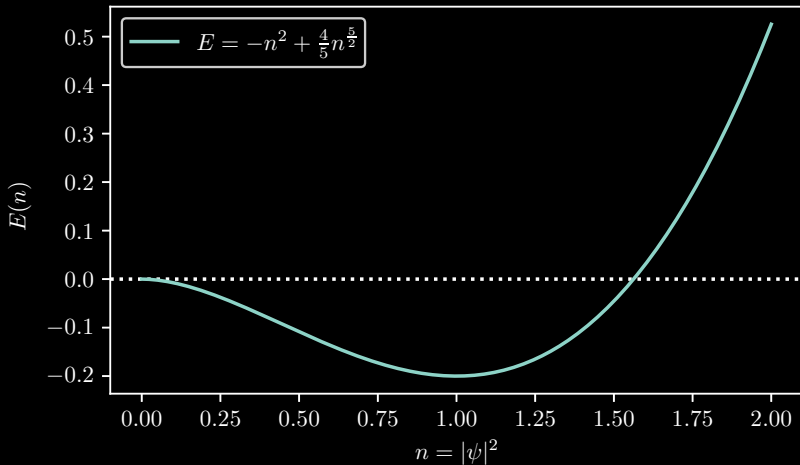
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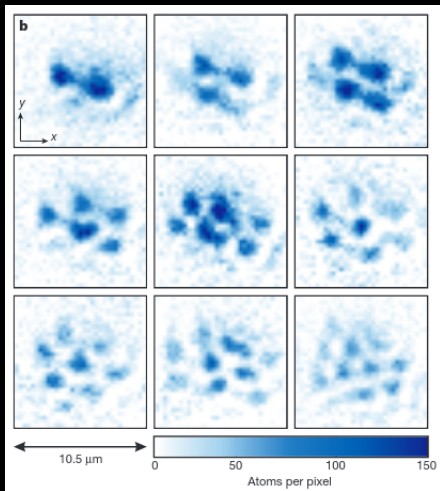
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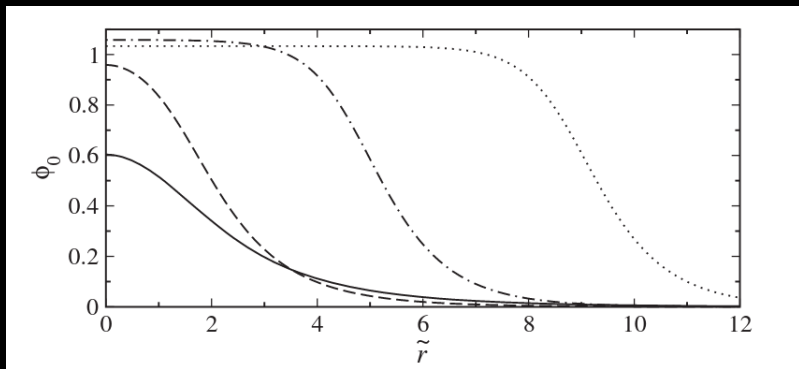
Finite number of particles at preferred density  $\implies$  finite volume

# Unexpected discovery



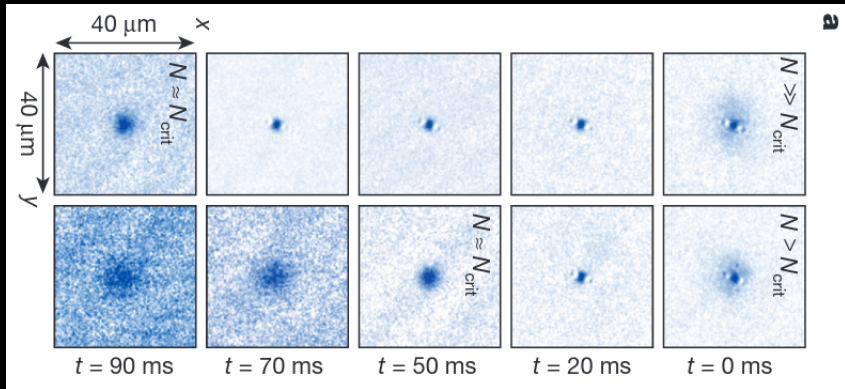
H. Kadau et al., Nature 530, 194 (2016)

# Theoretical prediction



D. S. Petrov, Phys. Rev. Lett. 115, 155302 (2015)

# Confirmation



M. Schmitt et al., Nature 539, 259 (2016)

# Extended Gross–Pitaevskii equation

$$i\partial_t\phi = \left(-\frac{\nabla^2}{2} - 3|\phi|^2 + \frac{5}{2}|\phi|^3 - \mu\right)\phi$$
$$\phi = \phi(x/\xi, t/\tau)$$

For  $^{39}\text{K}$  atoms:

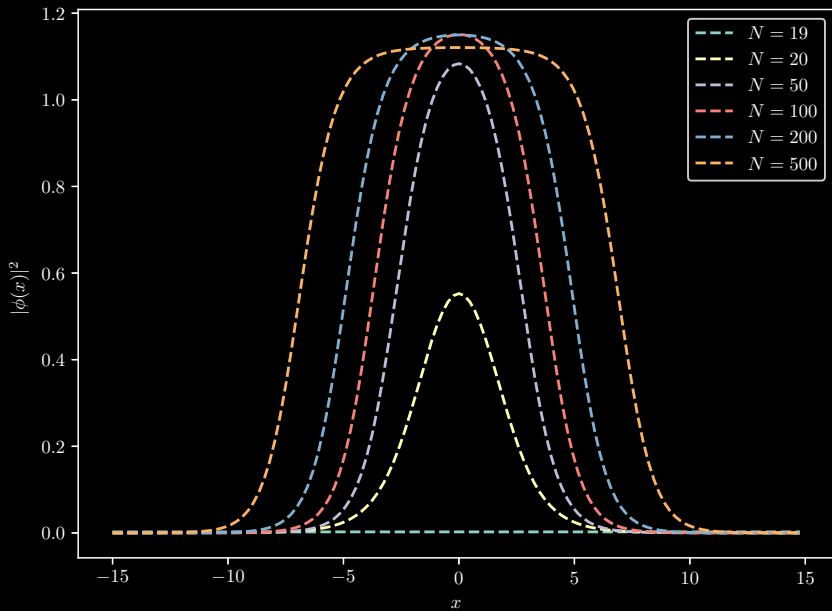
coordinate units:

$$\xi \approx 2\mu\text{m}$$

time units:

$$\tau \approx 2.5\text{ms}$$

# 3D droplets



2 component Bose-Bose mixture

$$E = E_0 + \frac{1}{2} \sum_{i,j} g_{ij} n_i n_j$$

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$$g_{ii} > 0 \wedge g_{12}^2 < g_{11}g_{22} \implies \text{stable (gas)}$$



## 2 components

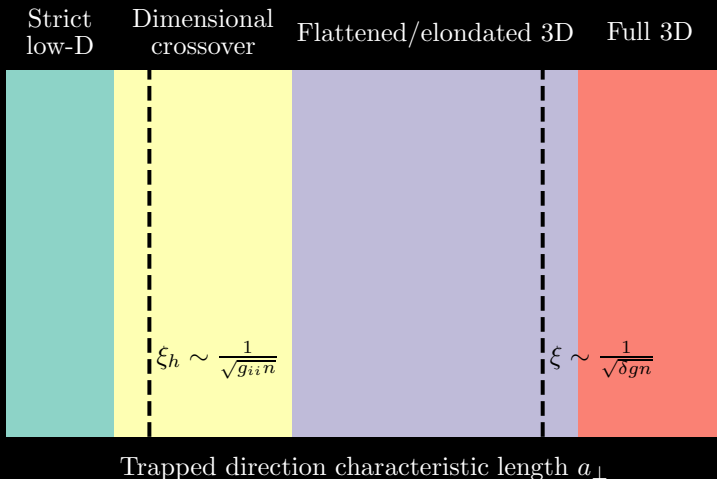
$$\delta g = g_{12} + \sqrt{g_{11}g_{22}}$$

$$\delta g < 0 \wedge g_{ii} > 0 \wedge |\delta g| \ll g_{ii}$$



Droplets!

# Sausages and pancakes: an effective low dimensional description



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D. S. Petrov and G. E. Astrakharchik, Phys. Rev. Lett. 117, 100401 (2016)

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Dimensional crossover

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Flattened/elongated 3D

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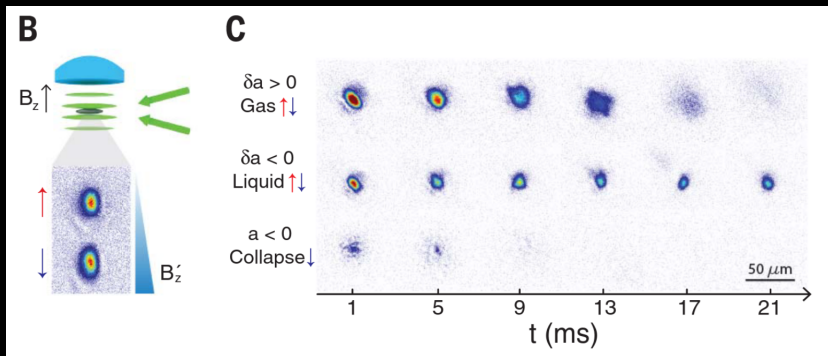
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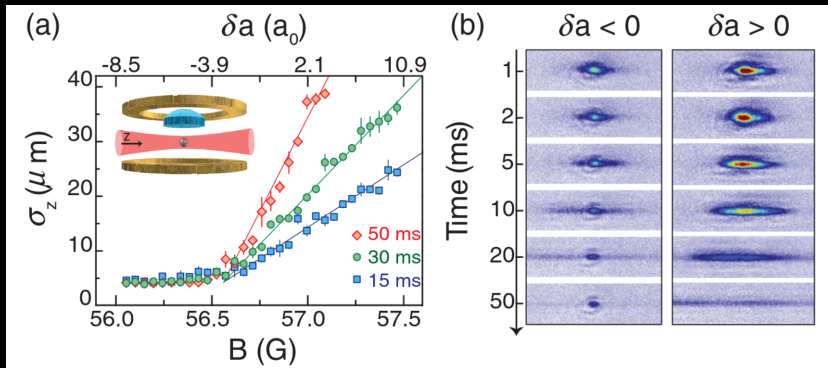
# Sausages and pancakes: an effective low dimensional description



C. R. Cabrera, L. Tanzi, J. Sanz, B. Naylor, P. Thomas, P. Cheiney, and L. Tarruell, *Science* 359, 301 (2018)

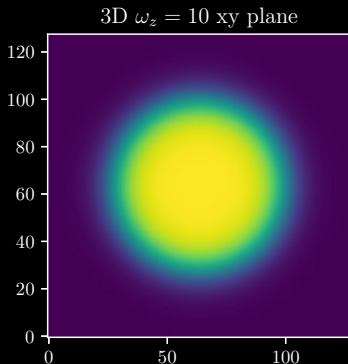
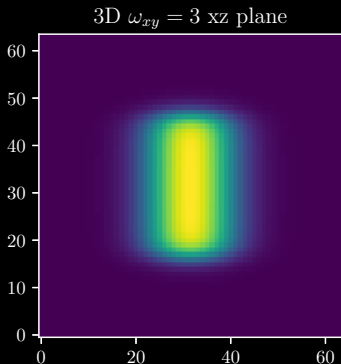


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P. Cheiney et al., Phys. Rev. Lett. 120, 135301 (2018)

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2D

$$\phi(x, y, z) = \phi_{2D}(x, y)\phi_{\text{gaussian}}(z)$$

1D

$$\phi(x, y, z) = \phi_{\text{gaussian}}(x, y)\phi_{1D}(z)$$

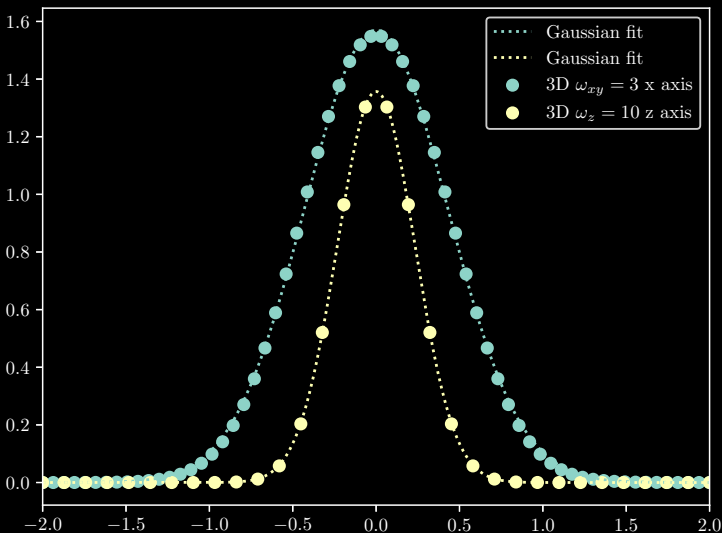
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$$i\partial_t\phi = \left(-\frac{\nabla^2}{2} - 3|\phi|^2 + \frac{5}{2}|\phi|^3 - \mu\right)\phi$$

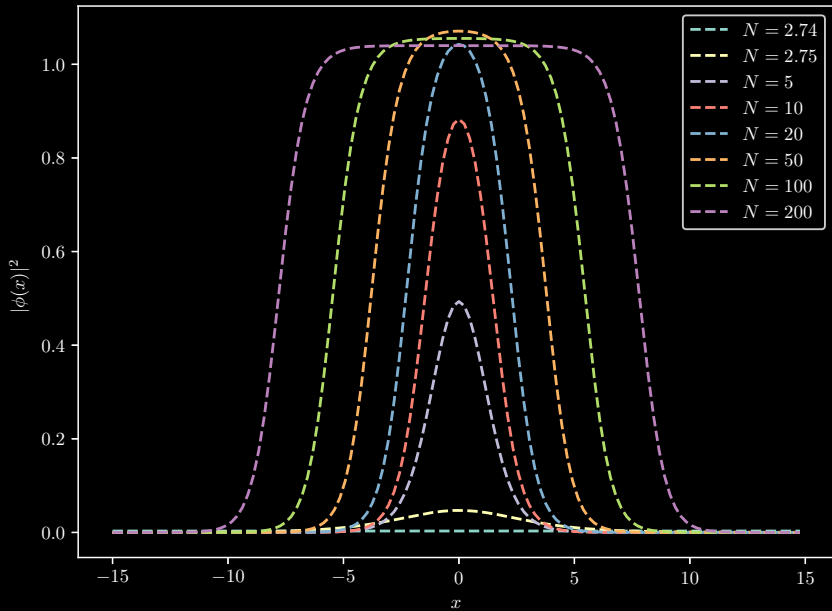
$$i\partial_t\phi_{2D} = \left(-\frac{\nabla^2}{2} - g_2(\omega)|\phi_{2D}|^2 + c_2(\omega)|\phi_{2D}|^3 - \mu\right)\phi_{2D}$$

$$i\partial_t\phi_{1D} = \left(-\frac{\nabla^2}{2} - g_1(\omega)|\phi_{1D}|^2 + c_1(\omega)|\phi_{1D}|^3 - \mu\right)\phi_{1D}$$

# Sausages and pancakes: an effective low dimensional description



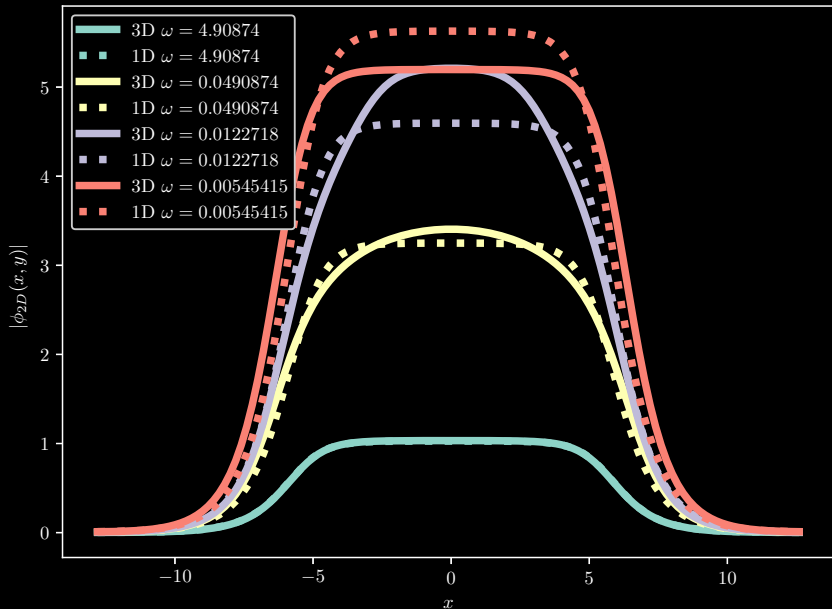
# Effective 2D (flattened 3D)



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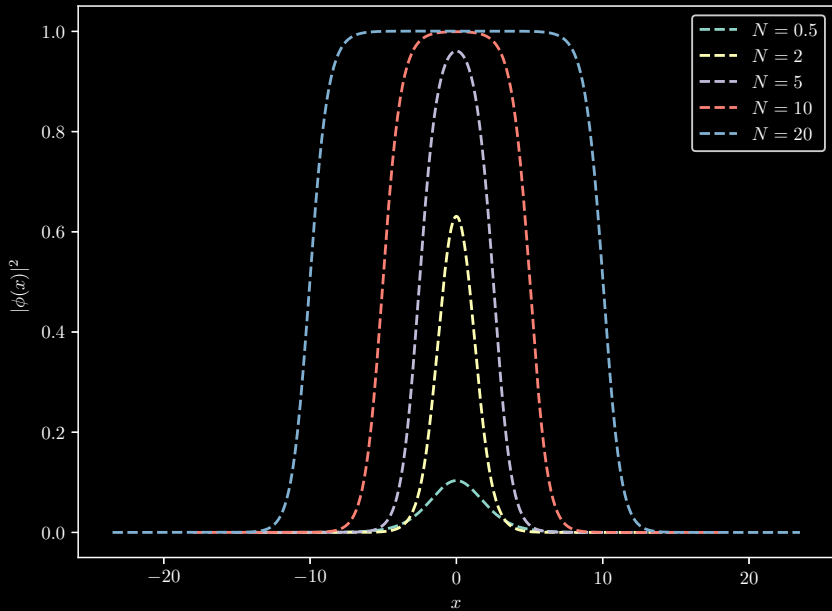
$$2.74 < N_c < 2.75$$

# Effective 2D (flattened 3D)

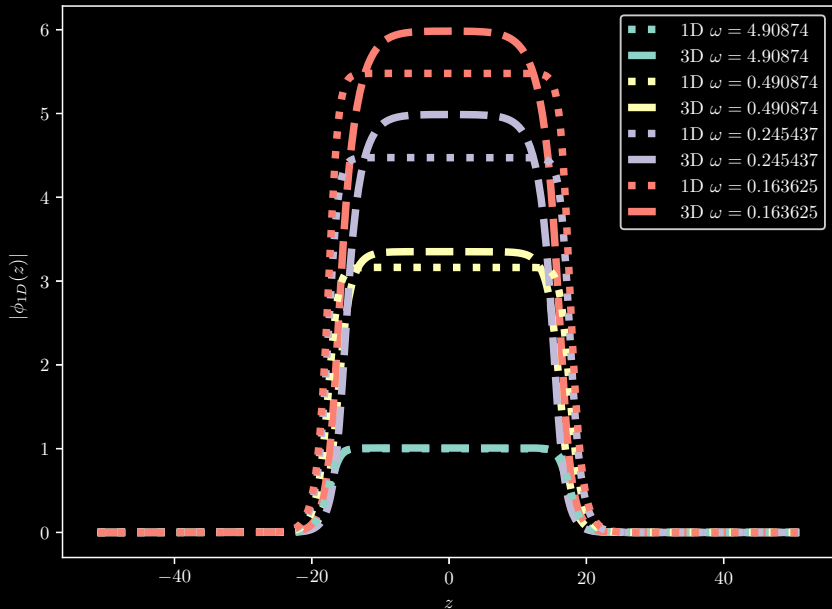




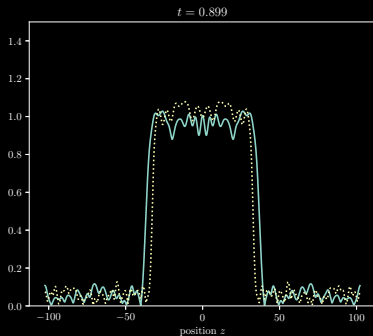
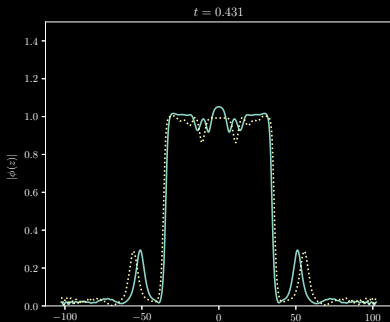
# Effective 1D (elongated 3D)



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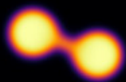


# Effective 1D dynamics

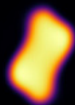
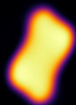
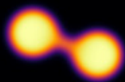


# Effective 2D dynamics

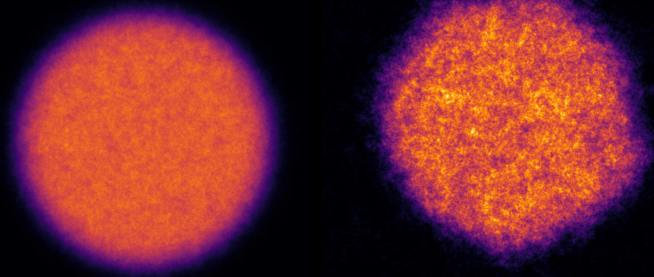
quasi-2D



3D



# Thermal properties



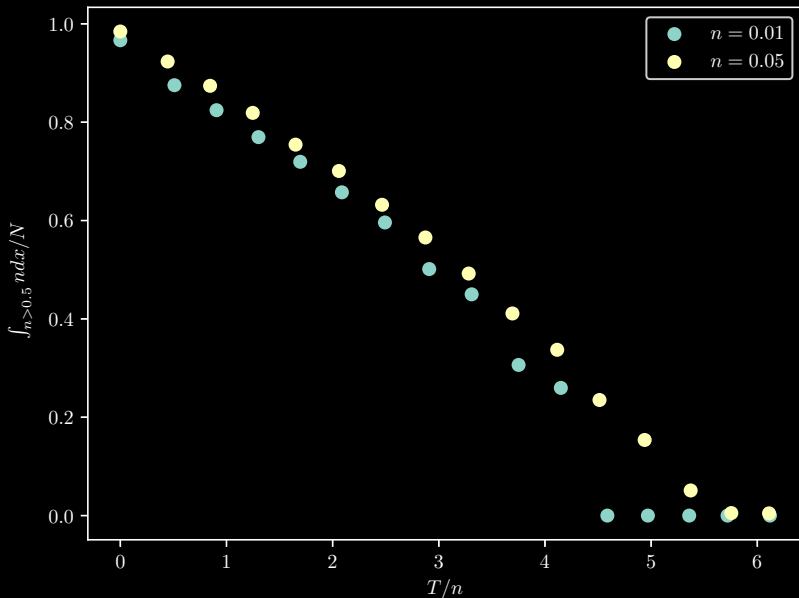
Modified Projected Stochastic GPE:

$$\begin{aligned}i\partial_t\phi &= \mathcal{P}_C\left((1 - i\gamma)\left(-\frac{\nabla^2}{2} - 3|\phi|^2 + \frac{5}{2}|\phi|^3 - \mu\right)\phi\right. \\ &\quad \left.- \frac{\gamma T}{\sigma^2}(N(\phi) - \bar{N})\phi\right. \\ &\quad \left.+ \sqrt{2\gamma T}\eta(x, t)\right)\end{aligned}$$

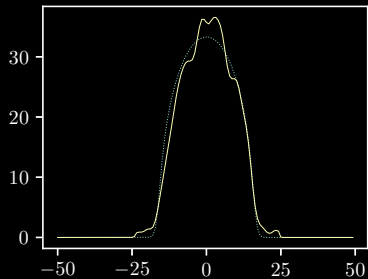
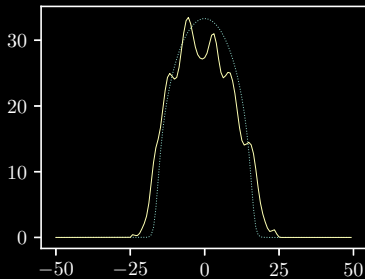
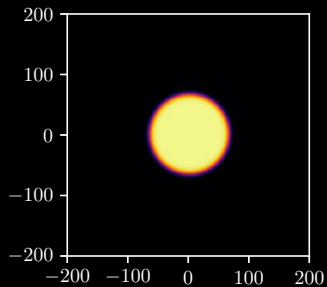
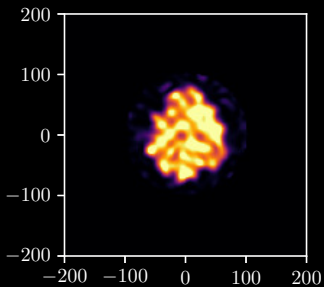
$$C = |\mu| + (\log 2)T$$

complex Gaussian noise:  $\eta, \langle \eta^*(x, t)\eta(x', t') \rangle = \delta(x - x')\delta(t - t')$

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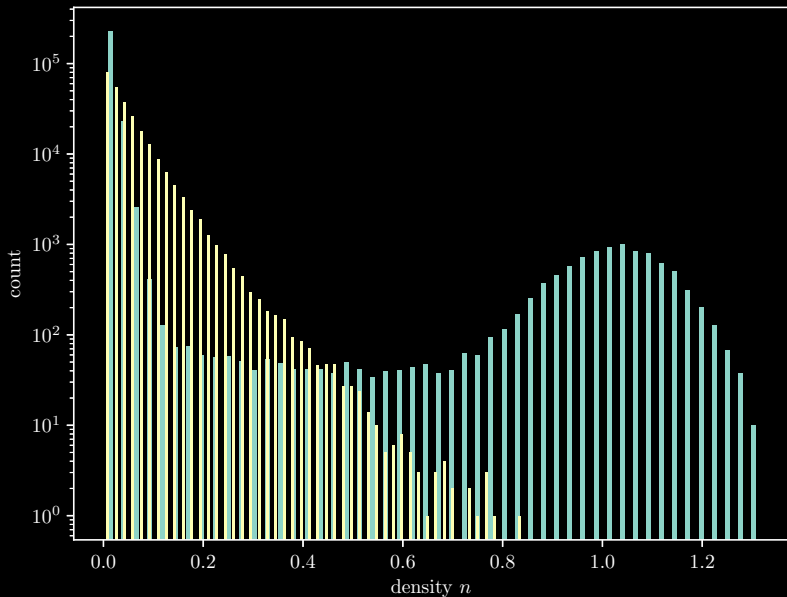


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