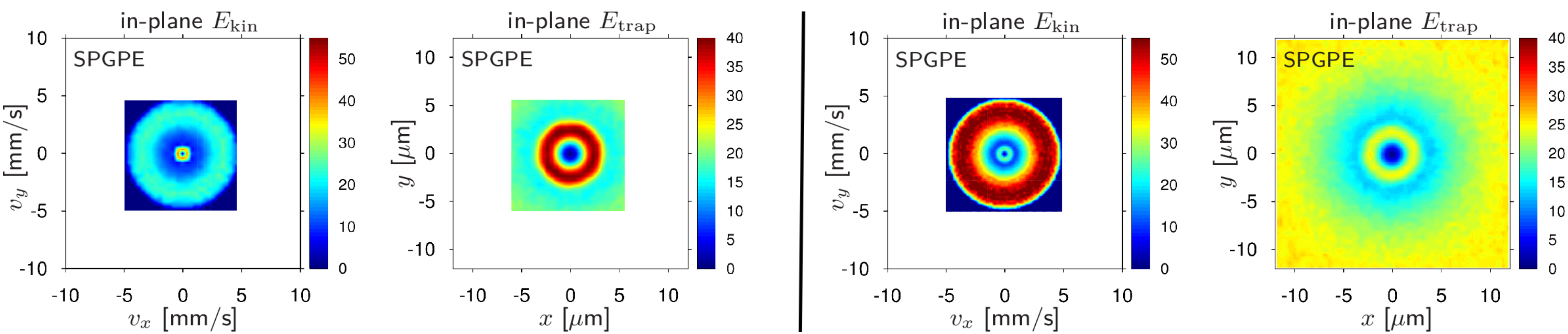
A semiclassical theory free of the curses of UV divergence and cutoff dependence

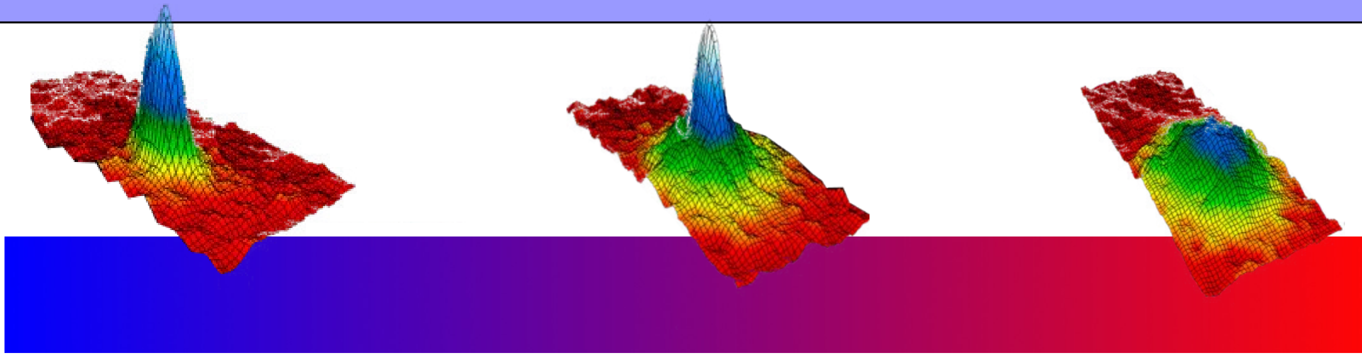
Piotr Deuar

Joanna Pietraszewicz

Institute of Physics, Polish Academy of Sciences, Warsaw, Poland



Thermal clouds - Classical fields essential



T=0	*Bogoliubov*	*Classical fields, ZNG*	*T=T$_c$*
1 mode GPE	few modes perturbative	zillions of modes non-perturbative	

Bose field

$$\hat{\Psi}(\mathbf{x}) = \sum_j \hat{a}_j \psi_j(\mathbf{x}) \longrightarrow \phi(\mathbf{x}) = \left\{ \sum_{j\in\mathcal{C}} \alpha_j \psi_j(\mathbf{x}) \right\}$$

Assuming high occupation:

$$\left[\hat{a}_j, \hat{a}_k^\dagger\right] = \delta_{jk}$$

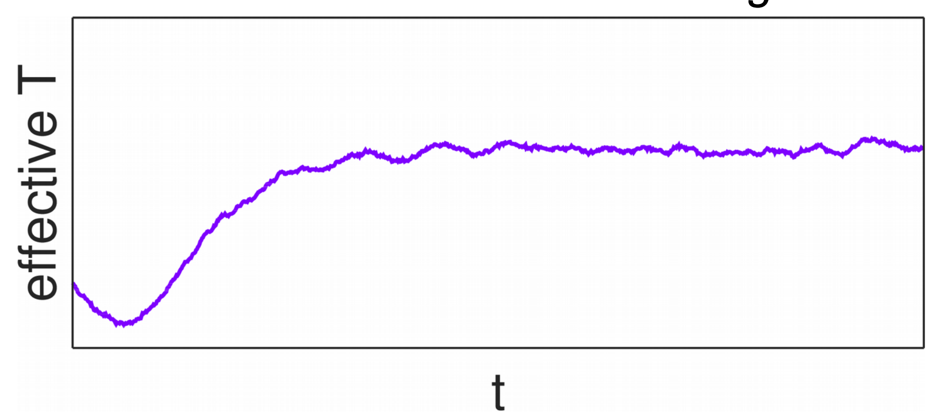
$$\hat{a}_j \gg 1 \;\rightarrow \hat{a}_j \approx \alpha_j$$

Evolution: GPE

$$\hbar \frac{d\phi(\mathbf{x})}{dt} = -i\mathcal{E}(\mathbf{x})\phi(\mathbf{x})$$

$$\mathcal{E}\phi(\mathbf{x}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g\,|\phi(\mathbf{x})|^2\right]\phi(\mathbf{x})$$

Thermalised ensemble at long time

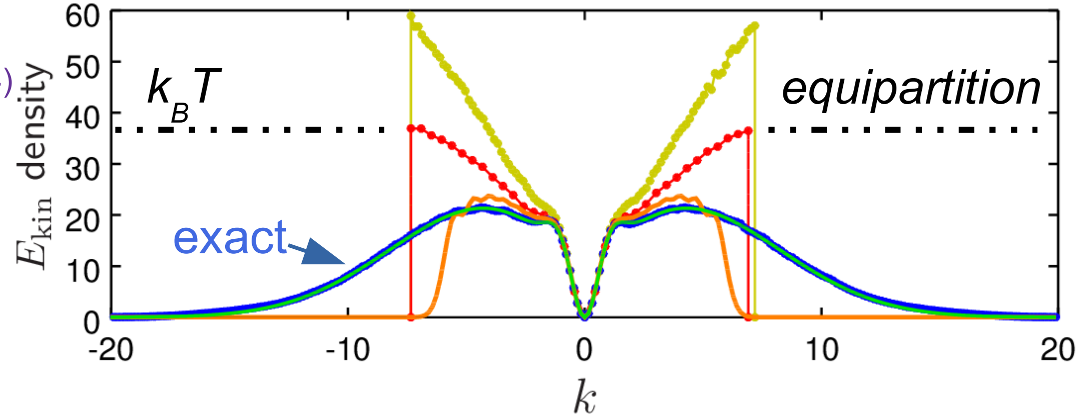


Age-old problem: cutoff dependence

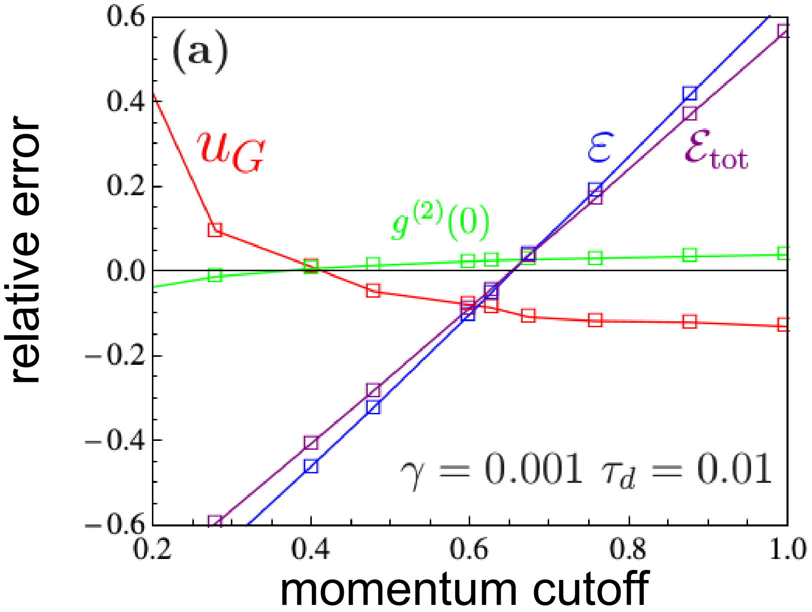
Studied by many:

Witkowska, Gajda, Rzążewski, PRA **79**, 033631 (2009)
Karpiuk, Brewczyk, Gajda, Rzążewski, PRA **81**, 013629 (2010)
Zawitkowski, Brewczyk, Gajda, Rzążewski, PRA **70**, 033614 (2004)
Bradley, Blakie, Gardiner, J Phys B **38**, 4259 (2005)
Cockburn, Proukakis, PRA **86**, 033610 (2010)
and the list goes on ...

Kinetic energy distribution, ideal gas in trap

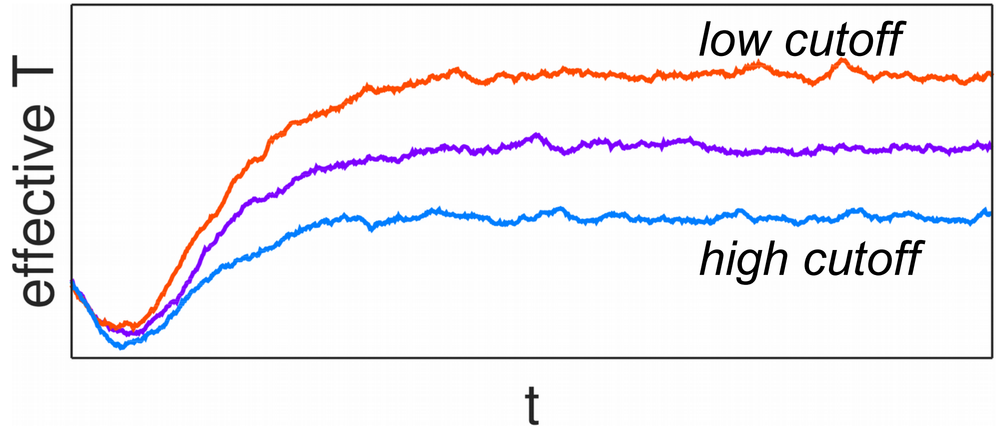


Most recent study: different observables



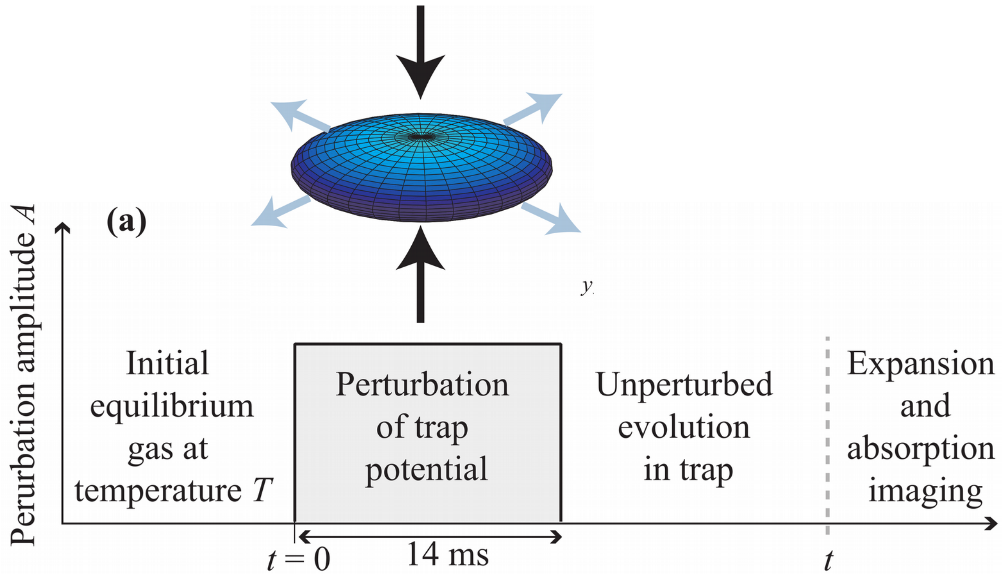
Pietraszewicz, PD, PRA **92**, 063620 (2015)
Pietraszewicz, PD, PRA **97**, 053607 (2018)
Pietraszewicz, PD, PRA **98**, 023622 (2018)

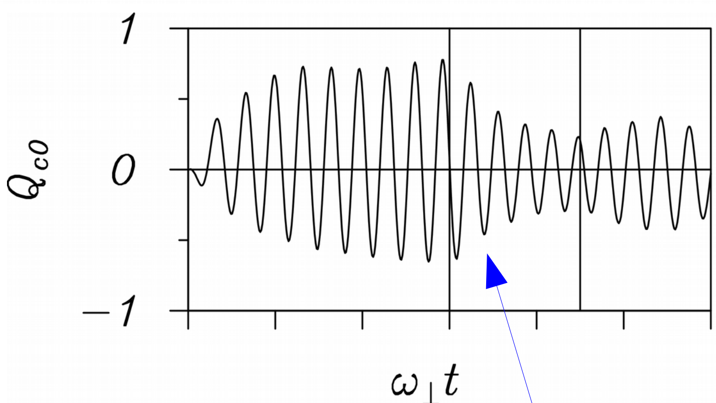
Equilibrium temperature depends on cutoff



Litmus test for thermal cloud: *m=0* collective mode

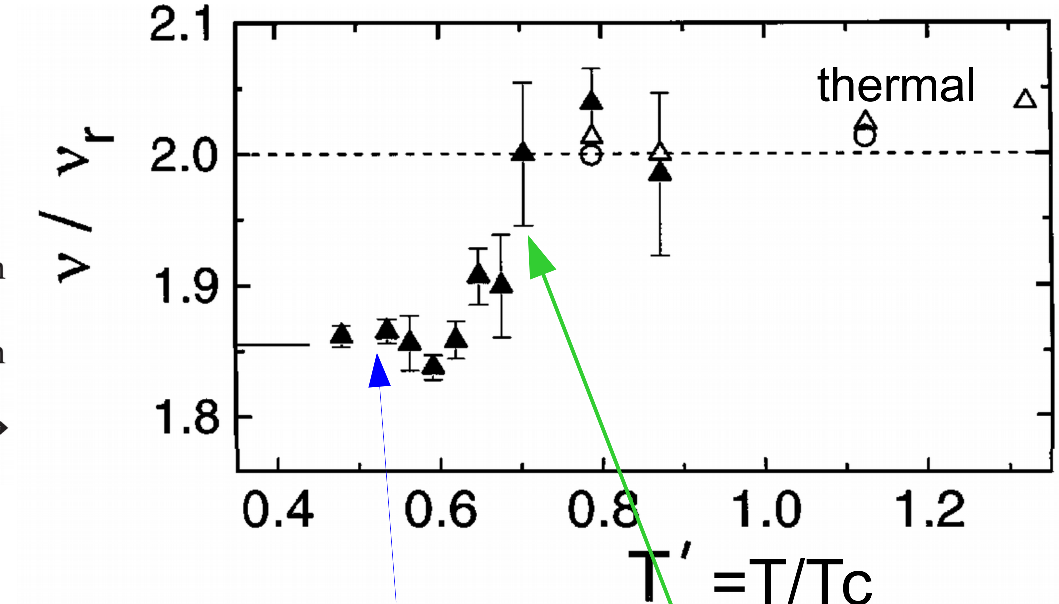
Ancient evil experiment: Jin, Matthews, Ensher, Wieman, Cornell, PRL 78, 764 (1997)





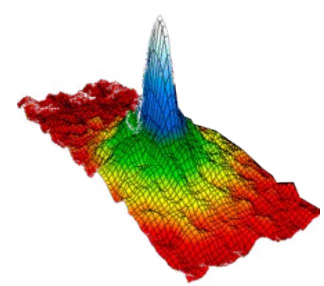
fit decaying sinusoid

adapted from: Jin *et al*, PRL 78, 764 (1997)



condensate oscillations

shift due to drag of thermal cloud on condensate



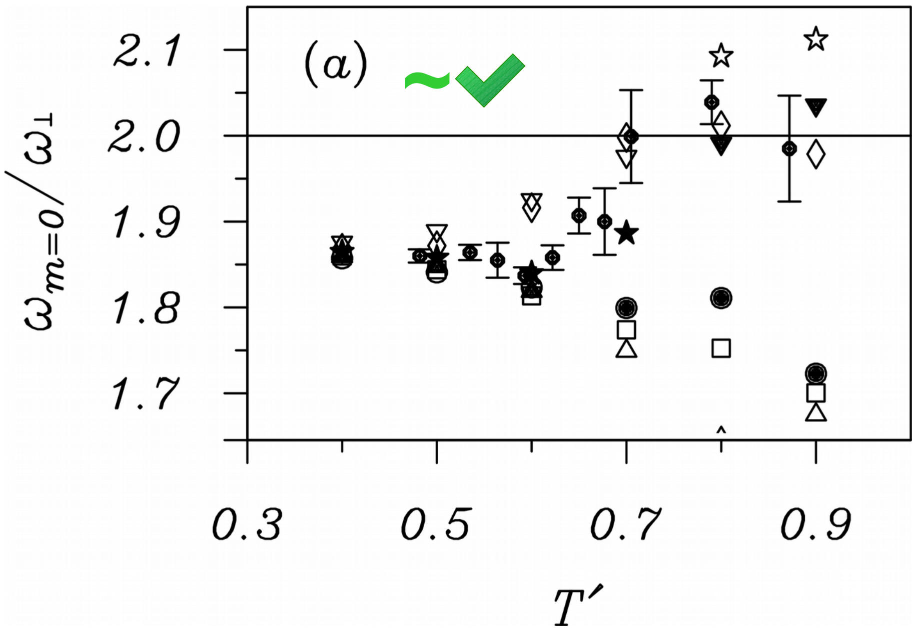
The above figures from:
Jackson, Zaremba, PRL 88, 180402 (2002)
Bezett, Blakie, PRA 79, 023602 (2009)

Results of *m=0* mode tests in the past

Targeted approaches

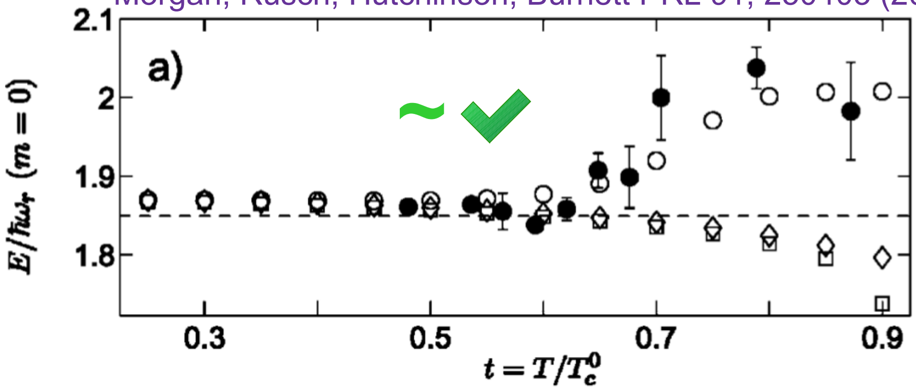
ZNG

Jackson, Zaremba, PRL 88, 180402 (2002)



Frequencies from "2nd order Bogoliubov"

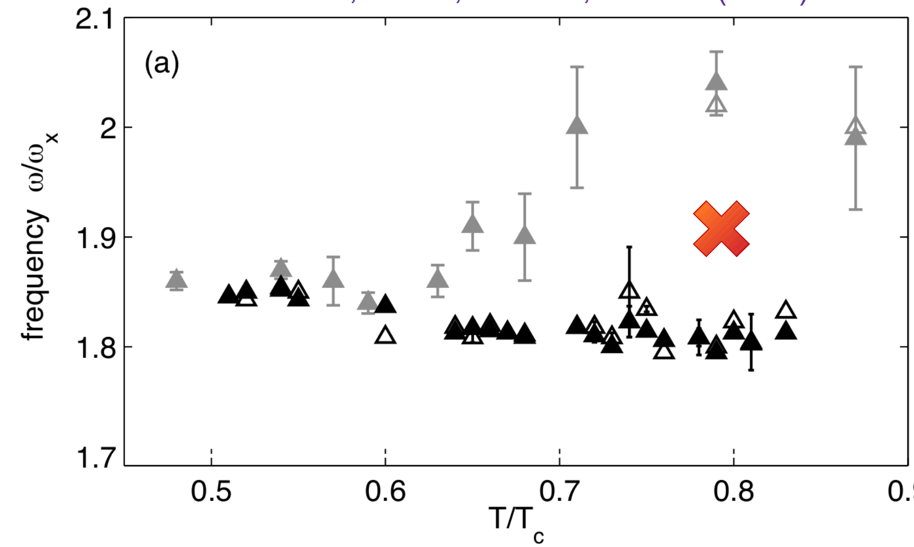
Morgan, Rusch, Hutchinson, Burnett PRL 91, 250403 (2003)



Flexible simulations

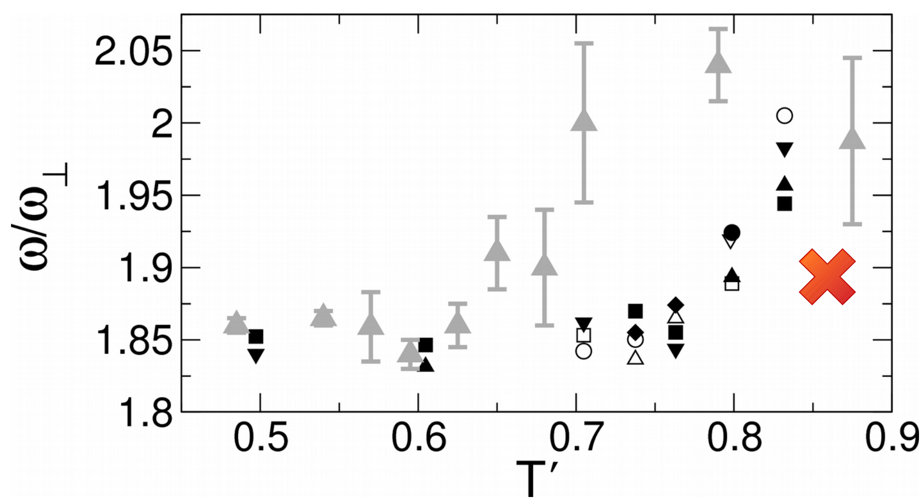
Classical fields + HF

Bezett, Blakie, PRA 79, 023602 (2009)



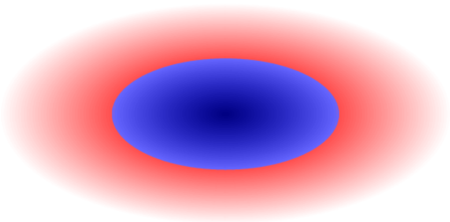
Classical fields (higher cutoff)

Karpiuk, Brewczyk, Gajda, Rzążewski, PRA 81, 013629 (2010)

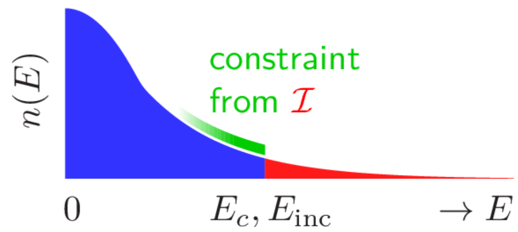


Stochastic Gross-Pitaevskii equation (SGPE)

real space



occupations



complex noise

$$\hbar \frac{\partial \phi(\mathbf{x})}{\partial t} = -i\mathcal{E}(\mathbf{x})\phi(\mathbf{x}) - \gamma\left[\mathcal{E}(\mathbf{x}) - \mu\right]\phi(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\,\eta(\mathbf{x},t)$$

Loss rate to “incoherent” tails

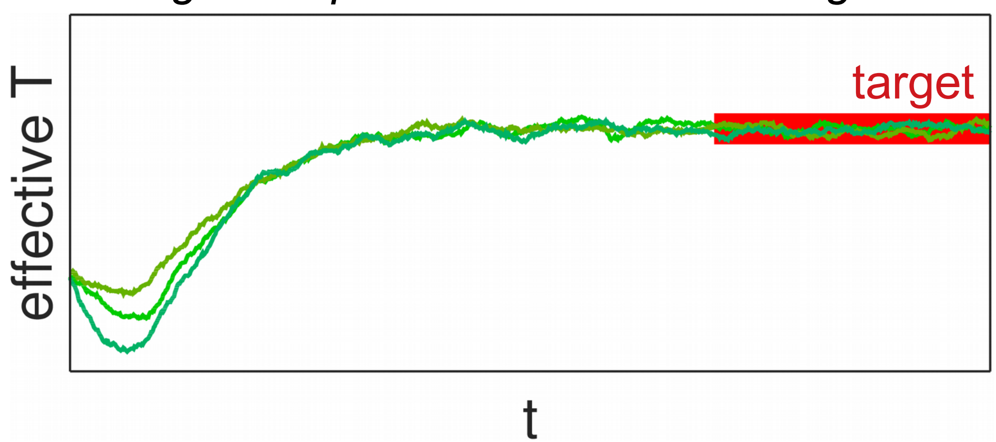
Incoherent growth from tails

Hamiltonian evolution of “coherent” field

$$\mathcal{E}\phi(\mathbf{x}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g\,|\phi(\mathbf{x})|^2\right]\phi(\mathbf{x})$$

BUT, still UV divergent

Target temperature reached at long time



Invokes a “classical field” linearisation of occupation in tails (correct when $\overline{N}(\omega) \gg 1$)

$$\overline{N}(\omega) \to \frac{k_B T}{\omega(\mathbf{x}) - \mu}$$

→ equipartition znowu :(

Use proper quantum occupations

$$\overline{N}(\omega) = N_{BE} = \left[e^{\frac{\omega(\mathbf{x})-\mu}{k_B T}} - 1 \right]^{-1}$$

real space

occupations

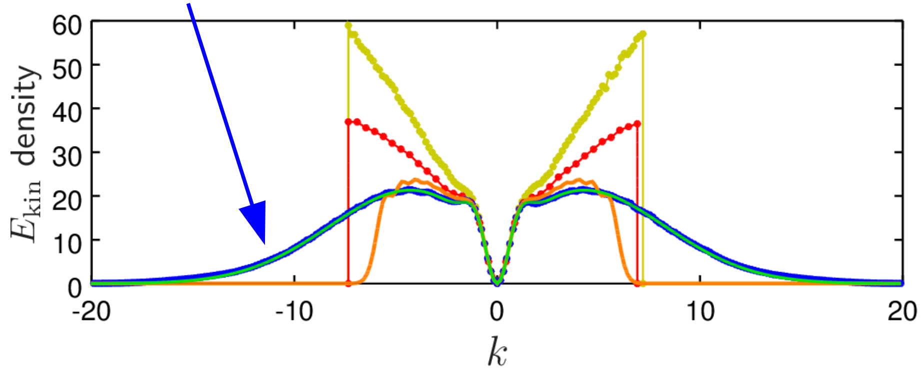
Obtain *regularised* SGPE

$$\hbar \frac{\partial \phi(\mathbf{x})}{\partial t} = -i\mathcal{E}(\mathbf{x})\phi(\mathbf{x}) - \gamma k_B T \left[e^{\frac{\mathcal{E}(\mathbf{x})-\mu}{k_B T}} - 1 \right] \phi(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\, \eta(\mathbf{x},t)$$

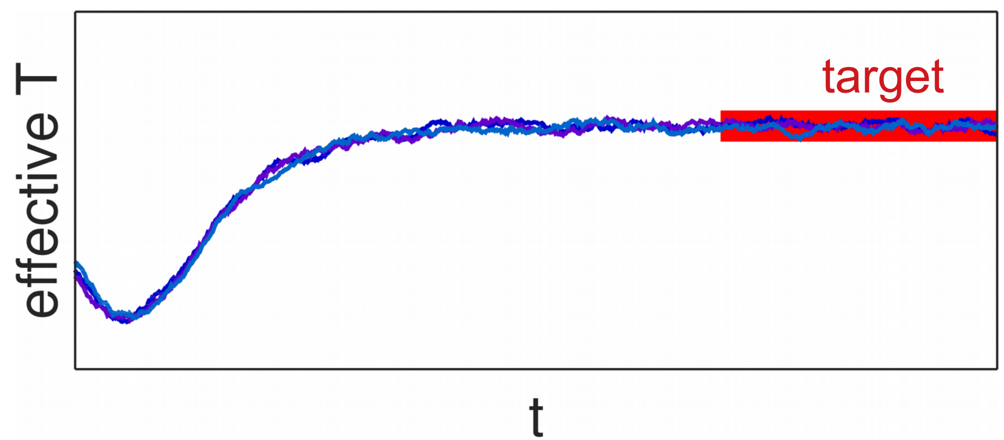
$$\mathcal{E}\phi(\mathbf{x}) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g\,|\phi(\mathbf{x})|^2 \right] \phi(\mathbf{x})$$

"Only" real difference

UV divergence is gone

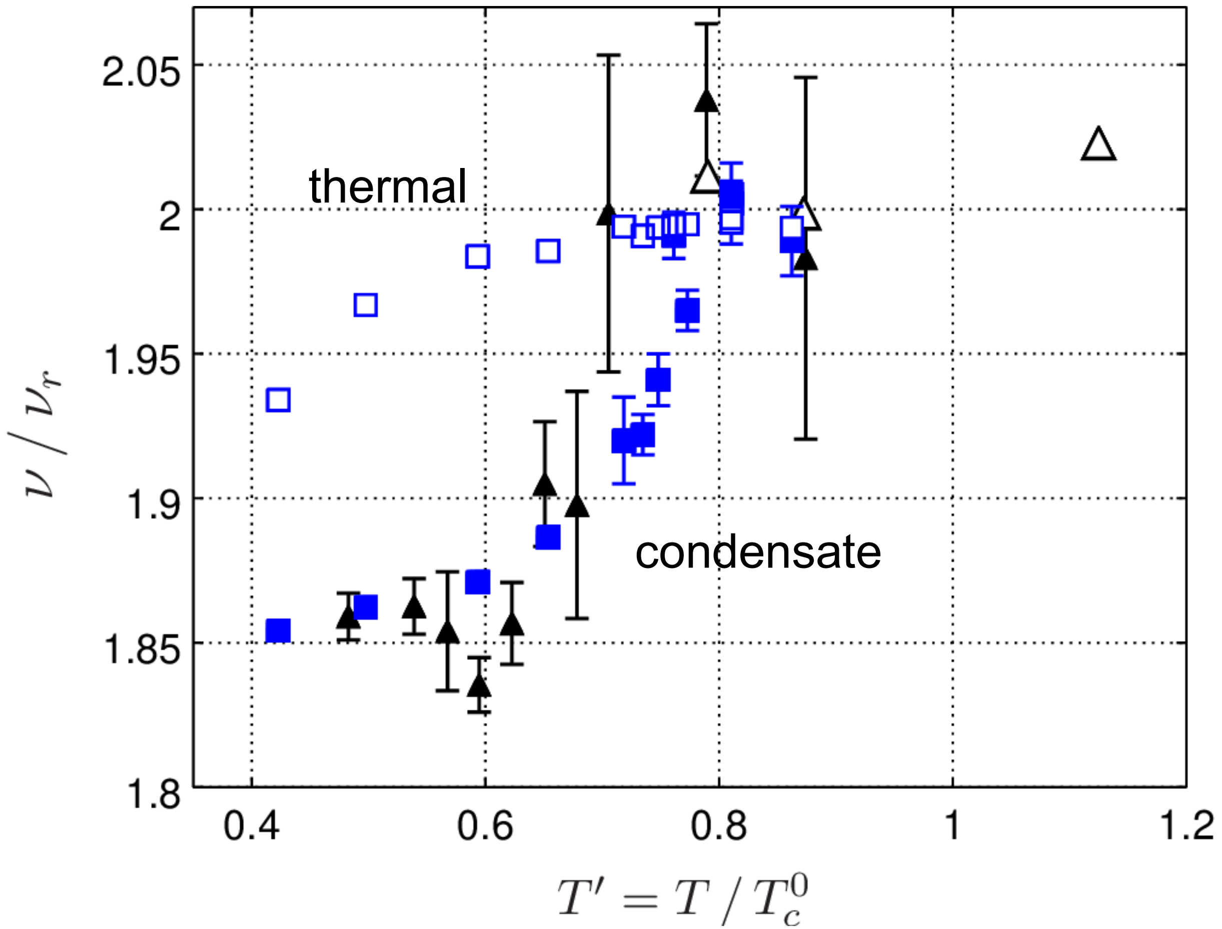


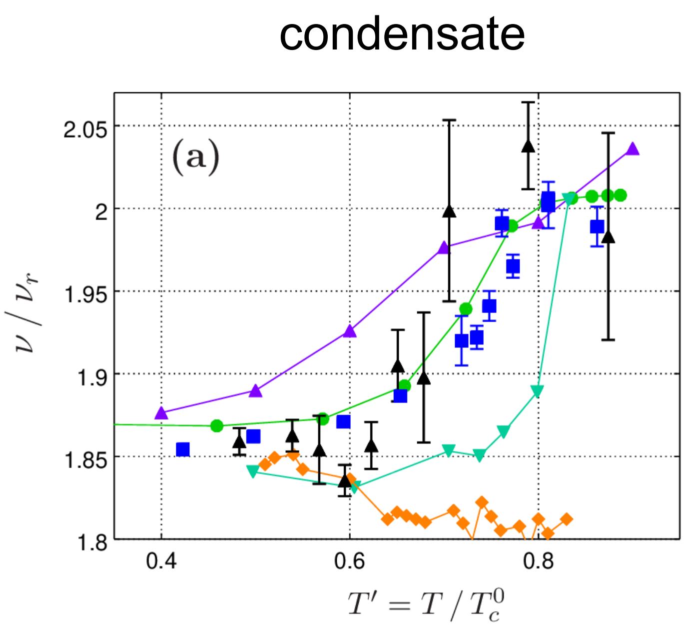
Evolution well controlled

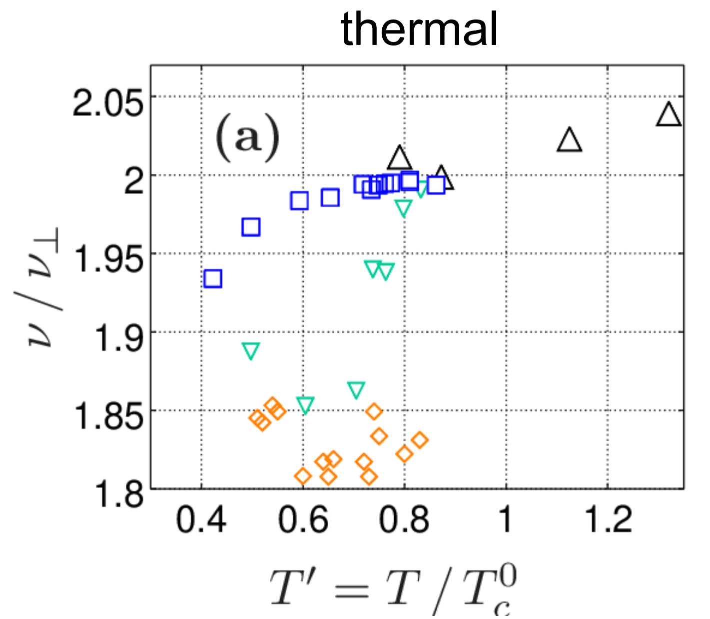


NARODOWE CENTRUM NAUKI

Results: *m=0* mode frequency







Experiment: Jin, Matthews, Ensher, Wieman, Cornell, PRL 78, 764 (1997)

This work (2018)

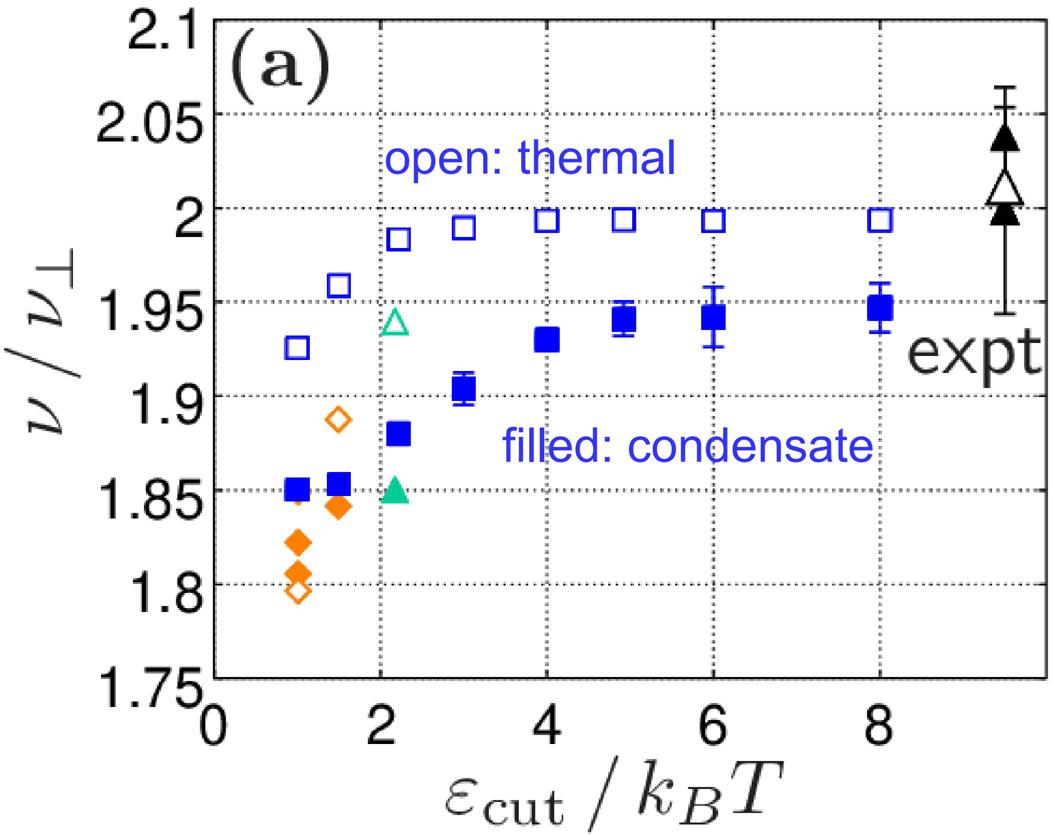
ZNG: Jackson, Zaremba, PRL 88, 180402 (2002)

2nd order Bogoliubov: Morgan, Rusch, Hutchinson, Burnett PRL 91, 250403 (2003)

PGPE+HF: Bezettt, Blakie, PRA 79, 023602 (2009)

PGPE: Karpiuk, Brewczyk, Gajda, Rzążewski, PRA 81, 013629 (2010)

Cutoff dependence

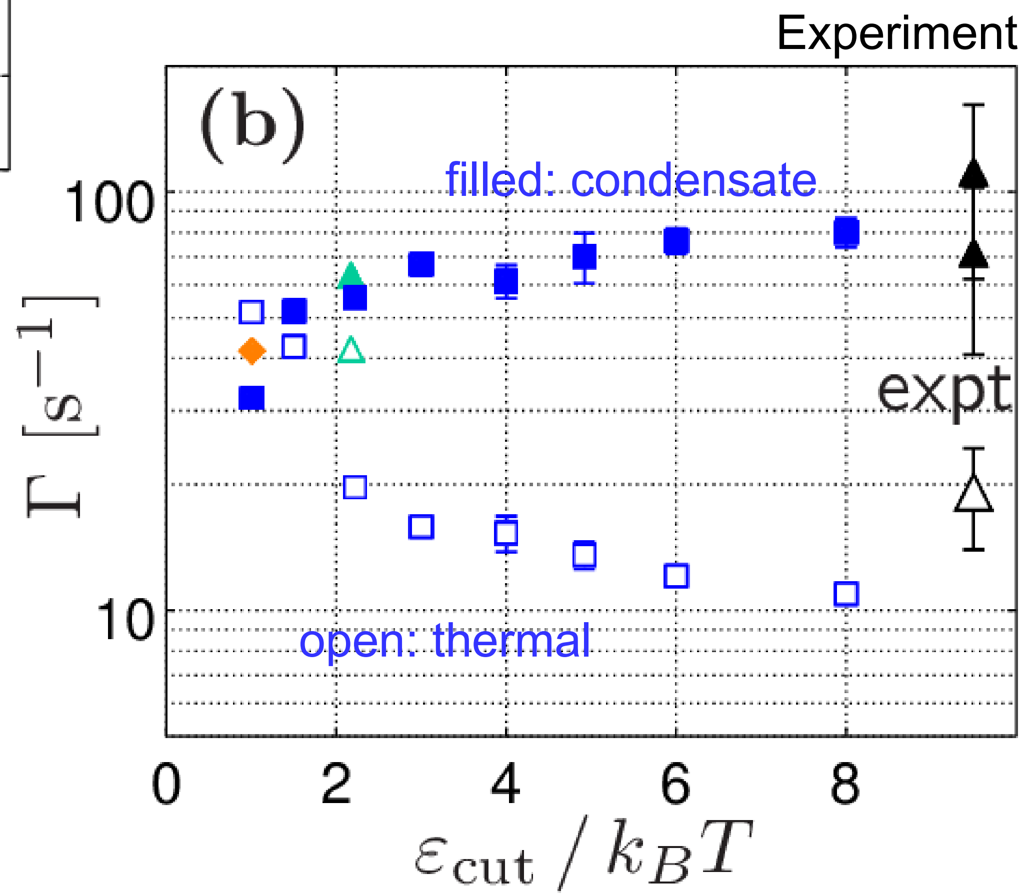


Experiment Jin *et al* 1997

Bezett + Blakie 2009

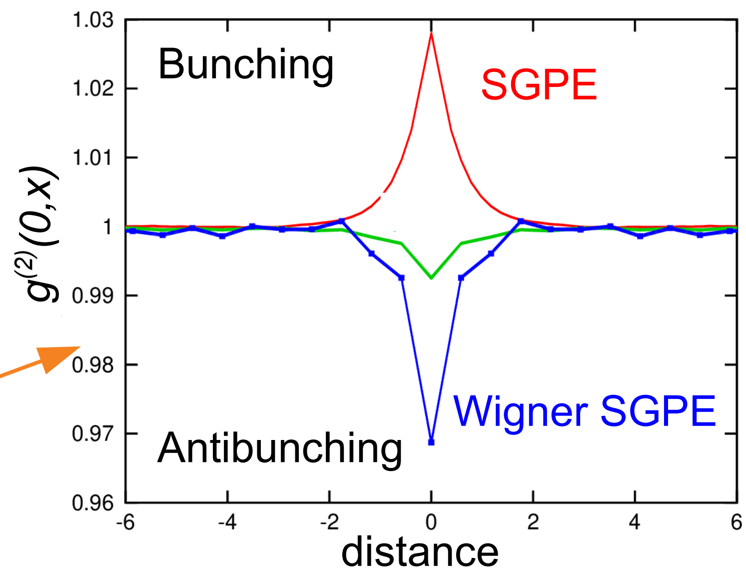
Karpiuk, Brewczyk, Gajda, Rzążewski 2010

This work



Outlook

- Quantitative results with classical fields
  Robust because no fitting of technical parameters
- Numerical effort comparable to SGPE
  though, lattice may need to be larger
- Still lacks wave-particle duality
  but this is harder to spot than expected
- NEXT: Wigner representation version
  to include quantum fluctuations



Thanks to discussions with

Nick Proukakis
Mariusz Gajda
Mirek Brewczyk
Kazimierz Rzążewski
Emilia Witkowska

Simon Gardiner
Crispin Gardiner
Matt Davis
Ashton Bradley
Blair Blakie

Krzysztof Gawryluk
Tomasz Karpiuk
Thomas Gasenzer
Andrew Daley
Krzysztof Pawłowski