Fate of the False Vacuum: A finite temperature stochastic model for the simulated early universe in BEC

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Introduction

Coleman's theory of quantum tunneling [1,2] predicts that a relativistic scalar quantum field from a metastable vacuum (i.e. false vacuum) decays to a stable vacuum (i.e. true vacuum). Applying Coleman's theory to the cosmological model of an inflationary universe, the decay of false vacuum can result in the formation of early universe "bubbles". The quantum-field-theory prediction of such false vacuum tunneling has not been tested. Here we study a proposed table-top experiment using a one-dimensional two-species Bose-Einstein condensate (BEC) to simulate the inflationary universe [3-6]. To include finite temperature effects that would be found under realistic conditions, we present a theoretical model which combines the Wigner stochastic phase-space method and the Bogoliubov theory [7] to numerically evaluate the dynamics of a coupled Bose field.

Hamiltonian of the two-species system

The Hamiltonian of a uniform one-dimensional two-species ($j = 1,2$) Bose gas is

$$\hat{H} = \sum_{j=1}^{2} \int dx \left\{ \left(-\hat{\Psi}_j^\dagger \frac{\hbar^2 \nabla^2}{2m} \hat{\Psi}_j + \frac{g}{2} \hat{\Psi}_j^{\dagger 2} \hat{\Psi}_j^2 \right) - \nu(t) \left(\hat{\Psi}_2^\dagger \hat{\Psi}_1 + \hat{\Psi}_1^\dagger \hat{\Psi}_2 \right) \right\},$$

where the s-wave intraspecies scattering strength $g = 2\hbar a \omega_\perp$. In this proposed experiment, the spin components are coupled by a cw microwave field with an additional amplitude modulation frequency ω. The interspecies coupling strength is given by $\nu(t) = \nu + \delta\hbar\omega\cos\omega t$. According to the concept of Stephenson-Kapitza pendulum, the sinusoidal time-dependent coupling creates an engineered phase potential $U(\phi_a) = \omega_0^2 \left[\cos(\phi_a) + \frac{\lambda^2}{2}\sin^2(\phi_a)\right]$, where $\omega_0 = 2\sqrt{\nu g |\psi_c|^2}/\hbar$ and $\lambda = \delta\sqrt{2g|\psi_c|^2/\nu}$ for an initial condensate density $|\psi_c|^2$. This creates a high-energy metastable state for the relative phase ϕ_a between the two components, here we define $\phi_a = \phi_1 - \phi_2 - \pi$.

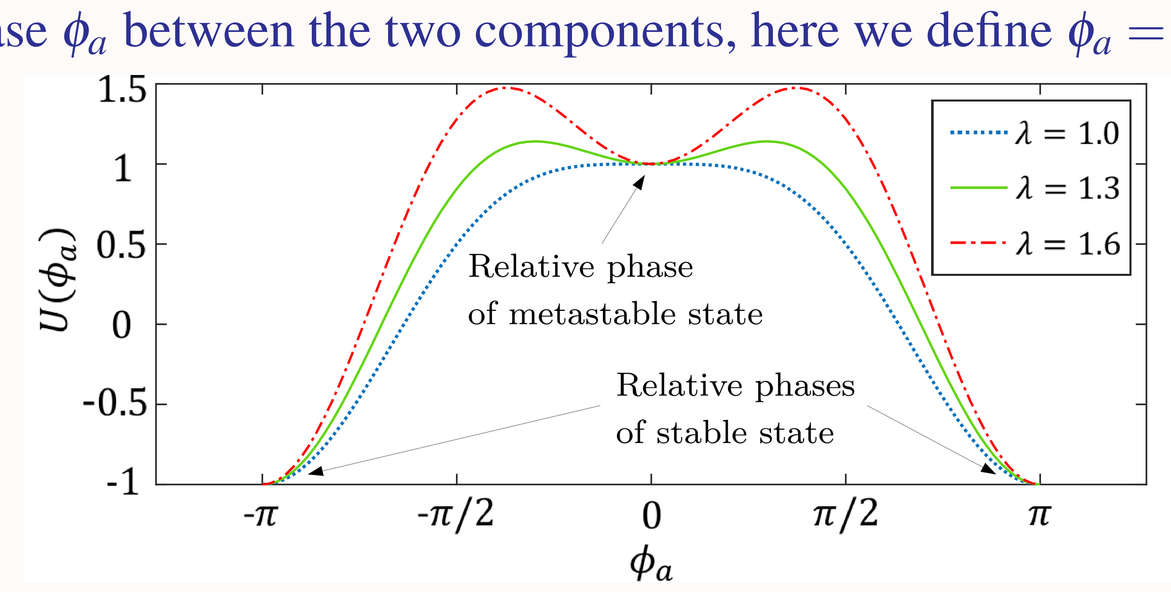

Figure 1: Illustration of the engineered phase potential creating a metastable state for $\lambda > 1$, the BEC is in false vacuum state when $\phi_a = 0$ and true vacua when $\phi_a = \pm\pi$.

Initial state at finite temperature

The BEC is initially prepared with single spin component $j = 1$. Following the Bogoliubov theory, the Bose field operator at finite temperature is expanded as

$$\hat{\Psi}_1(x,0) = \psi_c + \frac{1}{\sqrt{L}} \sum_k \left[u_k \hat{b}_k e^{ikx} - v_k \hat{b}_k^\dagger e^{-ikx} \right].$$

The Bogoliubov expansions, $u_k = \frac{\varepsilon_k + E_k}{2\sqrt{\varepsilon_k E_k}}$ and $v_k = \frac{\varepsilon_k - E_k}{2\sqrt{\varepsilon_k E_k}}$, have $u_0 = 1$ and $v_0 = 0$ at zero momentum [8], where $E_k = \hbar^2 k^2/2m$ and $\varepsilon_k = \sqrt{E_k(E_k + 2g|\psi_c|^2)}$. Applying the Wigner stochastic method, the phonon modes are represented as complex Gaussian random variables $\hat{b}_k \sim \beta_k$ and $\hat{b}_k^\dagger \sim \beta_k^*$ which follow $\langle|\beta_k|^2\rangle = \frac{1}{\exp(\varepsilon_k/k_B T) - 1} + \frac{1}{2}$ at the finite temperature T. The Wigner field expression for the $j = 1$ field operator is given by

$$\psi_1 = \psi_c + \frac{1}{\sqrt{L}} \sum_k \left(u_k \beta_k e^{ikx} - v_k \beta_k^* e^{-ikx} \right).$$

The spin component $j = 2$ is initially in a vacuum state $\psi_2 = \frac{1}{\sqrt{L}} \sum_k \alpha_k e^{ikx}$. The vacuum modes are represented as complex Gaussian random variables where $\langle|\alpha_k|^2\rangle = \frac{1}{2}$. The Wigner fields are then rotated to give the initial fields $\psi_{1,0}$ and $\psi_{2,0}$, where $\begin{pmatrix}\psi_{1,0}\\ \psi_{2,0}\end{pmatrix} = \begin{pmatrix}\cos(\pi/4) & -ie^{i\pi/2}\sin(\pi/4)\\ -ie^{-i\pi/2}\sin(\pi/4) & \cos(\pi/4)\end{pmatrix}\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$. This corresponds to applying a microwave pulse to Rabi rotate the BEC into two hyperfine levels, so that the two species have equal initial occupation with a relative phase of π (i.e. $\langle|\psi_{1,0}|^2\rangle = \langle|\psi_{2,0}|^2\rangle$, $\phi_a = 0$). The time evolution of the Wigner field trajectory (in dimensionless form) then follows [9]:

$$\frac{d\widetilde{\psi}_j}{d\widetilde{t}} = -i\left[-\sqrt{\widetilde{\nu}}\widetilde{\nabla}^2\widetilde{\psi}_j + \widetilde{g}\widetilde{\psi}_j|\widetilde{\psi}_j|^2\right] + i\frac{\sqrt{\widetilde{\nu}}}{2}\left(1 + \sqrt{2}\lambda\widetilde{\omega}\cos(\widetilde{\omega}\widetilde{t})\right)\widetilde{\psi}_{3-j}.$$

References

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Topological phases of true vacua

Starting with a $\phi_a = 0$ false vacuum initialized with finite-temperature effects, the following single-tracjectory example shows that three true vacua bubbles expand until they meet at the domain walls of false vacuum, or else form localized oscillons. The bubbles representing distinct universes each have phase either $-\pi$ or π.

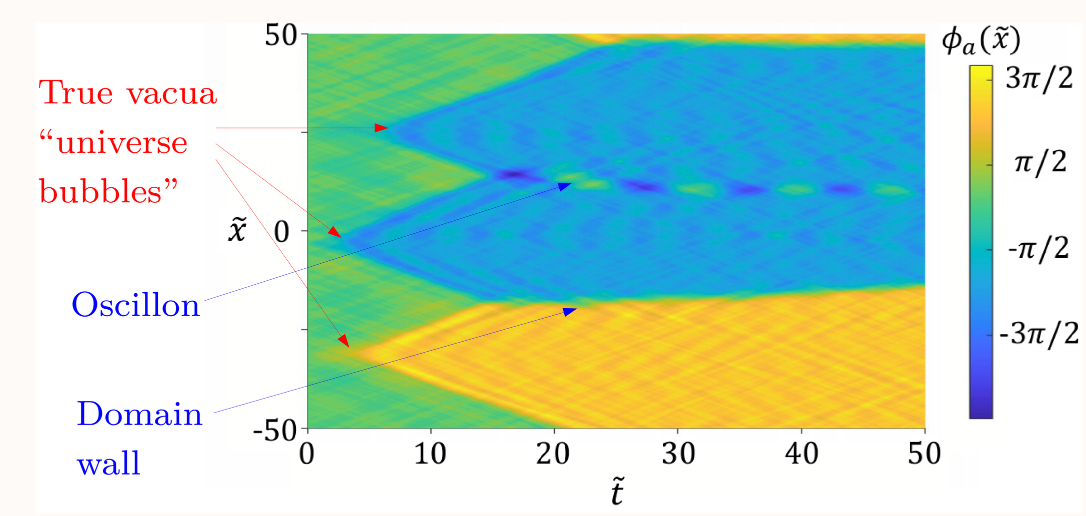

Figure 2: The false vacuum (green) decays to two distinct topological phases (blue or yellow). The numerical results are evaluated using an open-source software xSPDE [10].

Decay of false vacuum at finite temperature

Converting the relative phase into the relative number distribution p_z, where $p_z(\widetilde{x}) = \frac{|\widetilde{\psi}_2|^2 - |\widetilde{\psi}_1|^2}{|\widetilde{\psi}_2|^2 + |\widetilde{\psi}_1|^2}$. Results show that at higher reduced temperature $\tau = T/T_d$, thermal fluctuations reduce the survival time and disturb the clear structure of the true vacua bubbles.

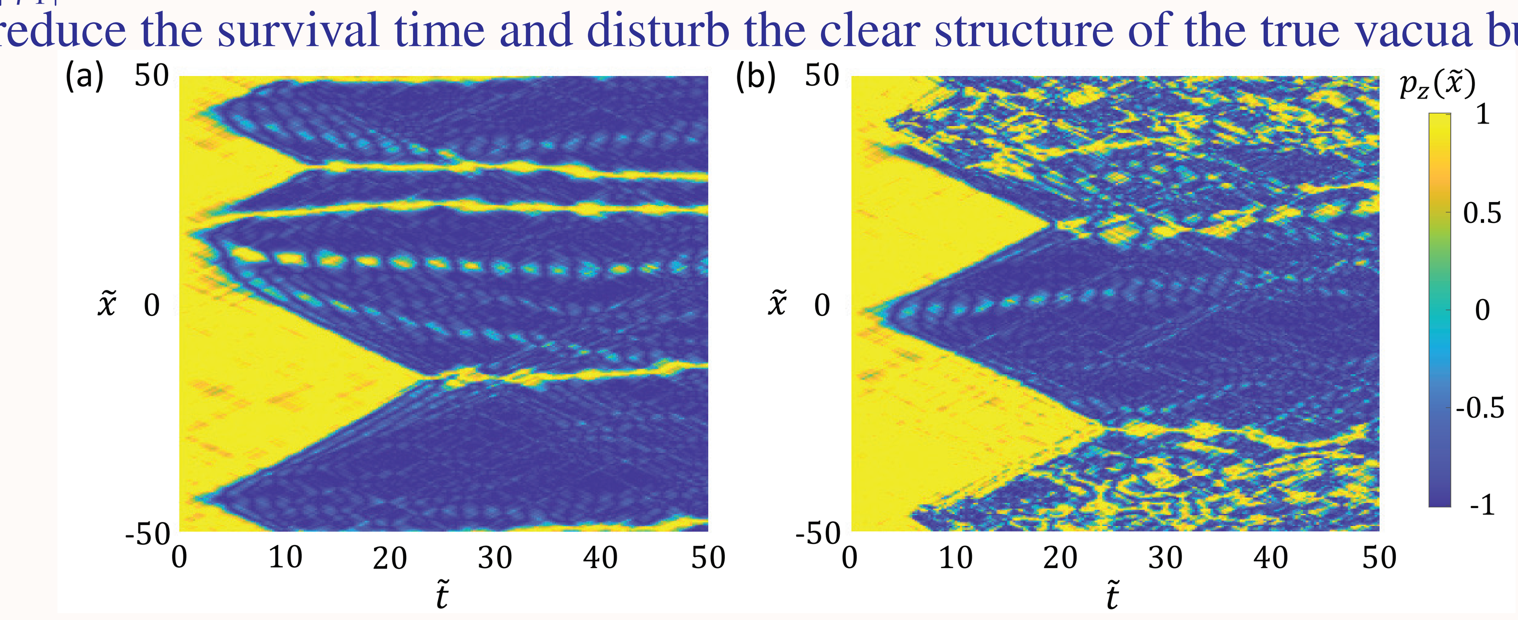

Figure 3: Decay of the false vacuum (yellow) to true vacua (blue): (a) at a lower reduced temperature $\tau = 1 \times 10^{-5}$ with four "universe bubbles" clearly formed and; (b) at a higher temperature $\tau = 3 \times 10^{-4}$, strong fluctuations occur between the false and true vacua.

Tunneling rate

Determining the tunneling rate Γ from the survival probability of the false vacuum, Γ shows a power-law dependence on $\widetilde{\nu}$. For a fixed coupling $\widetilde{\nu}$, the quantum vacuum nucleation is accelerated at higher temperature.

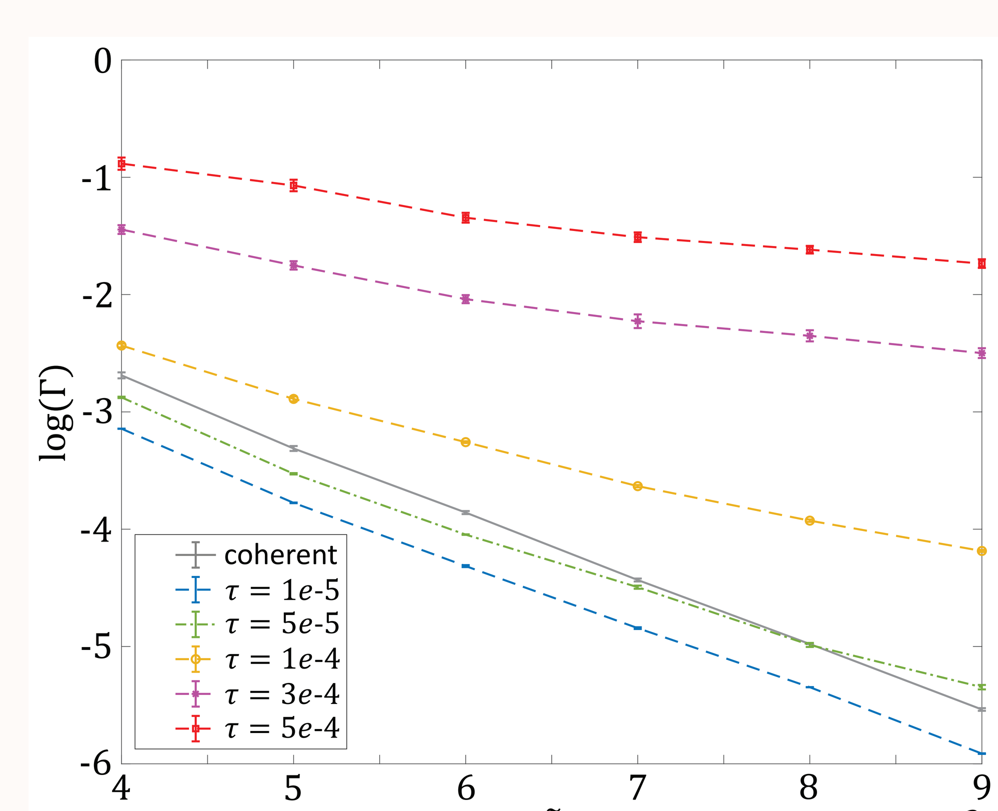

Figure 4: Tunneling rate Γ determined from the coherent state and thermal states.

Modulation instability

The simulation results below show that at the same reduced temperature, increasing the oscillator frequency $\widetilde{\omega}$ can suppress the short-wavelength fluctuations and stabilize the true vacua. This increases the feasibility of a false vacuum BEC experiment.

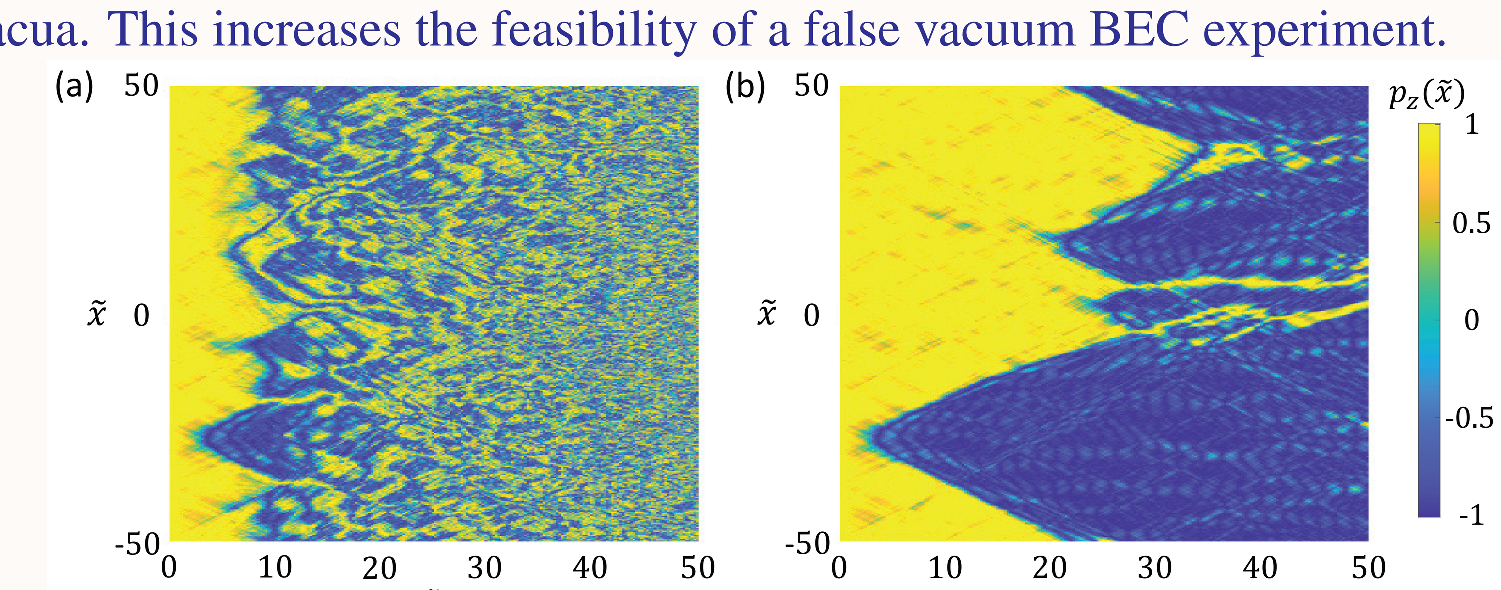

Figure 5: (a) Strong fluctuations present at a lower oscillator frequency $\widetilde{\omega} = 50$, and; (b) short-wavelength instabilities are removed at a higher oscillator frequency $\widetilde{\omega} = 200$.