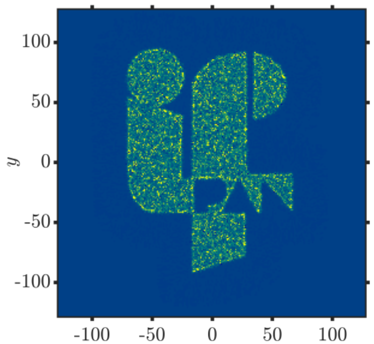
Simulating the complete quantum mechanics of very large driven-dissipative Bose-Hubbard systems

Um yeah... on a *classical* computer, with tricks



Piotr Deuar

Institute of Physics, Polish Academy of Sciences, Warsaw, Poland

Michał Matuszewski

Institute of Physics, Polish Academy of Sciences, Warsaw, Poland

Alex Ferrier, Marzena Szymańska

University College London, UK

Giuliano Orso

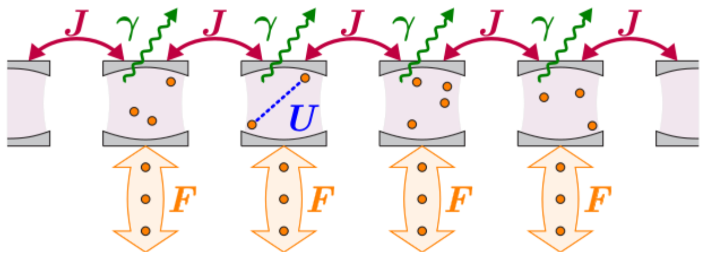
LMPQ, Universite de Paris 7, France

References:

- PD, Ferrier, Matuszewski, Orso, Szymańska, PRX Quantum **2**, 010319 (2021).
- PD, Quantum **5**, 455 (2021).

Driven dissipative Bose-Hubbard model

$$\widehat{H} = \sum_j \widehat{H}_j - \sum_{\text{connections } i,j} \left[J_{ij}\widehat{a}_j^\dagger \widehat{a}_i + J_{ij}^* \widehat{a}_i^\dagger \widehat{a}_j \right]$$



Vincentini, Minganti, Rota, Orso, Ciuti, PRA **97**, 013853 (2018)

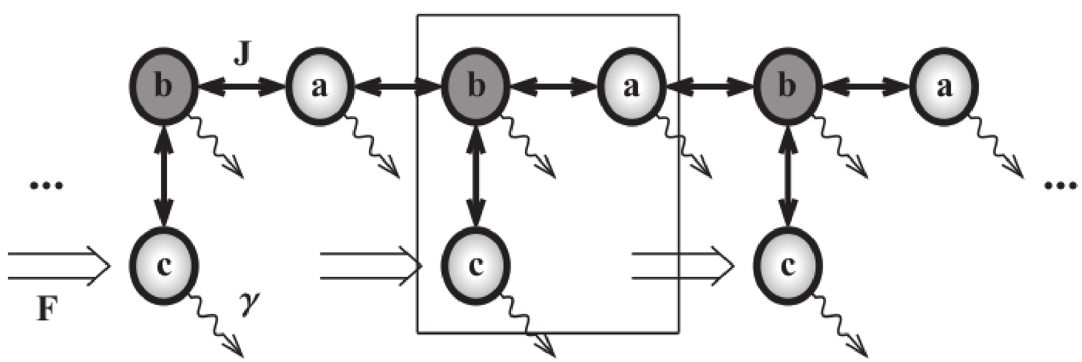
$$\widehat{H}_j = -\Delta_j \widehat{a}_j^\dagger \widehat{a}_j + \frac{U_j}{2} \widehat{a}_j^\dagger \widehat{a}_j^\dagger \widehat{a}_j \widehat{a}_j + F_j \widehat{a}_j^\dagger + F_j^* \widehat{a}_j$$

driving

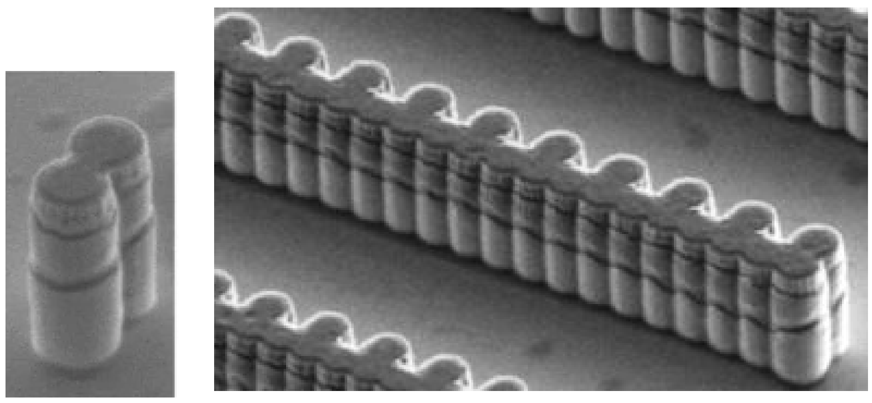
$$\frac{\partial \widehat{\rho}}{\partial t} = -i\left[\widehat{H}, \widehat{\rho}\right] + \sum_j \frac{\gamma_j}{2} \left[2\widehat{a}_j \widehat{\rho} \widehat{a}_j^\dagger - \widehat{a}_j^\dagger \widehat{a}_j \widehat{\rho} - \widehat{\rho} \widehat{a}_j^\dagger \widehat{a}_j \right]$$

dissipation

Also structured lattices – e.g. Lieb lattice



Casteels, Rota, Storme, Ciuti, PRA **93**, 043833 (2016)



Baboux, Ge, Jacqmin, Biondi, Galopin, Lemaitre, Le Gratiet, Sagnes, Schmidt, Tureci, Amo, Bloch, PRL **116**, 066402 (2016)

M subsystems (modes, sites, volumes) labeled by j

$$\hat{\rho} \sim \frac{1}{2} d^{2M}$$

Coherent state basis, complex, *local* $|\alpha_j\rangle_j = e^{-|\alpha_j|^2/2} e^{\alpha_j \hat{a}_j^\dagger} |\mathrm{vac}\rangle$

"ket" amplitude α_j
"bra" amplitude β_j^*

Local operator kernel $\hat{\Lambda}(\boldsymbol{\lambda}) = \bigotimes_j \frac{|\alpha_j\rangle_j \langle \beta_j^*|_j}{\langle \beta_j^*|_j |\alpha_j\rangle_j}$

full system configuration $\boldsymbol{\lambda} = \{\alpha_1, \ldots, \alpha_M, \beta_1, \ldots, \beta_M\}$

Full density matrix $$\hat{\rho} = \int d^{4M}\boldsymbol{\lambda}\, P_+(\boldsymbol{\lambda})\, \hat{\Lambda}(\boldsymbol{\lambda})$$

Quantum complexity is all in the distribution! ---> let's *SAMPLE IT !*

- Evolution equations for samples

$$\frac{\partial \alpha_j}{\partial t} = i\Delta_j \alpha_j - iU_j \alpha_j^2 \widetilde{\alpha}_j^* - iF_j - \frac{\gamma_j}{2}\alpha_j + \sqrt{-iU_j}\, \alpha_j\, \xi_j(t) + \sum_k iJ_{kj}\alpha_k,$$

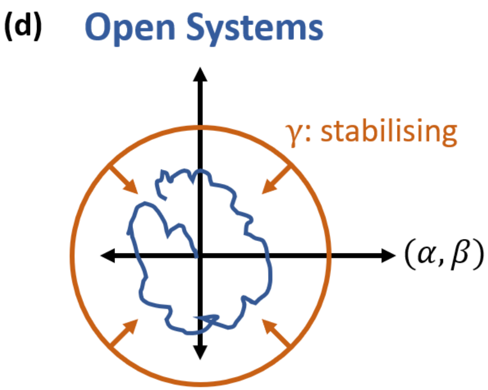
$$\frac{\partial \widetilde{\alpha}_j}{\partial t} = i\Delta_j \widetilde{\alpha}_j - iU_j \widetilde{\alpha}_j^2 \alpha_j^* - iF_j - \frac{\gamma_j}{2}\widetilde{\alpha}_j + \sqrt{-iU_j}\, \widetilde{\alpha}_j\, \widetilde{\xi}_j(t) + \sum_k iJ_{kj}\widetilde{\alpha}_k$$

White Gaussian noise deals with interparticle collisions

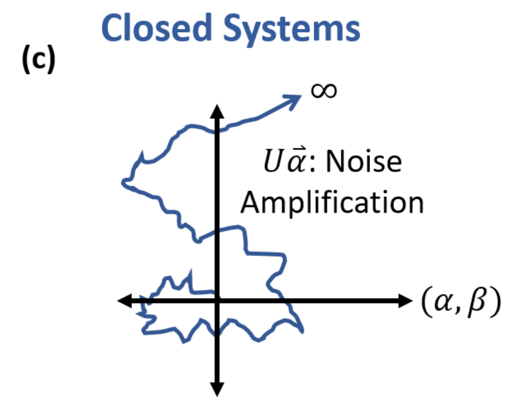
$$\langle \xi_j(t)\xi_k(t')\rangle_s = \delta(t-t')\delta_{jk}, \quad \langle \widetilde{\xi}_j(t)\widetilde{\xi}_k(t')\rangle_s = \delta(t-t')\delta_{jk}$$

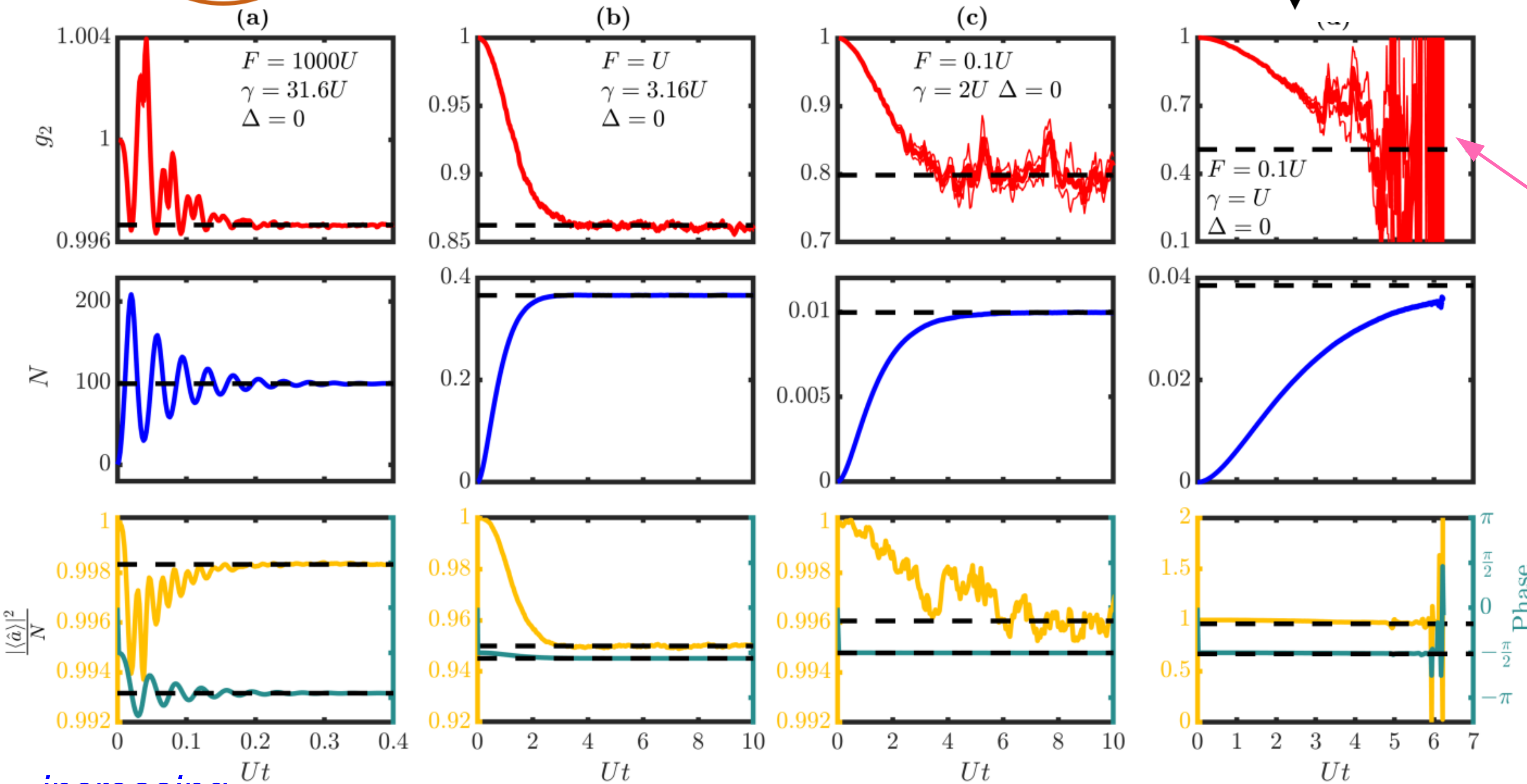
The rest of the equations is basically mean field

Positive-P simulations stabilised by the dissipation



stationary state can be reached
and studied
in a full quantum description

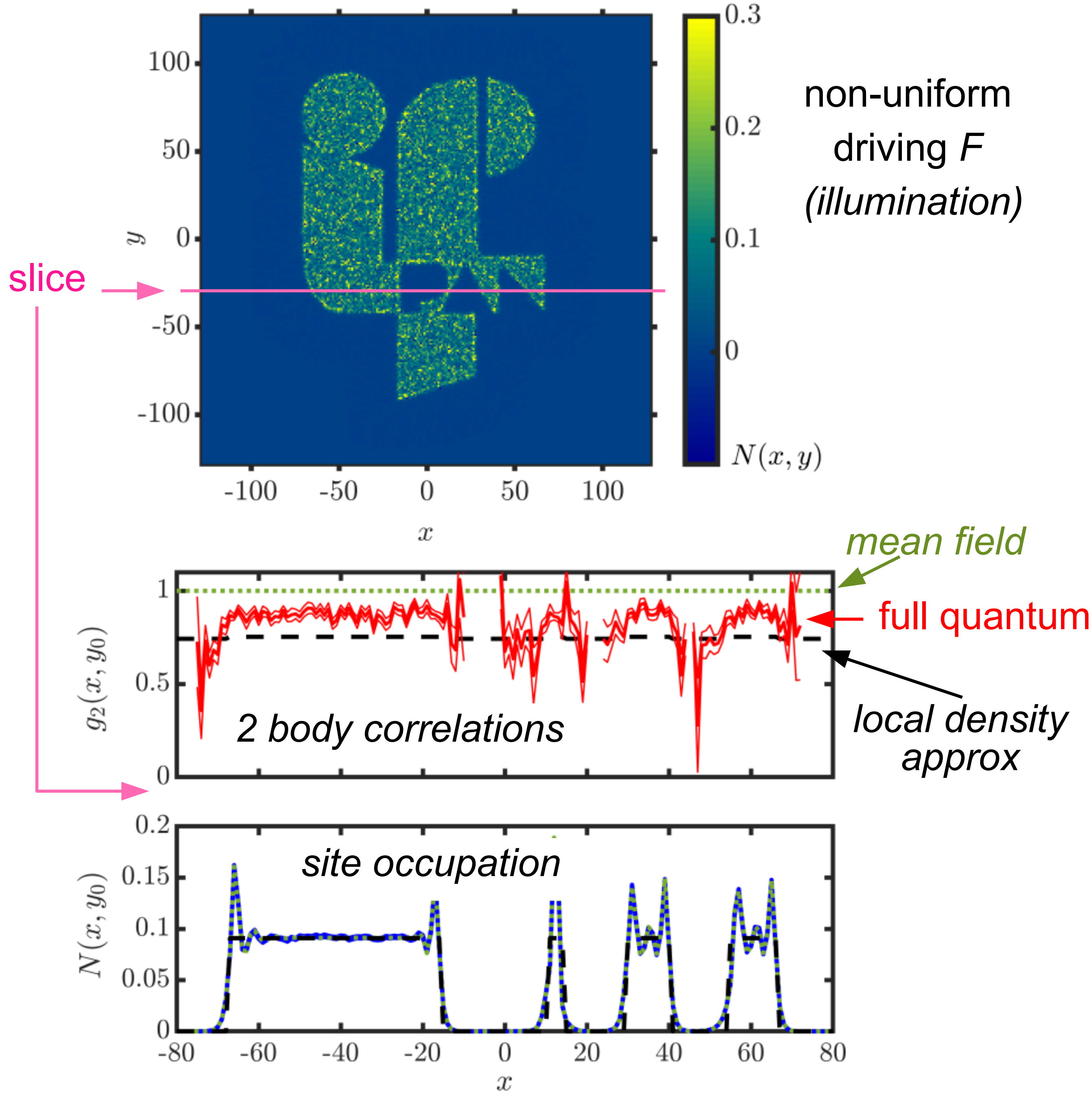




instability triggered γ too low

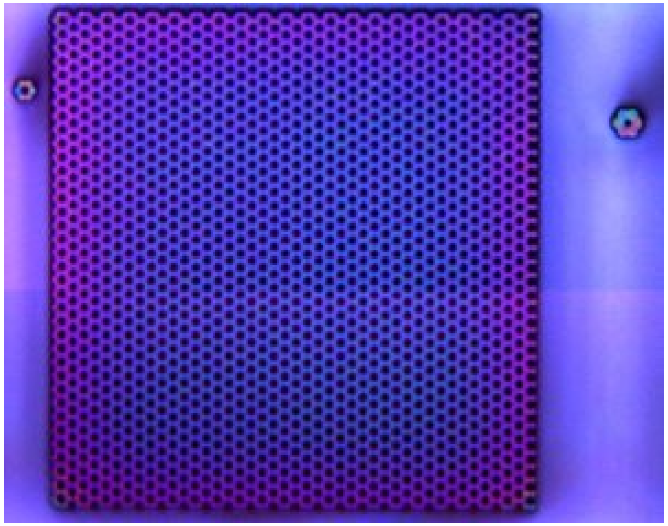
increasing dissipation

Large systems – example simulation 256 x 256 lattice



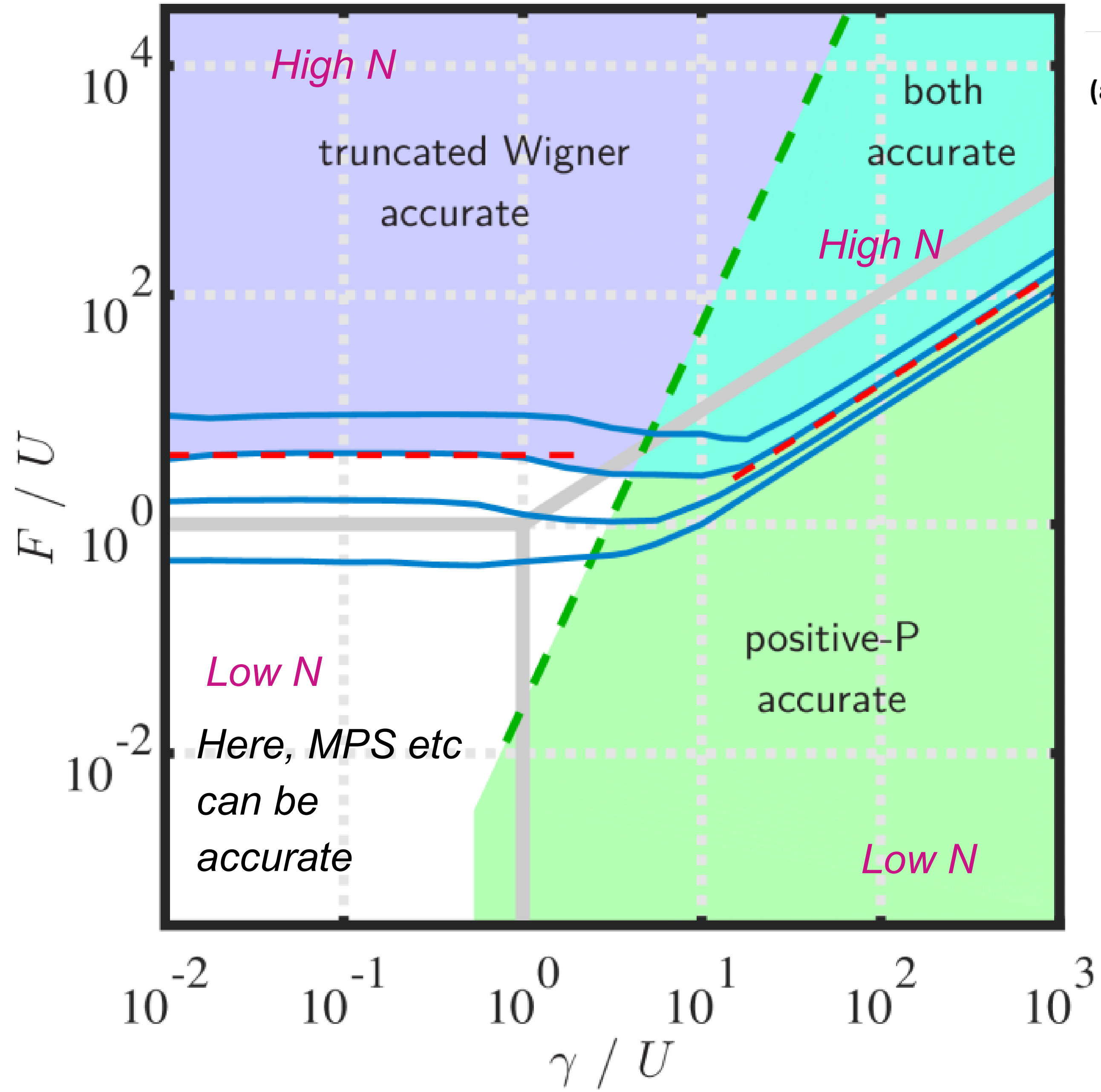
PD, Ferrier, Matuszewski, Orso, Szymańska, PRX Quantum **2**, 010319 (2021).

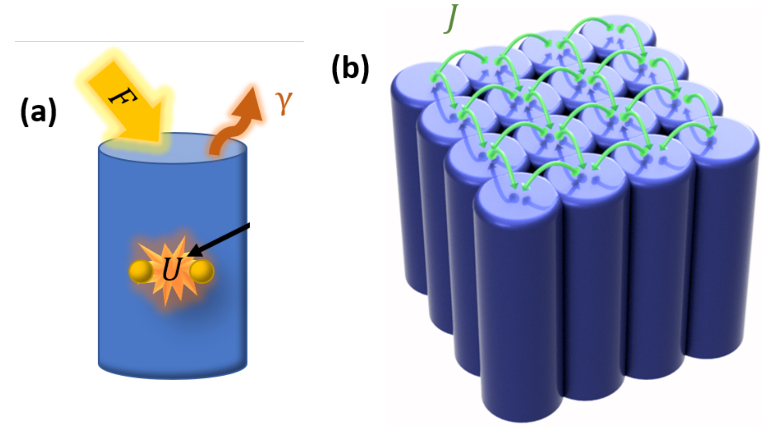
e.g. photonic quasicrystals micropillars



Milicevic, Ozawa, Montambaux, Carusotto, Galopin, Lemaitre, Le Gratiet, Sagnes, Bloch, Amo, PRL **118**, 107403 (2017)

Phase space methods - regimes of applicability





many tests were done
all confirm positive-P
accuracy as long as there is
an exact method to compare

Have a dissipative system you want to simulate?

non-uniform ?
time-dependent ??

Contact us ;-)

PD, Ferrier, Matuszewski, Orso, Szymańska, PRX Quantum **2**, 010319 (2021).