Single shot phenomena in quantum atom optics

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Outline

• Single shot physics
• Spontaneous solitons
• Classical field method
• “Equilibrium dynamics”
• Judging the accuracy of classical fields
• Extension to include quantum fluctuations
Underlying fact

Ultracold atom experiments do not usually measure the simple density

\[ n(x) = \langle \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \rangle \]

that we are familiar with from basic quantum mechanics and most calculations

- Absorption imaging actually measures a very high order object:

  " ...... "

  (Read: the position of all indistinguishable atoms in the many-body wavefunction OR: the number of atoms at each pixel simultaneously)

- Often it doesn't matter that "......" \( \neq n(x) \)
  (e.g. when resolution is worse than density correlations)

- However, it is interesting, when it does.
Warm-up example

Attractively interacting Bose gas in a trap


Density

\[ \rho^{(1)}(x;x) = \left( \frac{\omega_1}{\pi} \right)^{3/2} e^{-\omega_1 x^2/2}. \]

COM fluctuations

Measurement 1

\[ X_0 \]

Measurement 2

\[ X_0 \]

Measurement 3

\[ X_0 \]

Conditional density

\[ \rho^{(1|1)}_\text{cond}(x;x|x_0) = \left( \frac{\omega}{2\pi} \right)^{3/2} e^{-\omega(x-x_0)^2/2}. \]

\( \omega \gg \omega_1 \)
Graying of dark solitons

Generation by phase imprinting

\[ \Delta \Phi \approx - \frac{U(x) t}{\hbar} \]

Burger, Bongs, Dettmer, Ertmer, Sengstock, Sanpera, Shlyapnikov, Lewenstein, PRL 83, 5198 (1999)

Simulation of density at \( T=0 \)

Mishmash, Carr, PRL 103, 140403 (2009)

Filling in?
By excitations?
Quantum fluctuations?
Actually – random position

One-particle density $n(x)$: “greying”

Histogram of atom positions measured sequentially from wavefunction


Wigner simulation at $T=0$

Correlations: false friends

Two body correlation

Looks like greying+broadening

Mishmash, Carr, *PRL* 103, 140403 (2009)

Counterexample

$g^{(2)}$ is not a good indicator of the shape of the dip!

Defect formation during evaporative cooling

Expected a Kibble-Zurek mechanism:
*Formation of condensate grains, and defects on the boundaries*

We saw defects, but according to a rather different picture:

Początek termalizacji

Cooling ramp

Thermalization

Ramp beginning

Formation of solitons

End of ramp
Low order correlations do not describe the state

DENSITY  PHASE  COHERENCE $g^{(1)}(x,-x)$

End of cooling ramp

Long times

Temperature quench - experiment

Temperature quench in a sodium condensate

Different shots

Analysis of each shot separately

**Soliton counting statistics**


**Kibble-Zurek-type scaling**
Solutions induced by a disturbance are common.

**Evaporative cooling (temperature quench)**


**Interaction quench**


**Uniform gas after chemical potential quench**


**Growth of quasicondensate during thermalization**

Swisłocki, PD, arXiv: 1409.0146
Classical fields approximation

Gives access to single realizations

Full quantum field $\rightarrow$ Ensemble of complex-fields

$\hat{\Psi}(x) = \sum_k \hat{a}_k \psi_k(x) \rightarrow \left\{ \sum_{k \in C} \xi_k \psi_k(x) \right\}$

N-body state

Assume highly occupied modes

Replace mode amplitude operators $\hat{a}_k$ with complex number amplitudes $\xi_k$

“Quantum field theory, without discretized particles”

Developed by many authors:
A. Sinatra, M. Brewczyk, M. Gajda, M. Davis, K. Rzazewski, K. Burnett, E. Witkowska, … (no particular order)

Two ways to thermal equilibrium (GCE)

Metropolis sampling:

\[ P(\Psi(x)) \propto \exp\left\{ -\frac{1}{T} \int dx \Psi(x)^* \left[ -\frac{\hbar^2}{2m} \nabla^2 + V(x) + \frac{g}{2} |\Psi(x)|^2 - \mu \right] \Psi(x) \right\} \]

Witkowska, Gajda, Rzążewski, Optics Communications 283, 671 (2010)

Stochastic Gross-Pitaevskii equation (SGPE):

\[ i\hbar \frac{\partial \Psi}{\partial t} = (1 - i\gamma) \left( -\frac{\hbar^2 \nabla^2}{2m} + V - \mu + g|\Psi|^2 \right) \Psi + \sqrt{2\hbar\gamma k_B T} \eta, \]

reservoir coupling (weak \( \gamma \ll 1 \))

Gross-Pitaevskii equation (GPE)

Complex white noise (thermal fluctuations)

Solitons in thermal equilibrium state

Solitons in equilibrium with phonons

Swisłocki, Nowicki, Pietraszewicz PD, *in preparation*

**Lieb-Liniger excitations:**
- Type I (phonons)  
- Type II (solitons)
Equilibrium dynamics - uniform gas
Issue 1: Qualitative or quantitative?

- For many problems, classical fields (c-fields) are the only viable method.
  * Especially when single realizations are needed

- Perennial questions:
  * Fine, but, are the effects real?
  * is it quantitative or only qualitative?
  * what was the cutoff used?

- Perennial answers:
  * It's okay if there are many particles
  * Can work very well

Local density fluctuations in a trapped 1D bose gas

Cutoff benchmarking

Canonical ensemble, trapped ideal Bose gas

Witkowska, Gajda, Rzazewski, PRA 79, 033631 (2009)
However, it depends on the observable...

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<td>SGPE calculations of interacting gas</td>
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<td>Match energy in high E modes to $k_B T$ (equipartition) $\sim 1$ particle in high E modes</td>
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<td>$k_B T$ + chemical potential</td>
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**Initial plan:** benchmark 1D quasicondensate with exact solution


**Realization:** even ideal gas is not well understood
Generic case: uniform section of gas

- Local Density approximation (LDA)
  → Grand Canonical ensemble
  (rest of gas acts as a reservoir)

Units:

Dimensionless temperature

\[ \tau = \frac{T}{T_d} \]

Ideal gas degeneracy temperature

Dimensionless cutoff

\[ f_c = \frac{k_c}{k_T} \]

\[ k_T = \frac{2\pi}{\Lambda_T} \]

Thermal de Broglie wavelength

\[ \Lambda_T = \sqrt{\frac{2\pi \hbar^2}{mk_B T}} \]
Cutoff optimum for different observables

Pietraszewicz, PD, arXiv:1504.06154

- $E_{\text{kin}}$ Kinetic energy per particle
- $\text{var}N / N$ Coarse-grained fluctuations
- $l_{\text{pg}}$ phase grain volume (~ coherence length $l_\Phi$)
- Half-width of $g^{(1)}(x)$
- $\rho_0$ condensate fraction

Most extreme behaviour
Accuracy

Pietraszewicz, PD, arXiv:1504.06154

Single observable error

$$\delta_{\alpha}(\tau, f_c) := \frac{\Delta \alpha}{\alpha} = \left( \frac{\alpha^{(cf)}(\tau, f_c)}{\alpha^{(id)}(\tau)} - 1 \right)$$

Global error

$$RMS_{\alpha, \beta, \ldots}(\tau, f_c) = \sqrt{\left(\delta_{\alpha}\right)^2 + \left(\tilde{\delta}_{\beta}\right)^2 + \ldots}$$

Error in any observable will be < RMS

- Kinetic energy and coarse-grained fluctuations capture most extreme behaviour

→ use these only
Recommendation:
Accuracy better than 10% for $T < 0.007 \ T_d$
Use $f_c = 0.65$ (Energy cutoff = $1.3 \ k_B T$)

Pietraszewicz, PD, arXiv:1504.06154

~ independent of density ($\tau$) → very good!
Recommendation:
Accuracy better than 10% for $T < 0.49 \, T_c$
Use $f_c = 0.78$ (Energy cutoff = $1.9 \, k_B T$)
Recommendation:
Don't use classical fields, at the least not near the ideal gas regime

Pietraszewicz, PD, arXiv:1504.06154
Interacting gas benchmarking

Comparison to Yang & Yang exact solution


\[
\gamma = \frac{g}{n} = 0.005
\]

\[
\tau = \frac{T}{T_d} = 0.0016
\]

interaction strength

Quasicondensate:

\[
g(2)(0) = 1.06
\]

Quite similar to ideal gas case:

10% accuracy

Same cutoff \(1.3 k_B T\)
Thank you

Collaboration

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Summary

• Cutoffs and accuracy depend strongly on the observable
  Kinetic energy and density fluctuations are most incompatible

• We found the temperatures and best cutoff for which a
  consistent and accurate c-field description exists in 1D and 3D.
  However, the 2D ideal gas is never well described

• Preliminary results in the interacting quasicondensate:
  Same cutoff as ideal gas, 10% accuracy also possible.