Spin distillation of ultracold Bose gases

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Support:

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QUANTERA

Outline

- Spin distillation cooling – what's that about?
- Our model and simulations
- The case of Chromium
- The case of Sodium
- Conclusions

Collaboration

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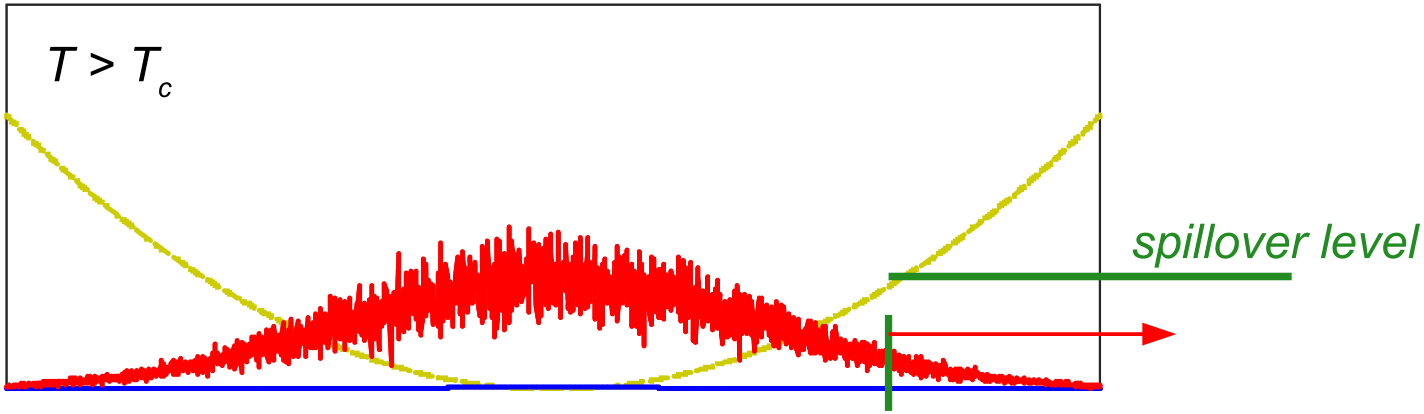
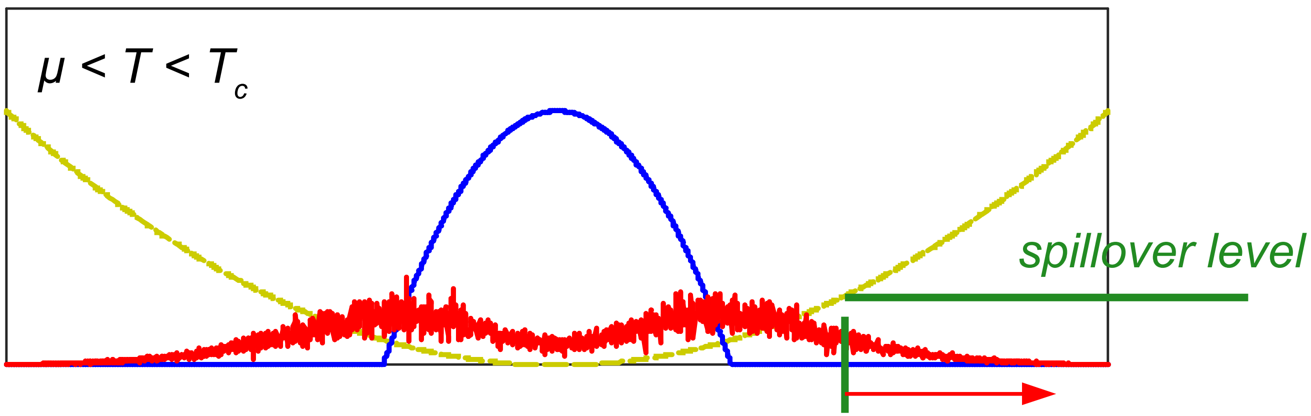
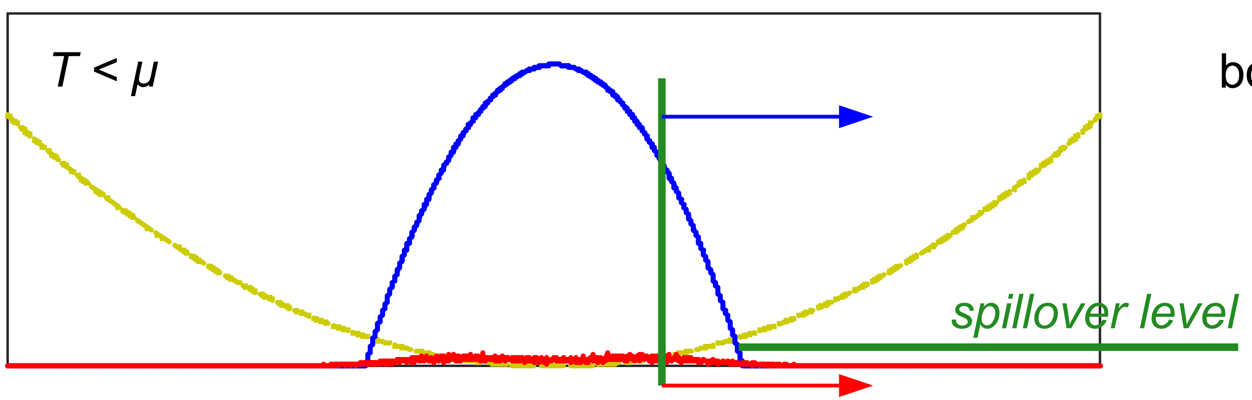
Evaporative cooling



good cooling



good cooling



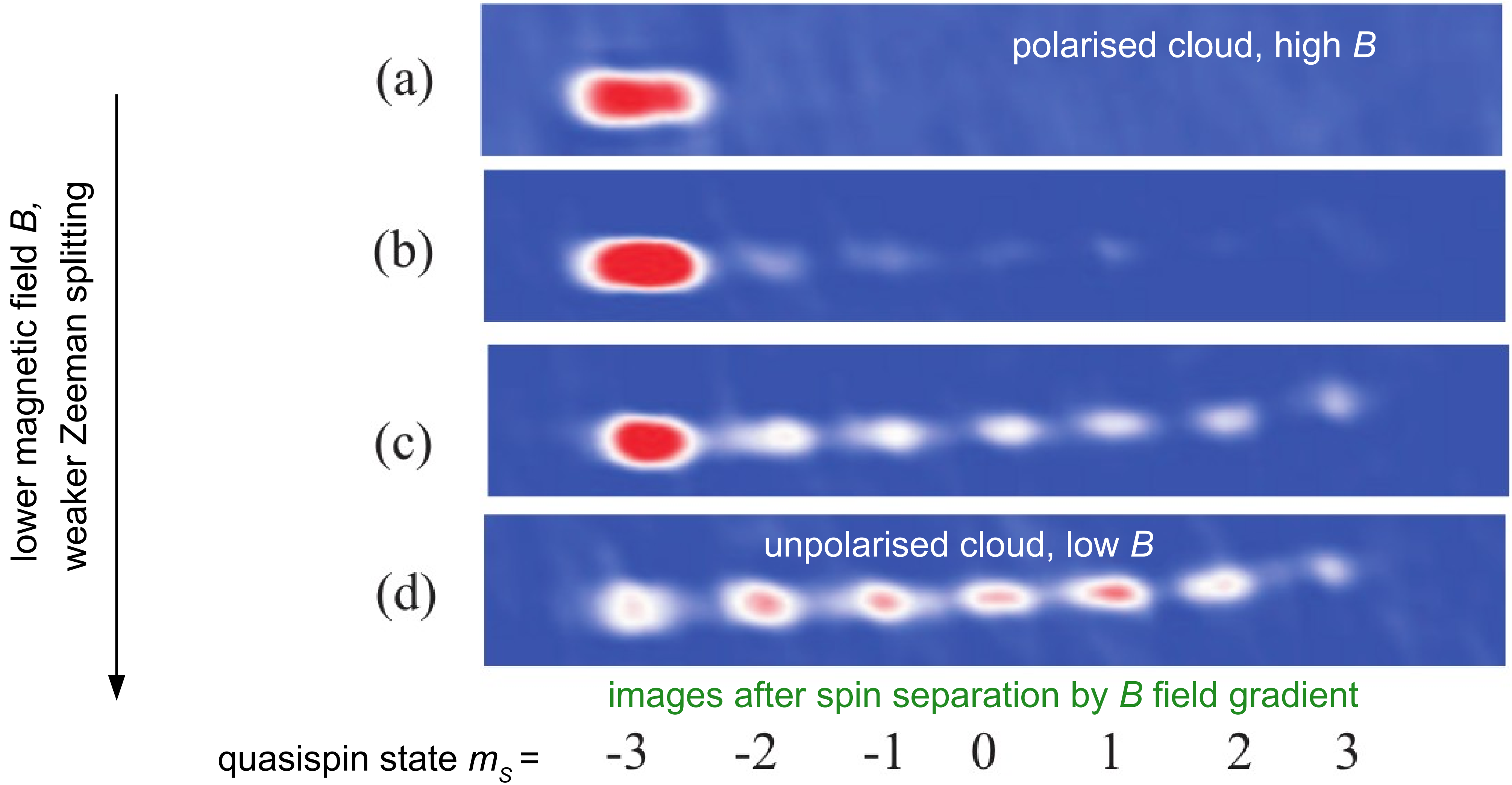
both thermal cloud and condensate

are evaporated

no more cooling, just loss

Spinor condensate – more tricks to choose from

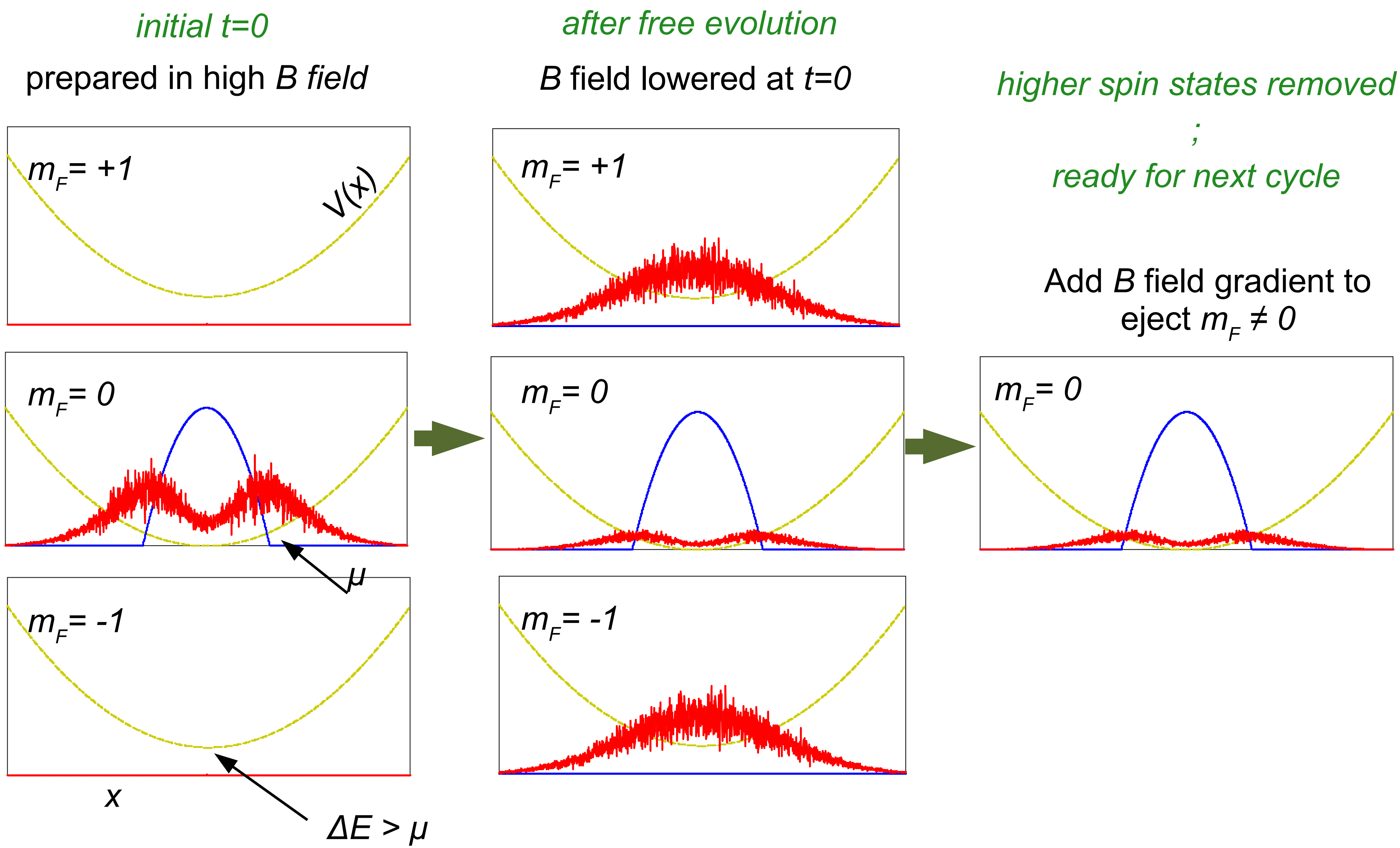
- Several hyperfine species of the same atom
- Atoms can change of species



Pasquiou, Marechal, Bismut, Pedri, Vernac, Gorceix, Laburthe-Tolra, PRL **106**, 255303 (2010)

Spin distillation cooling – concept

- schematic for the case of ^{23}Na (discussed in detail later)



Spin distillation cooling – Paris experiment

PRL **115**, 243002 (2015) PHYSICAL REVIEW LETTERS week ending 11 DECEMBER 2015

Cooling of a Bose-Einstein Condensate by Spin Distillation

B. Naylor,[1,2] E. Maréchal,[2,1] J. Huckans,[3,1] O. Gorceix,[1,2] P. Pedri,[1,2] L. Vernac,[1,2] and B. Laburthe-Tolra[2,1]
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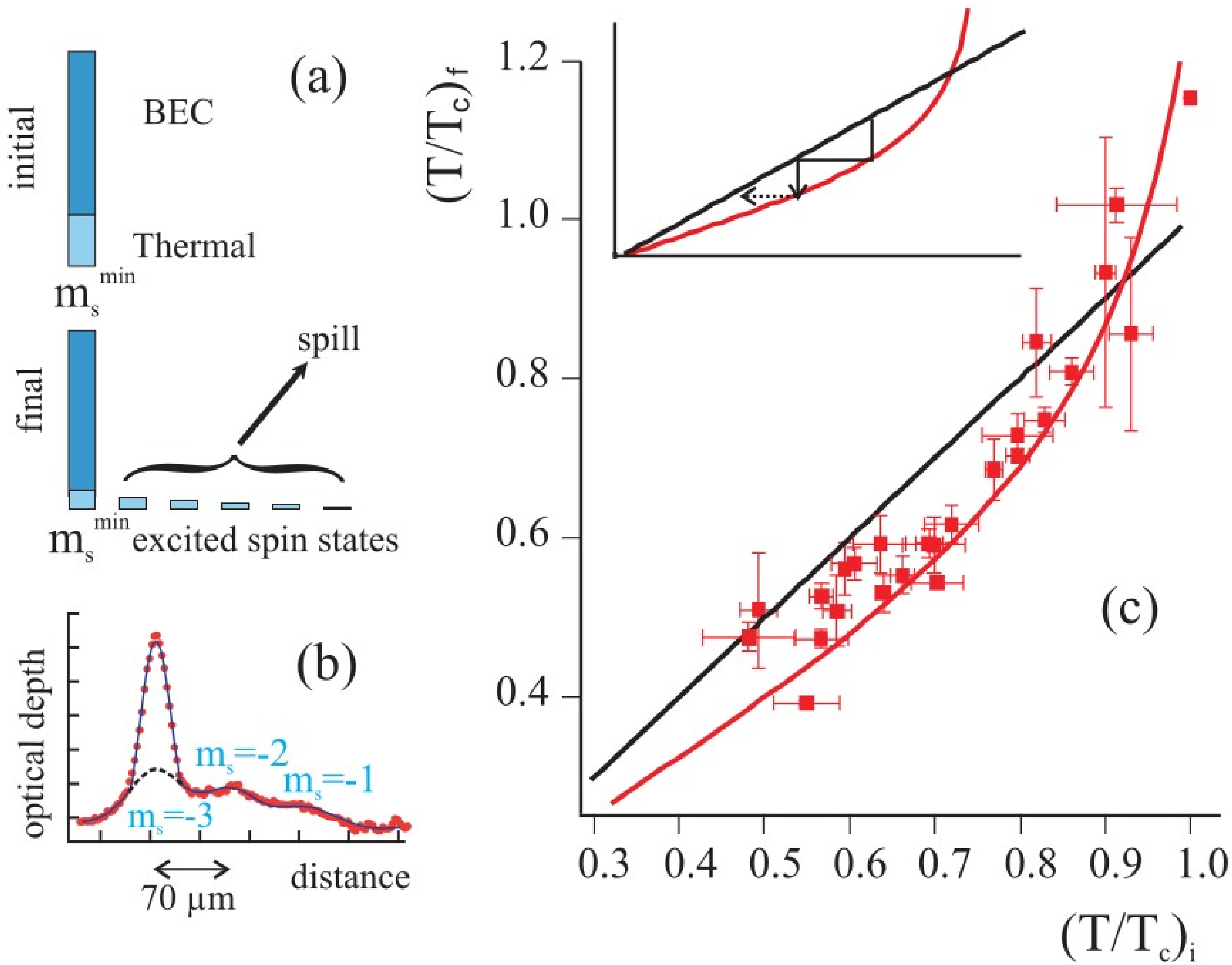
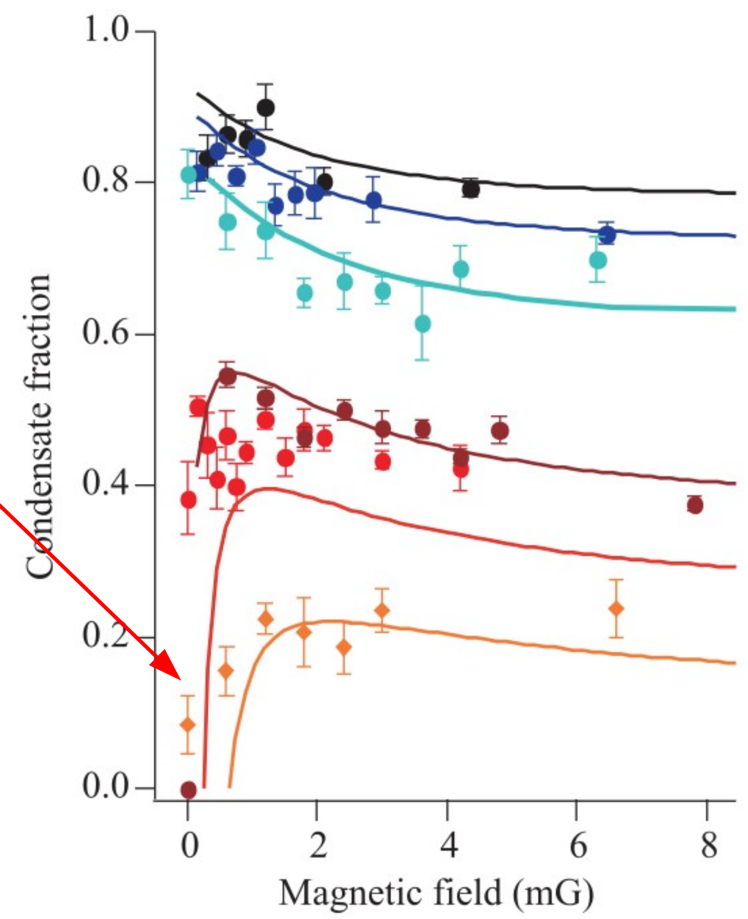
^{52}Cr



Open questions

- Can successive cycles lead to more cooling?
- What are the limitations / conditions needed?
    - how should magnetic field be changed in successive cycles?
- does it also work for ^{23}Na (suggested in the paper)
    - contact interactions only
    - using quadratic Zeeman effect
- why is there a worsening at small magnetic field?



Semiclassical field theory

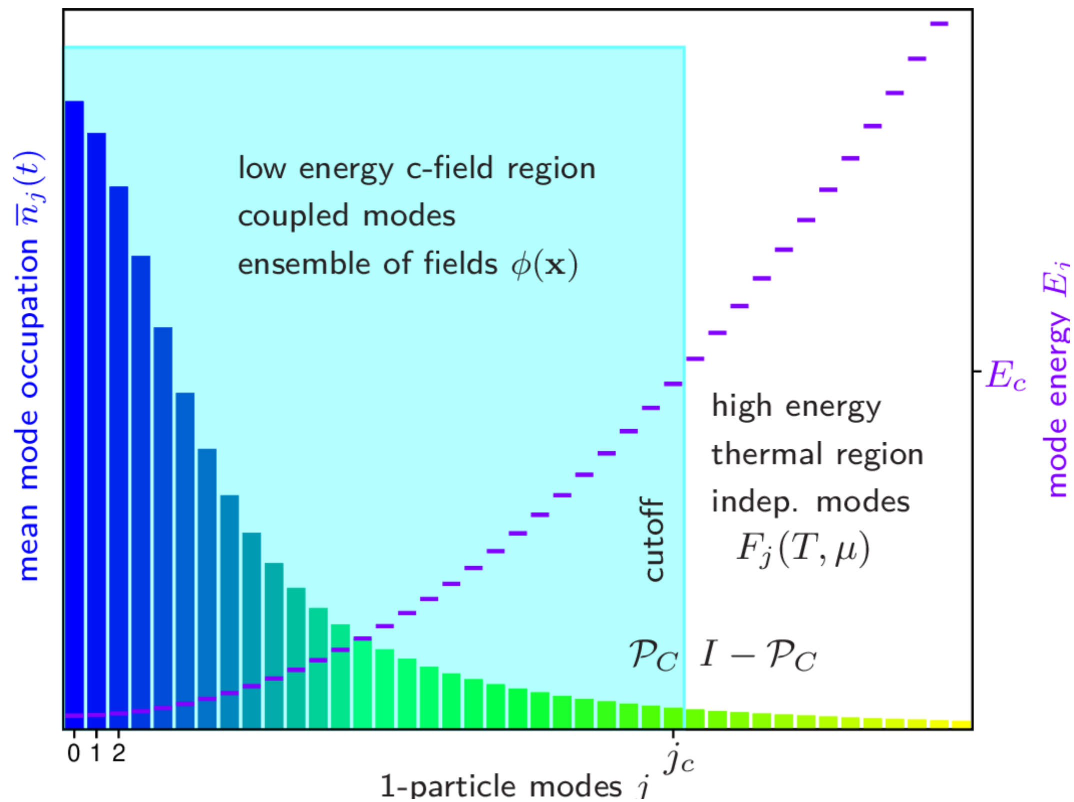
Bose field — semiclassical replacement

$$\widehat{\Psi}(\mathbf{x}) = \sum_j \widehat{a}_j \psi_j(\mathbf{x}) \longrightarrow \phi(\mathbf{x}) = \left\{ \sum_{j \in \mathcal{C}} \alpha_j \psi_j(\mathbf{x}) \right\}$$

$$\left[\widehat{a}_j, \widehat{a}_k^\dagger\right] = \delta_{jk}$$

$$\widehat{a}_j \gg 1 \ \rightarrow \widehat{a}_j \approx \alpha_j$$

Assuming high occupation:



Developed by many authors:

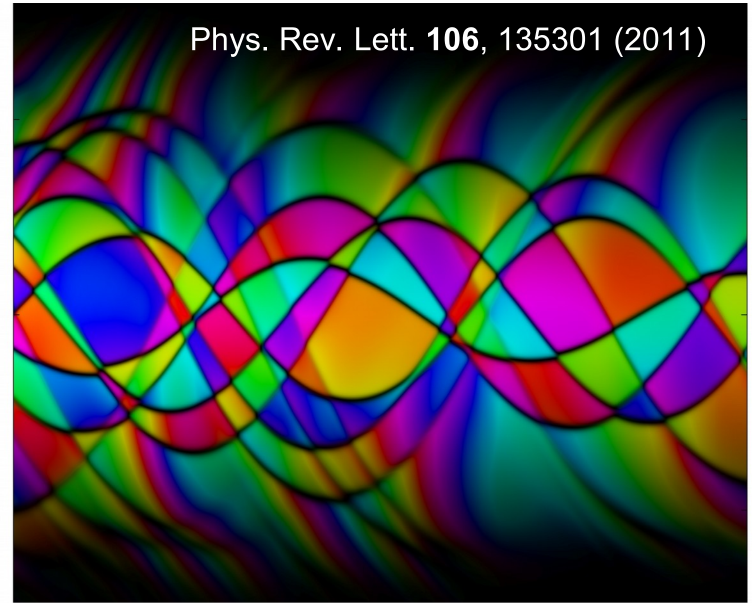
M. Brewczyk, M. Gajda, M. Davis, K. Rzazewski, A. Sinatra, K. Burnett, E. Witkowska, … (no priority implied)

*Useful Reviews: M. Brewczyk et al, J. Phys B **40,** R1 (2007); P. Blakie et al. Adv. Phys. **57**, 363 (2008)*

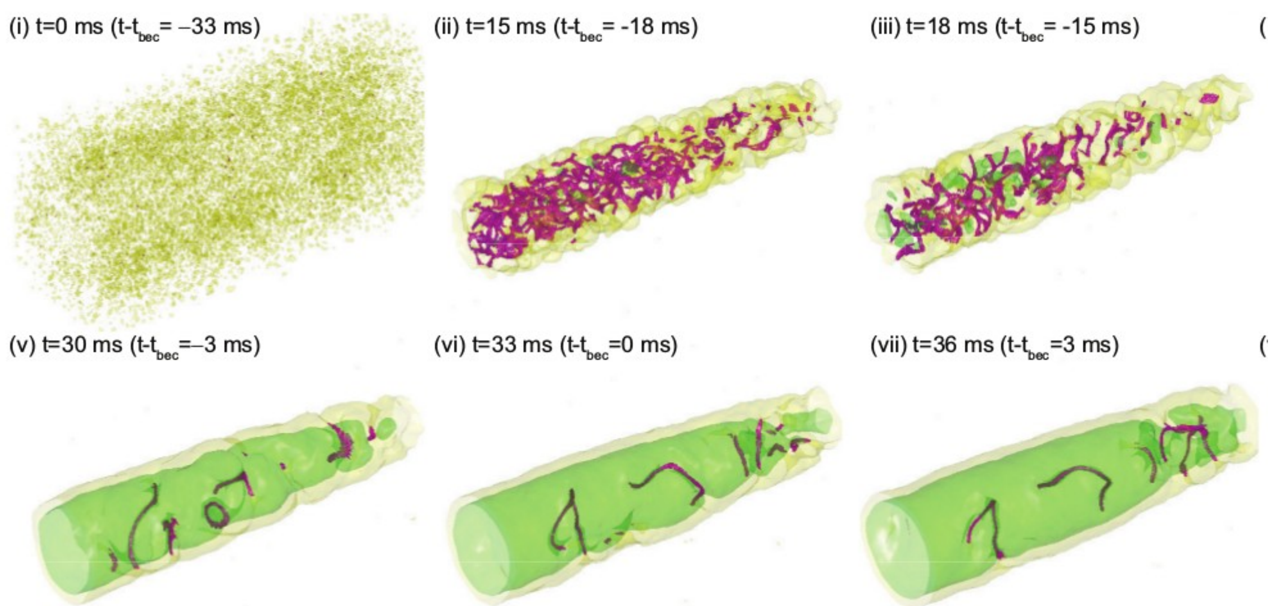
Notable qualities of the classical field method

- Treats both condensate and thermal part
- Good scaling: makes *10⁷* modes tractable
- Nonperturbative
- Single shots ~ single experimental realizations

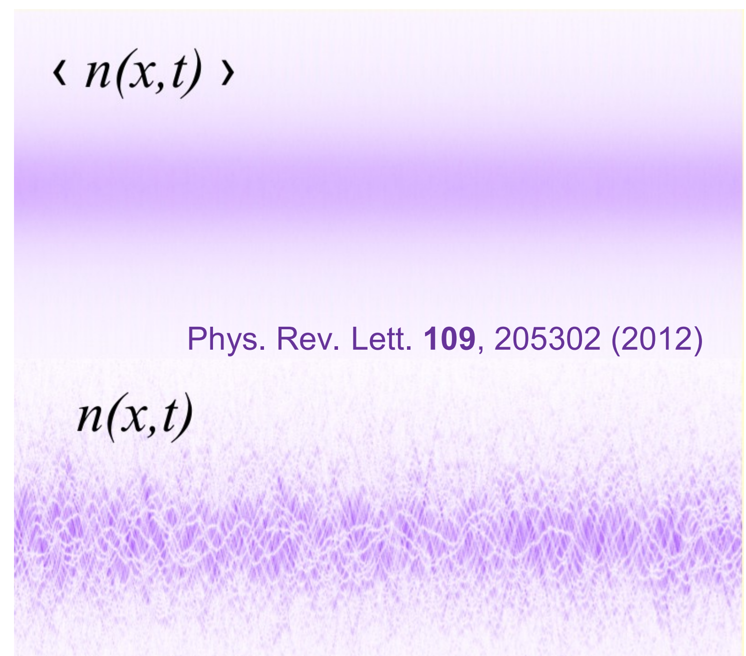
HERE ~ 4x10^5 sites in each spin component
~ 3x10^6 modes



1d gas – phase domains after cooling



Liu, Donadello, Lamporesi, Ferrari, Gou, Dalfovo, Proukakis, Commun. Phys. **1**, 24 (2018)



1d gas – thermal equilibrium

Initial state generation: 1 component Stochastic GPE

$$\hbar\frac{d\psi_C(\mathbf{x})}{dt} = \mathcal{P}_C\Big\{ -iH_{\rm GPE}\,\psi_C(\mathbf{x}) - \gamma\,(H_{\rm GPE}-\mu)\,\psi_C(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\,\eta(\mathbf{x},t)\Big\}$$

Projection onto low energy subspace (cutoff)

Hamiltonian dynamics

loss of particles to tails (dissipation)

chempot sets *N*

gain of particles from tails (fluctuation)

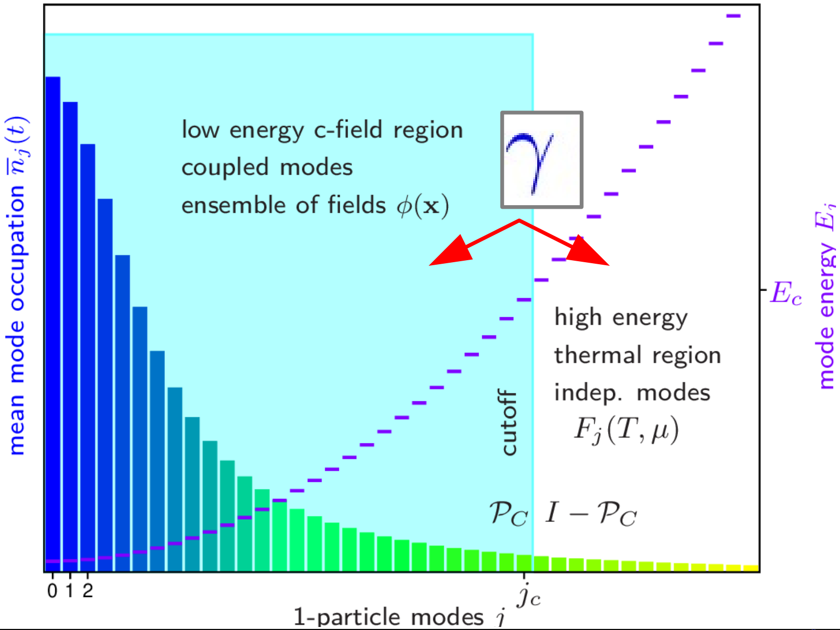
$$H_{\rm GPE} = -\frac{\hbar^2}{2m}\nabla^2 + V_{\rm trap}(\mathbf{r}) - \boldsymbol{\mu}\cdot\mathbf{B} + g|\Psi(\mathbf{r},\tau)|^2 + H_d(\Psi)$$

complex noise

$$\langle\eta(\mathbf{x},t)^*\eta(\mathbf{y},t')\rangle = \delta^d(\mathbf{x}-\mathbf{y})\delta(t-t')$$

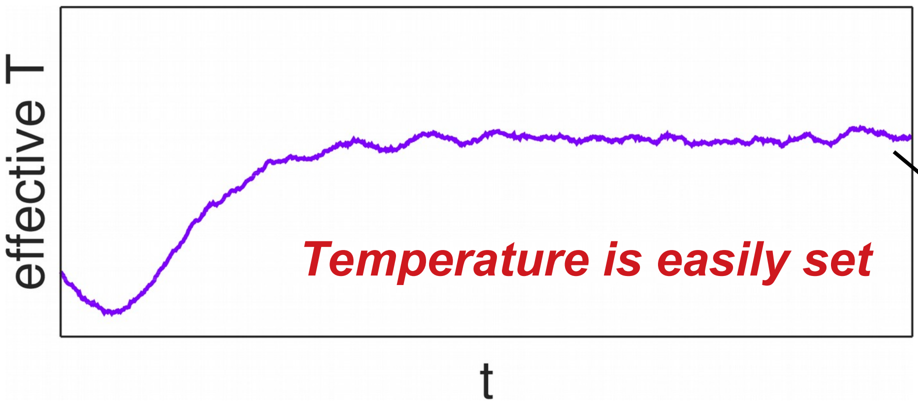
Compare to GPE:

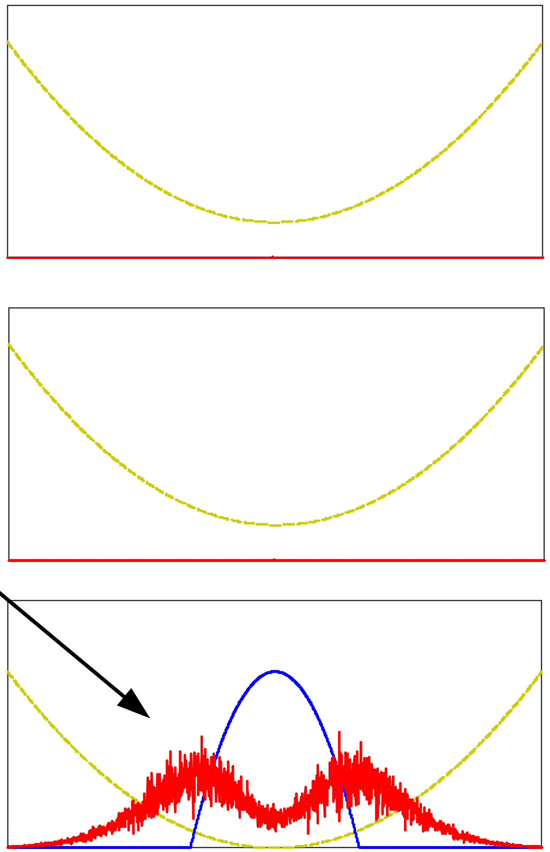
$$i\hbar\frac{d\psi_C(\mathbf{x})}{dt} = \mathcal{P}_C\Big\{H_{\rm GPE}\,\psi_C(\mathbf{x})\Big\}$$



Initial state generation: SGPE

Thermalised ensemble at long time



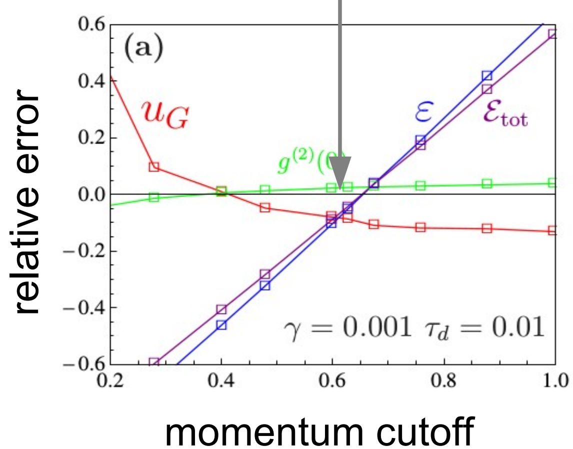


Cutoff and numerical lattice:

- k^{max} : optimised for several variables according to

  Pietraszewicz, PD, PRA **92**, 063620 (2015)
  Pietraszewicz, PD, PRA **98**, 023622 (2018)

- volume: chosen to match known condensate fraction



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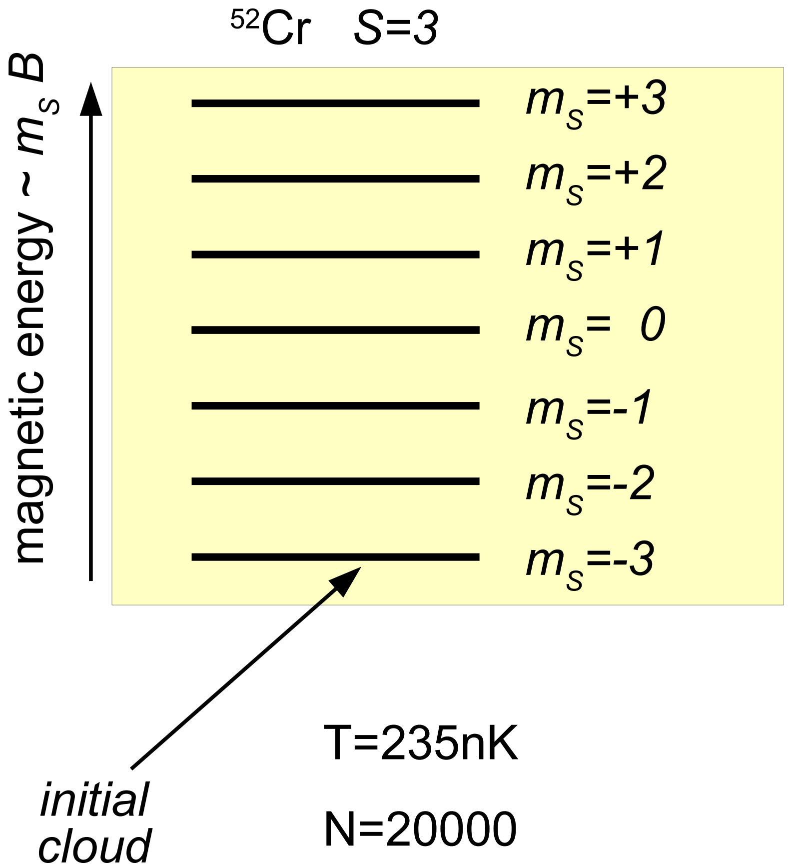
Chromium ^{52}Cr   S=3

Parameters as in experiment



T=235nK

N=20000

cond frac 55%

Seven spinor components

$$\boldsymbol{\psi}(\mathbf{r}) = (\psi_3(\mathbf{r}), \psi_2(\mathbf{r}), \psi_1(\mathbf{r}), \psi_0(\mathbf{r}), \psi_{-1}(\mathbf{r}), \psi_{-2}(\mathbf{r}), \psi_{-3}(\mathbf{r}))'$$

$$H_{\rm sp} = -\frac{\hbar^2}{2m}\nabla^2 + V_{\rm trap}(\mathbf{r}) - \boldsymbol{\mu}\cdot\mathbf{B}$$

$$\boldsymbol{\mu} = g_L \mu_B m_s$$

Linear Zeeman effect

Trap frequencies $(250, 300, 215)\,\mathrm{Hz}$

Evolution during cooling simulation

According to “plain” 7-component GPE with dipolar interactions:

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{\psi}(\mathbf{r}) = (H_{\mathrm{sp}} + H_c + H_d)\,\boldsymbol{\psi}(\mathbf{r})$$

$$\boldsymbol{\psi}(\mathbf{r}) \quad = \quad (\psi_3(\mathbf{r}), \psi_2(\mathbf{r}), \psi_1(\mathbf{r}), \psi_0(\mathbf{r}), \psi_{-1}(\mathbf{r}), \psi_{-2}(\mathbf{r}), \psi_{-3}(\mathbf{r}))'$$

$$H_{\mathrm{sp}} = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}}(\mathbf{r}) - \boldsymbol{\mu}\cdot\mathbf{B}$$

H_c is a 7×7 matrix in spinor components

H_d is the dipolar interaction term. It will not be written down today.

Two conditions on *B* for successful cooling

1) Thermal energy should be sufficient to overcome the magnetic energy barrier

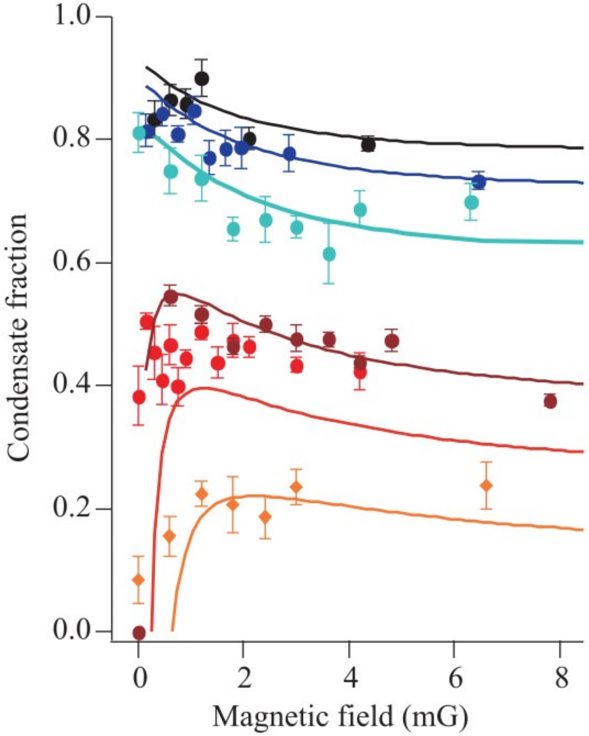
$$2\mu_B \mathrm{B} \lesssim k_B T,$$

2) The magnetic field should be high enough that the condensate ground state remains polarised and **cannot** overcome the magnetic energy barrier

$$\mathrm{B} > B_{\mathrm{th}} = 0.68 \frac{2\pi\hbar^2(a_6 - a_4)}{2m\mu_B} n$$

Santos, Pfau, PRL **96**, 190404 (2006)
Diener, Ho, PRL **96**, 190405 (2006)

scattering lengths in the total spin 6 and 4 channels



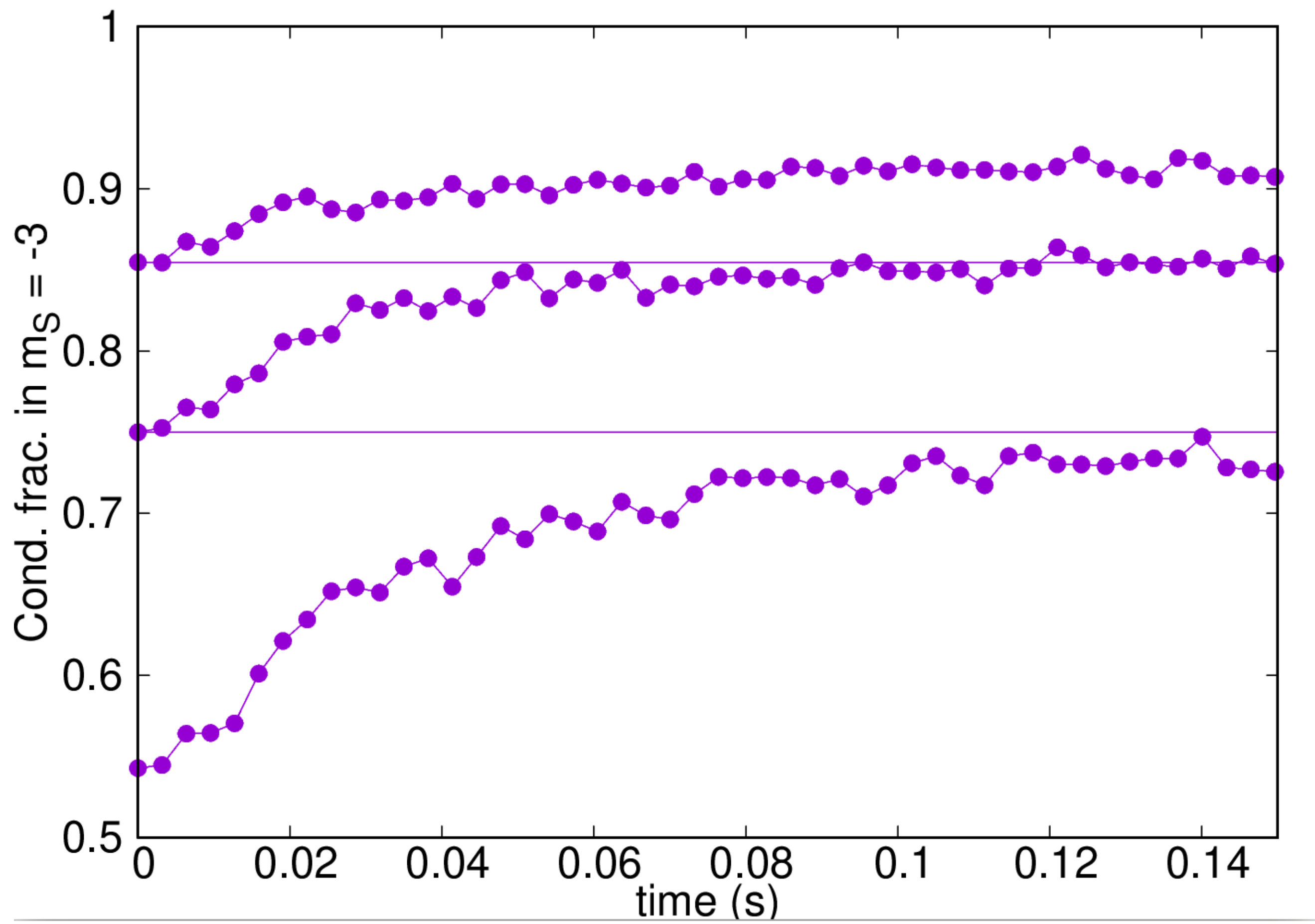
Combining conditions leads to

$$k_B T \gtrsim 0.34 \left(\frac{a_6 - a_4}{a_6} \right) \mu$$

for our system this gives *0.05Tc, 0.13 μ* --- very low

Simple cooling cycles

simple choice of magnetic field using initial *T* $\mathrm{B} = k_B T / 2\mu_B$

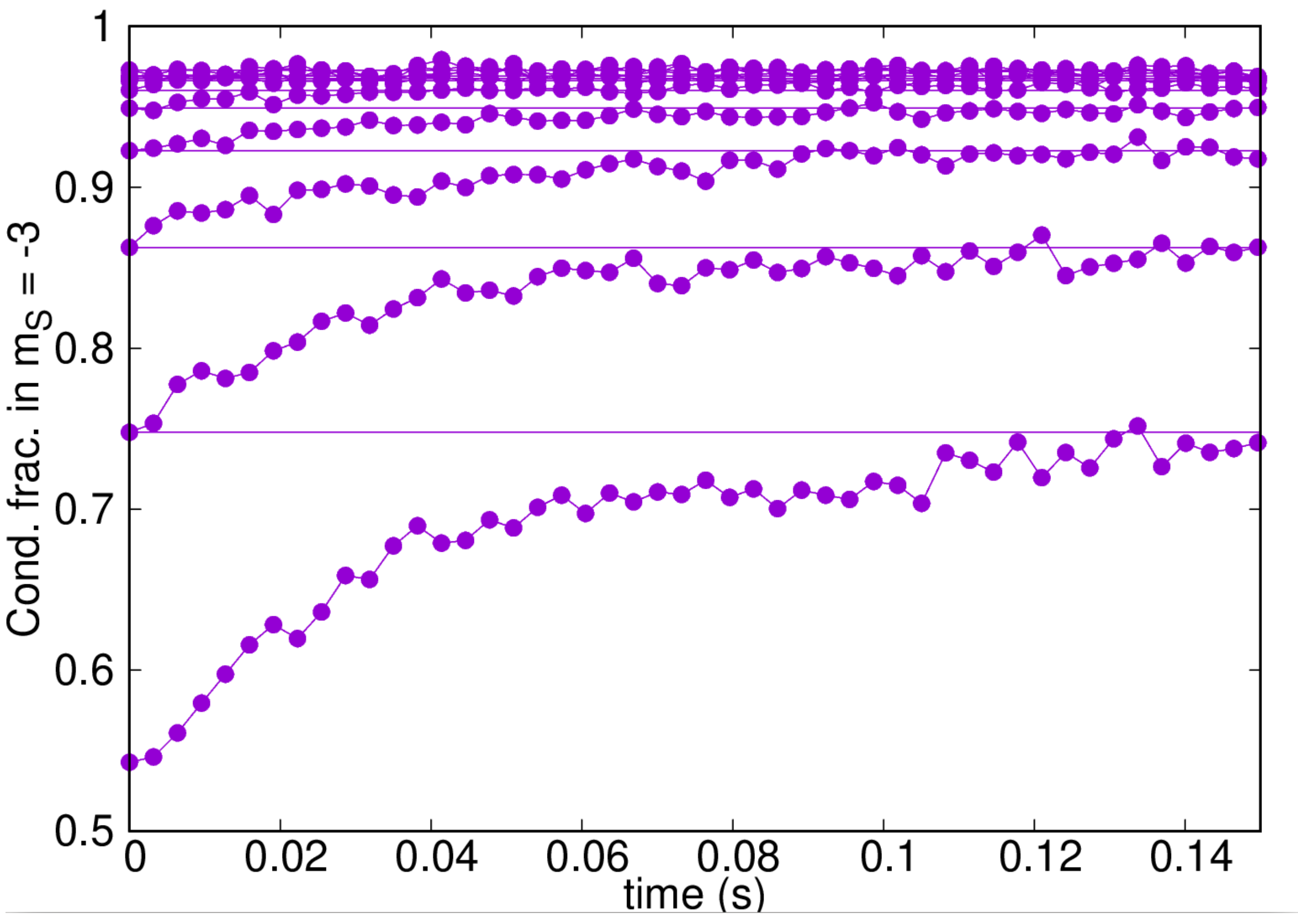


So .... successive cooling cycles give more cooling

but limited by $2\mu_B \mathrm{B} \lesssim k_B T$.

adaptive *B*

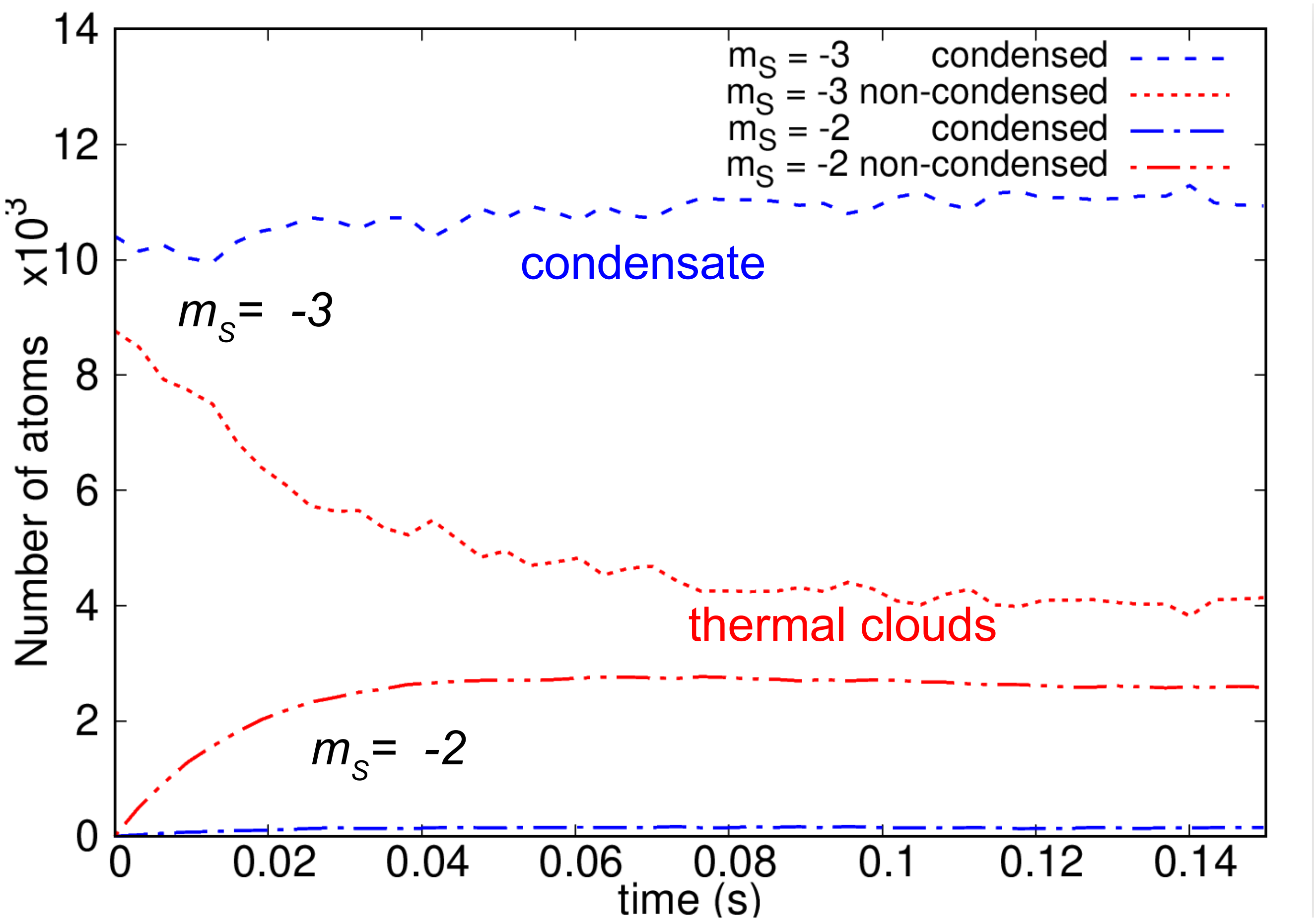
Adapting *B* after every cycle $\mathrm{B} = k_B T / 2\mu_B$



Limited by minimum *B*

$$\mathrm{B} > B_{\mathrm{th}} = 0.68 \frac{2\pi\hbar^2 (a_6 - a_4)}{2m\mu_B} n$$

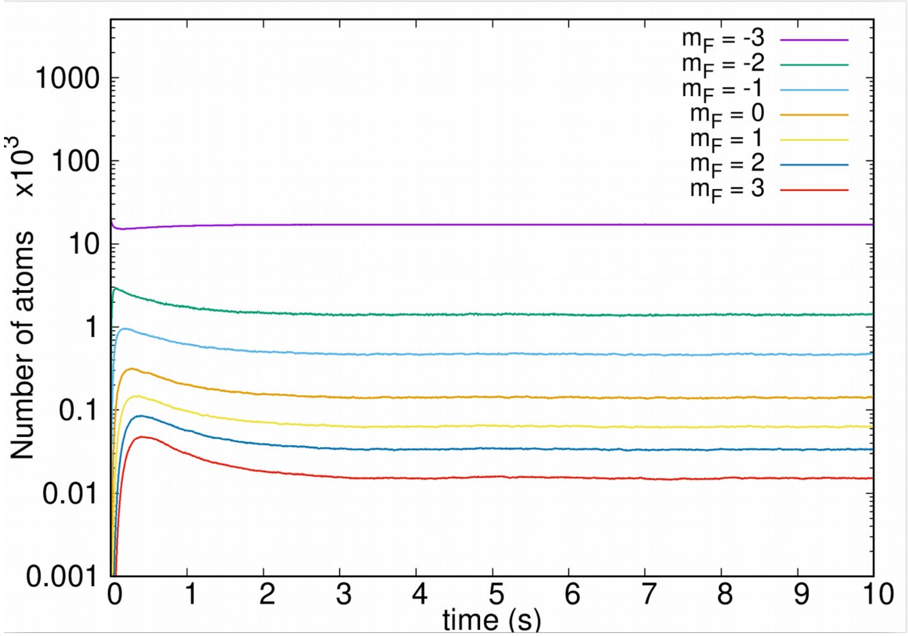
Single cooling cycle - details



Threshold vs no-threshold

populations of spin states

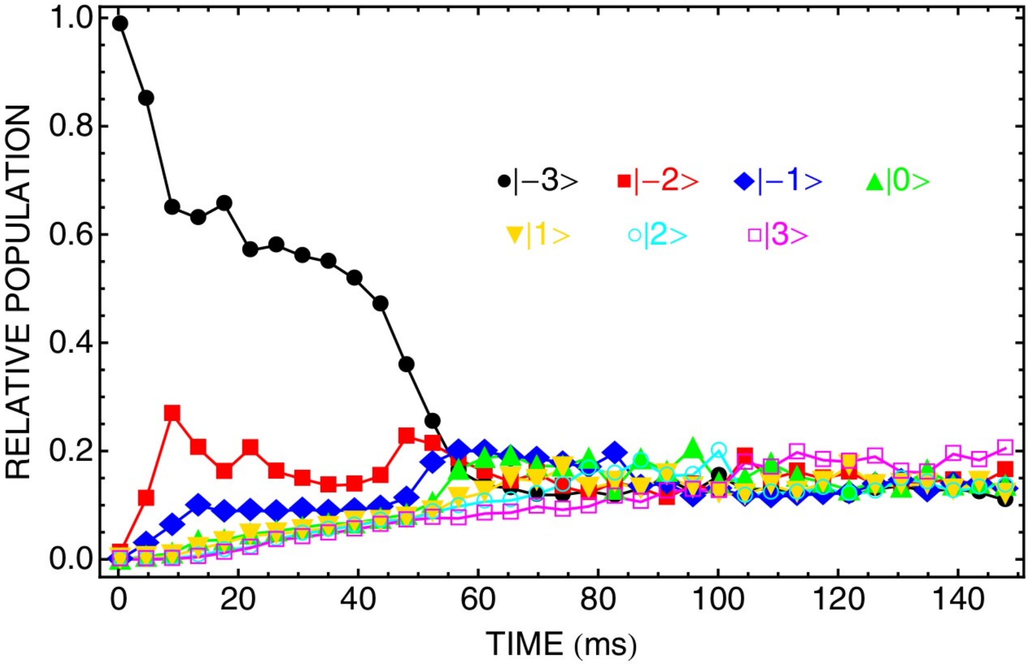
HERE: magnetic threshold



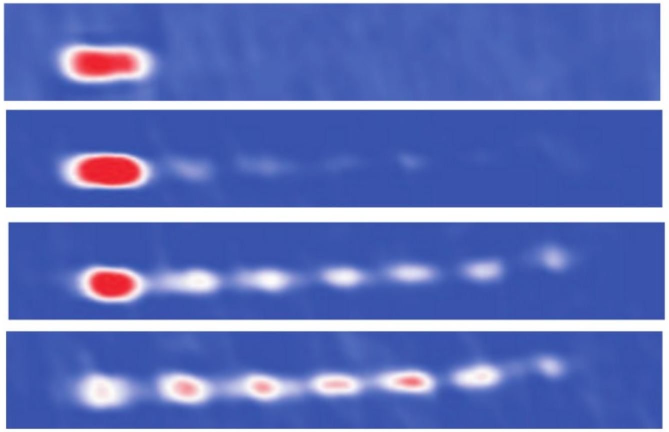
Stationary but not thermalised

no threshold

Swisłocki, Bauer, Gajda, Brewczyk PRA **89**, 023622 (2014)



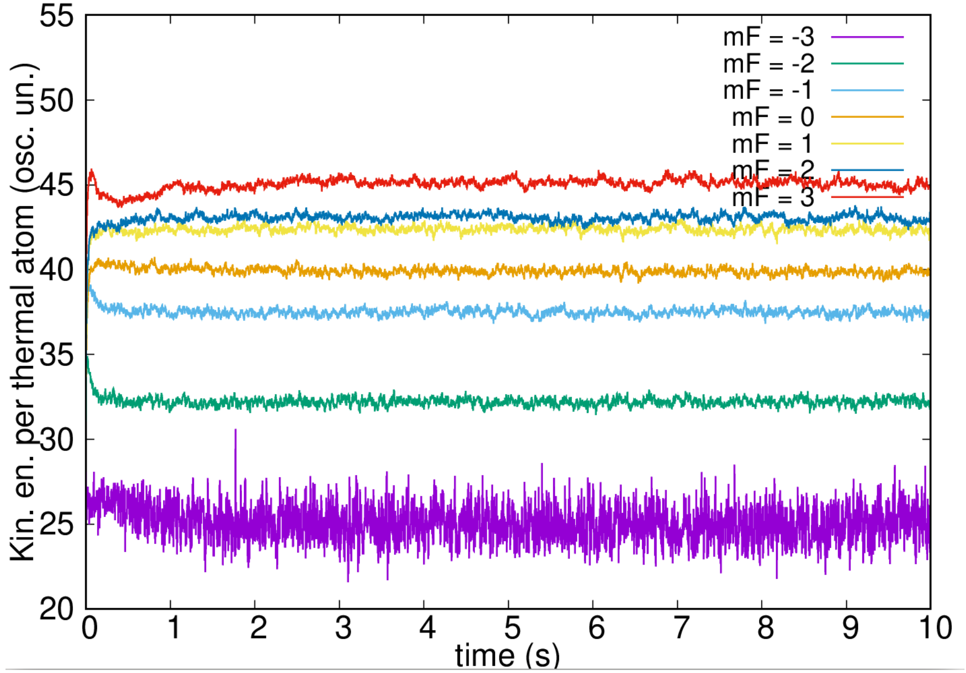
BEC components mutually thermalise



→ different mechanisms

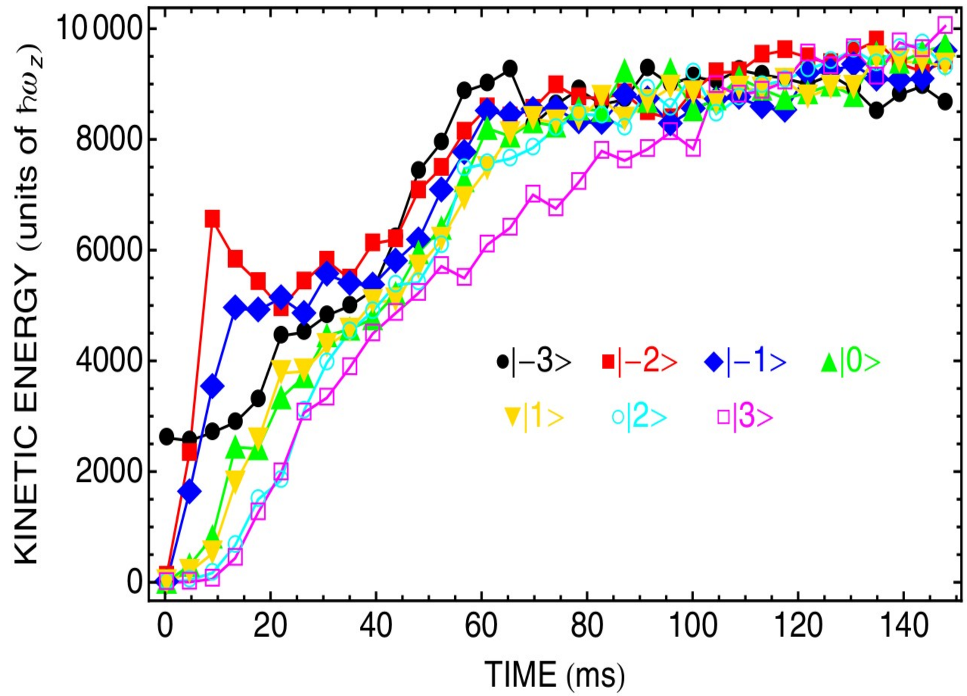
mean kinetic energy of thermal atoms

HERE: magnetic threshold



Stationary but not thermalised

no threshold

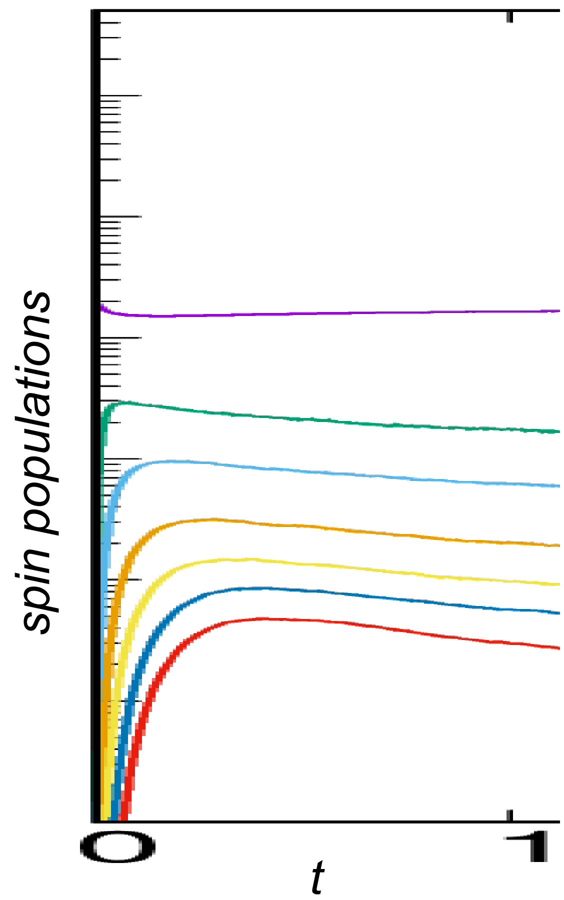


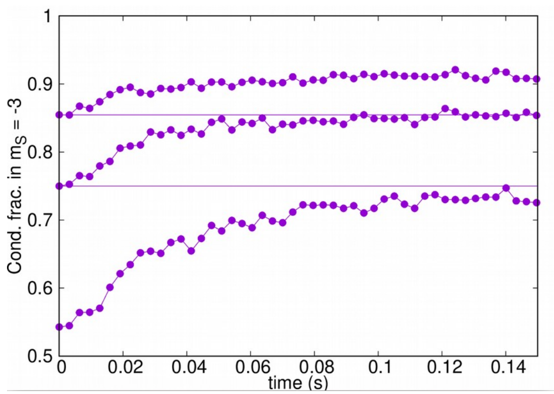
thermalised

What's the mechanism here? It's not thermalisation.....

hypotheses for the mechanism

- no threshold: thermalised condensates formed in all spin states
  Swisłocki, Bauer, Gajda, Brewczyk PRA **89**, 023622 (2014)
- here:
  - BEC cannot jump the threshold energy
  - "thermal clouds" not mutually thermalised.
  - which process is responsible for populating higher spin states?
- no contact interaction process for transfer from m_S=-3 (m_S must be conserved)
  - → only leaves dipole scattering
- Thermal-thermal scattering timescale: at least several seconds (too long)
- Timescales (rough estimate) for scattering off condensate mode:
  - $m_s=-3_{BEC}$ & $m_s=-3_{BEC}$ → $m_s=-3_{BEC}$ & $m_s=-2_{therm}$ ~ 1s (need better estimate)
  - $m_s=-3_{BEC}$ & $m_s=-2_{therm}$ → $m_s=-3_{BEC}$ & $m_s=-1_{therm}$ ~ 100 s (ruled out)
  - Thermal-thermal scattering timescale: at least several seconds (too long)
- Therefore: all processes involving two thermal atoms are far too slow
- Simulations suggest similar timescales for population of all higher spin states. Perhaps populations decrease because less states are energetically available.
- Hypothesis is that non-thermalised one-time scattering off condensate mode is responsible for the transfer:
  - $m_s=-3_{BEC}$ & $m_s=-3_{BEC}$ → $m_s=-3_{BEC}$ & $m_s>-3_{therm}$
- more quantitative analysis in progress.....



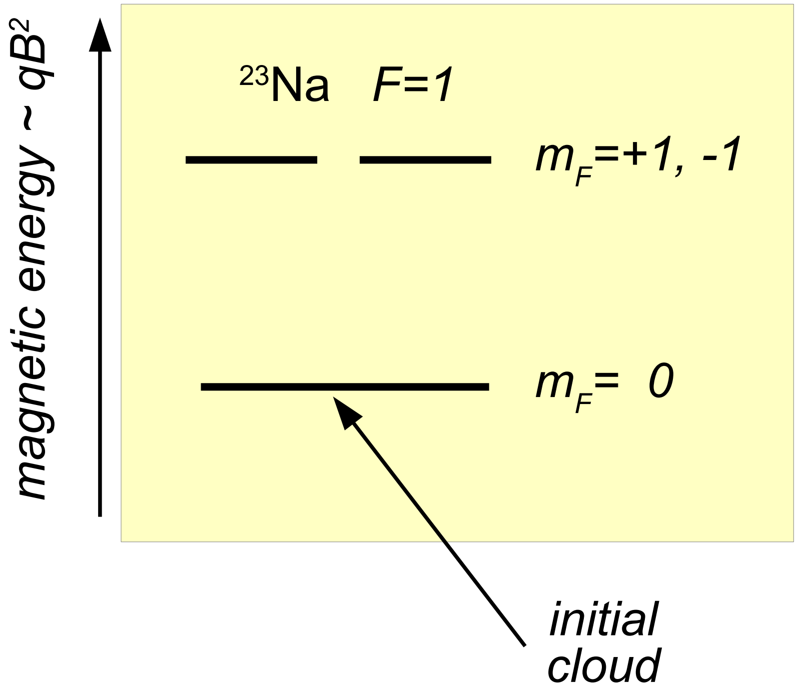


Was conjectured to also allow cooling via the quadratic Zeeman effect,

purely through contact spin-dependent interactions

Naylor, Marechal, Hackens, Gorceix, Pedri, Vernac, Laburthe-Tolra, PRL **115**, 243002 (2015)

Three quasispin components $\boldsymbol{\psi} = (\psi_1, \psi_0, \psi_{-1})$



Quadratic Zeeman effect is relevant here

$$H_{\mathrm{QZE}} = -q \int d\mathbf{r}\; n_0(\mathbf{r})$$

$$q = \alpha_q |\mathbf{B}|^2$$

$$\alpha_q / 2\pi\hbar \approx 277\mathrm{Hz}/G^2$$

The rest of the ^{23}Na Hamiltonian

$$H_s = \int d\mathbf{r} \sum_{m_F=-1}^{1} \psi^*_{m_F}(\mathbf{r}) \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{trap}} \right) \psi_{m_F}(\mathbf{r})$$

$$+ \int d\mathbf{r} \left(\frac{c_0}{2} n(\mathbf{r})^2 + \frac{c_2}{2} \mathbf{F}(\mathbf{r})^2 \right). \quad (7)$$

$$\mathbf{F} = (\psi^\dagger F_x \psi, \psi^\dagger F_y \psi, \psi^\dagger F_z \psi)$$

$$n(\mathbf{r}) = \sum_{m_F} n_{m_F}(\mathbf{r}) = \sum_{m_F} |\psi_{m_F}(\mathbf{r})|^2$$

$$c_0 = 4\pi\hbar^2(2a_2 + a_0)/3m$$ spin-independent; large

$$c_2 = 4\pi\hbar^2(a_2 - a_0)/3m$$ spin-dependent; small

$$a_0 = 50 \text{ and } a_2 = 55$$

Equations of motion

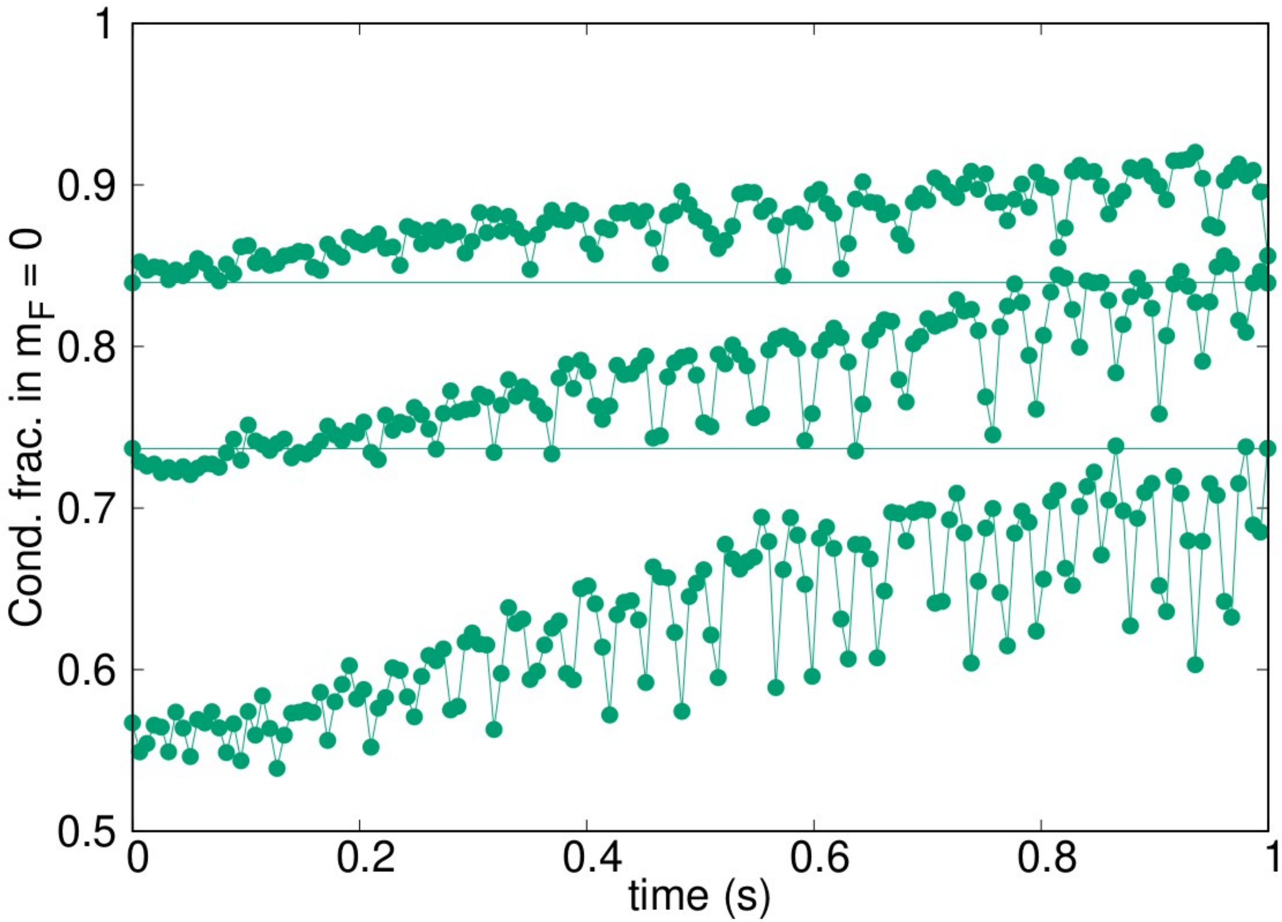
$$i\hbar\frac{\partial\psi_0}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{trap}} + c_0\, n - q\right)\psi_0 +$$

$$+ c_2\left[(n_1 + n_{-1})\psi_0 + 2\psi_0^*\psi_1\psi_{-1}\right],$$

$$i\hbar\frac{\partial\psi_{\pm1}}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\text{trap}} + c_0\, n\right)\psi_{\pm1} +$$

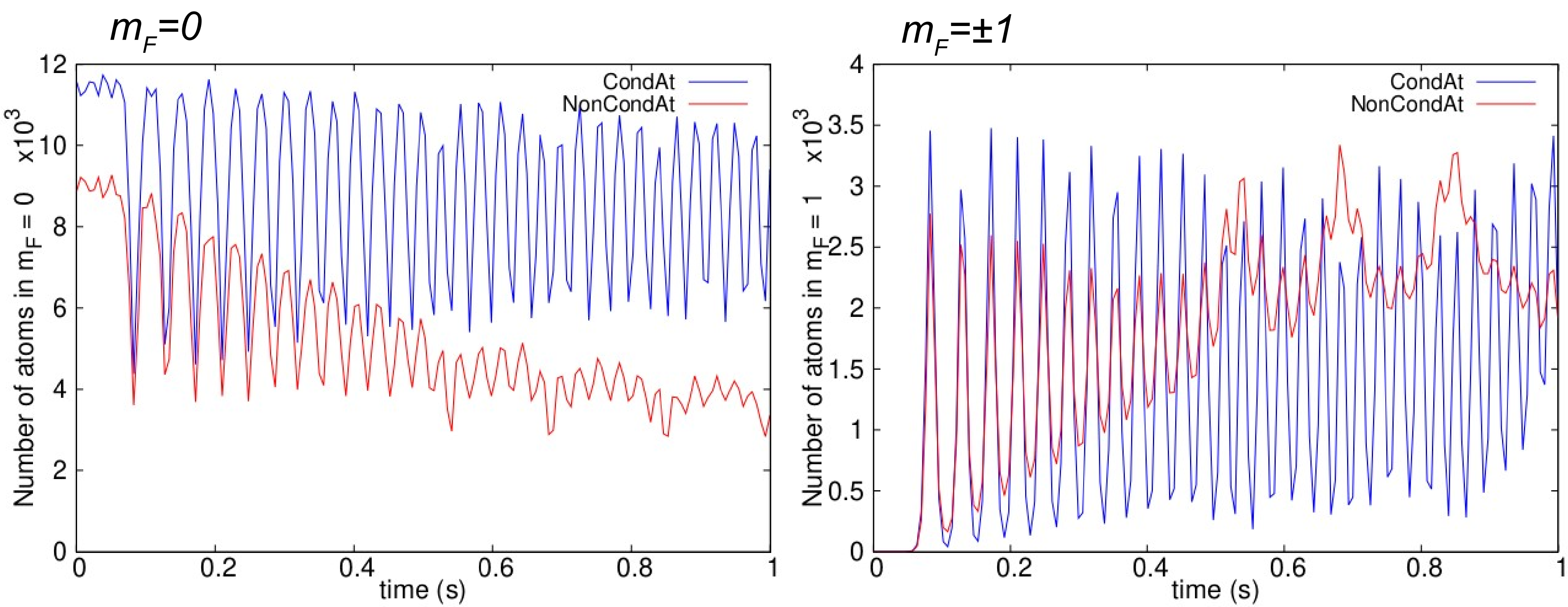
$$+ c_2\left[(n_{\pm1} - n_{\mp1} + n_0)\psi_{\pm1} + \psi_{\mp1}^*\psi_0^2\right]$$

Sodium cooling cycles



q=0.011 B=100 mG

A closer look

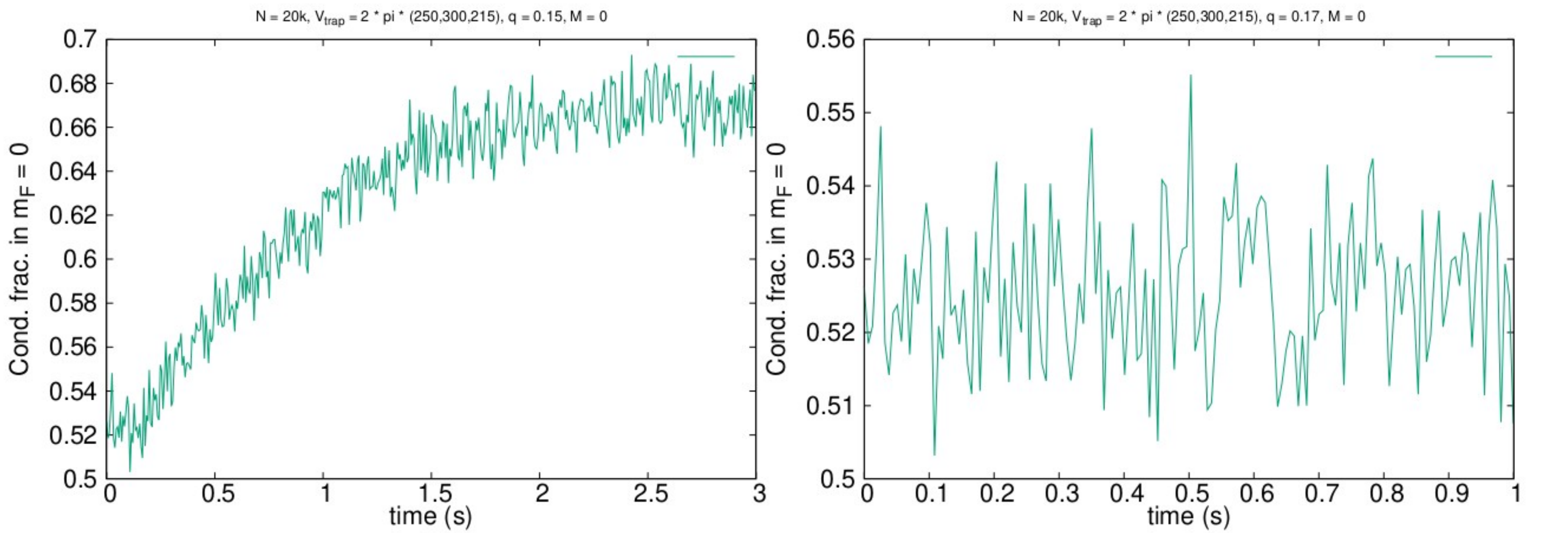


condensed and thermal atoms

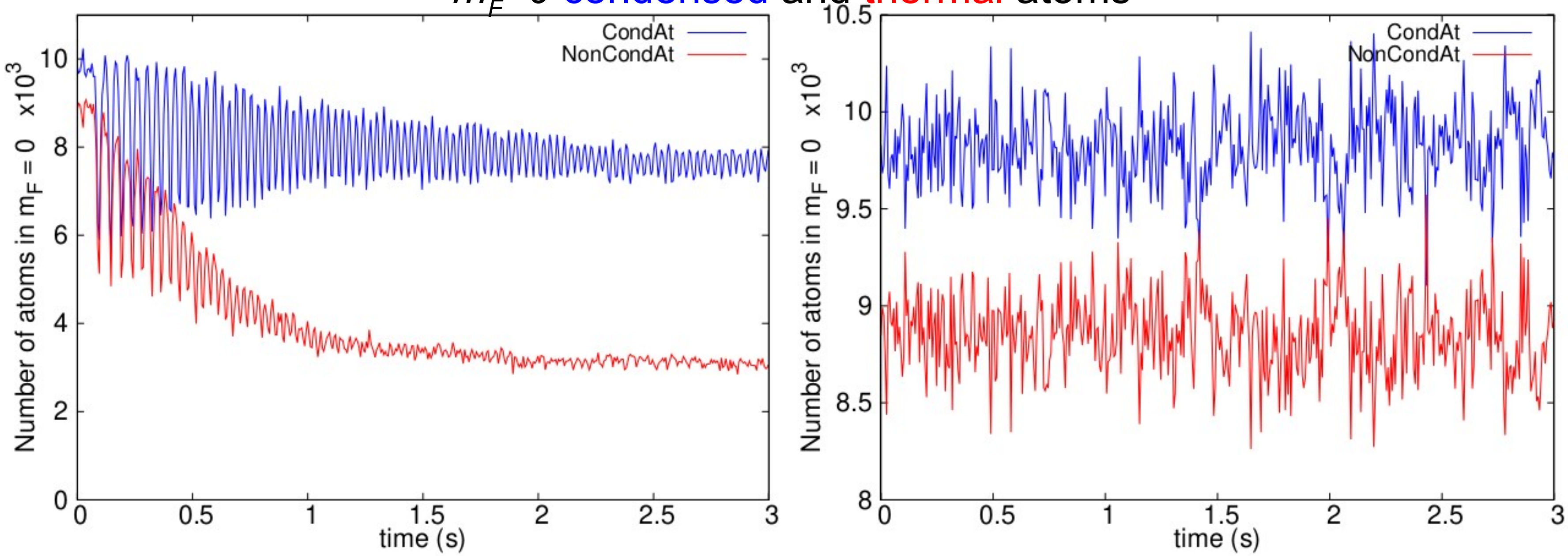
- Hmm, kind of different to Cr ....
- Fast Rabi oscillation timescale
- Condensate spreads easily into all spin states

max B threshold for cooling

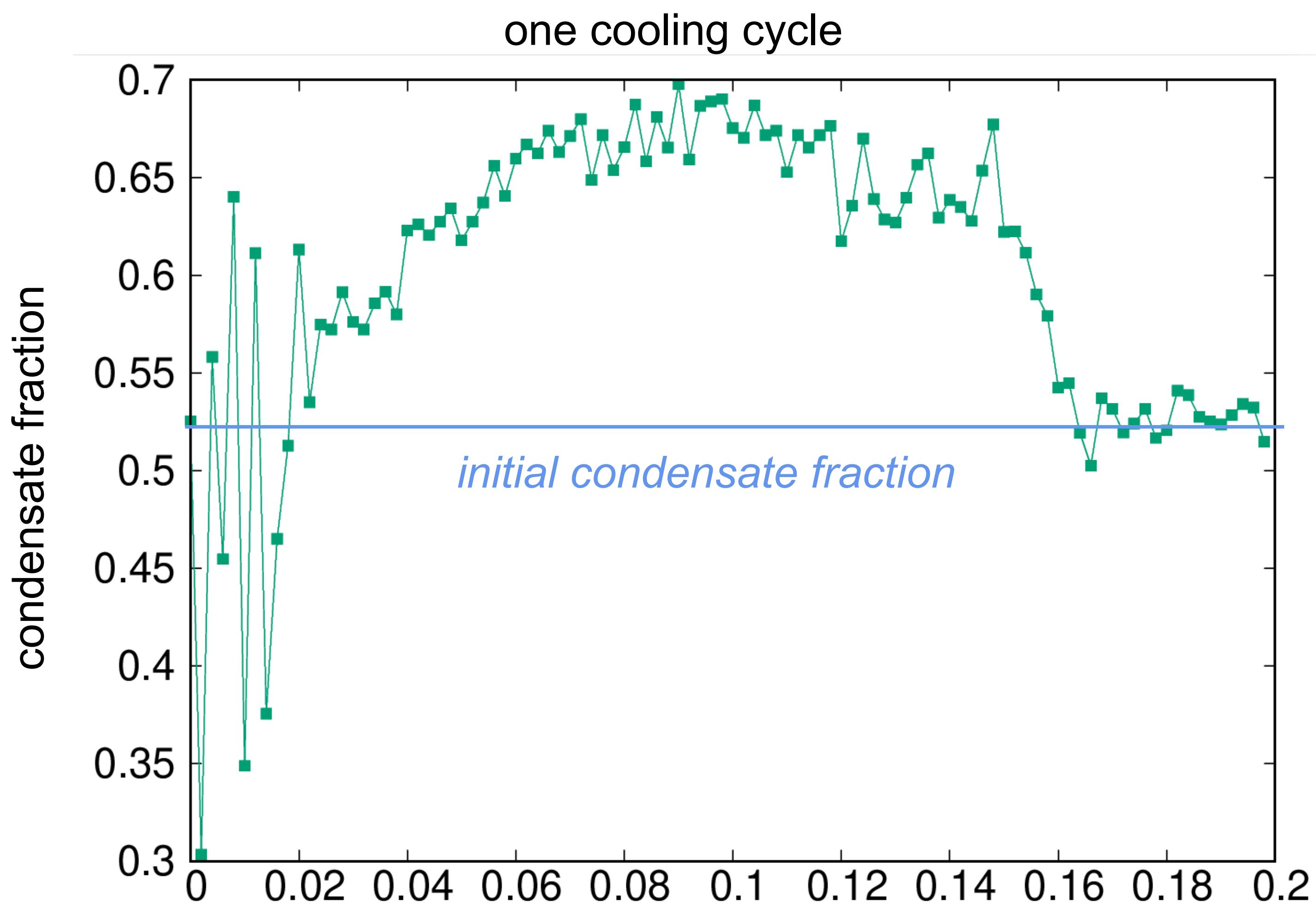
q=0.15 B=368 mG | *q=0.17 B=392 mG*



$m_F=0$ condensed and thermal atoms



Magnetic field dependence



$$q = \alpha_q |\mathbf{B}|^2$$

$k_B T/q \approx 100$, or more, in this whole range.

→ magnetic energy is tiny compared to thermal!

why does it cool at all?

Why a threshold at low magnetic field?

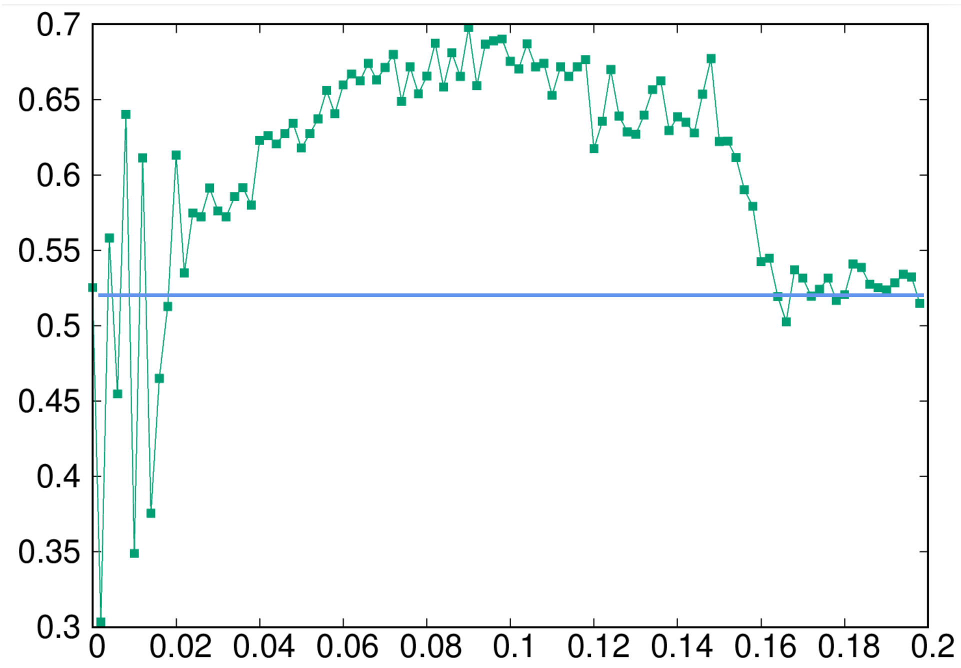
- No spin changing collisions near ground state, zero magnetization

unless $$q < \frac{c_2 n}{2}$$

Stenger, Inouye, Stamper-Kurn, Miesner, Chikkatur, Ketterle, Nature **396**, 345 (1998)

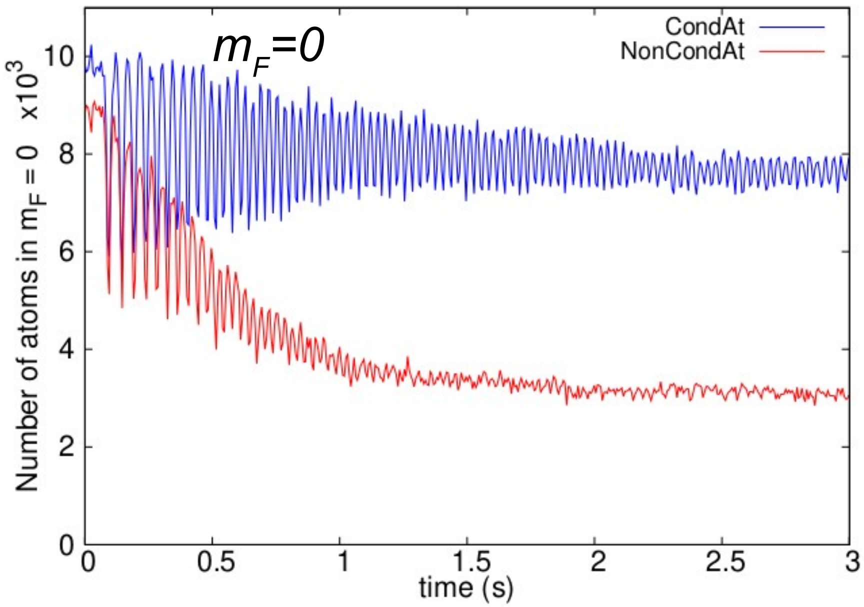
plugging in, get

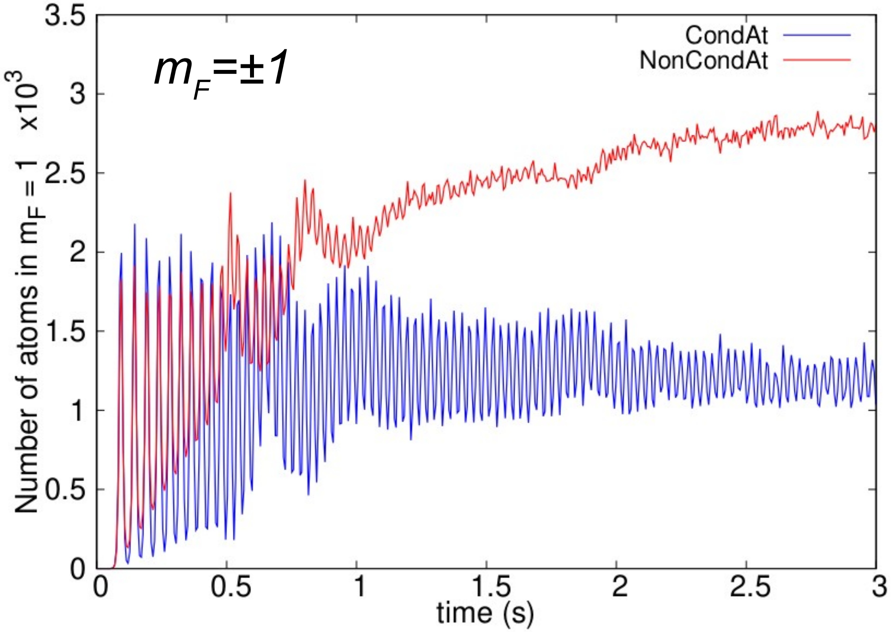
$$q < 0.1$$



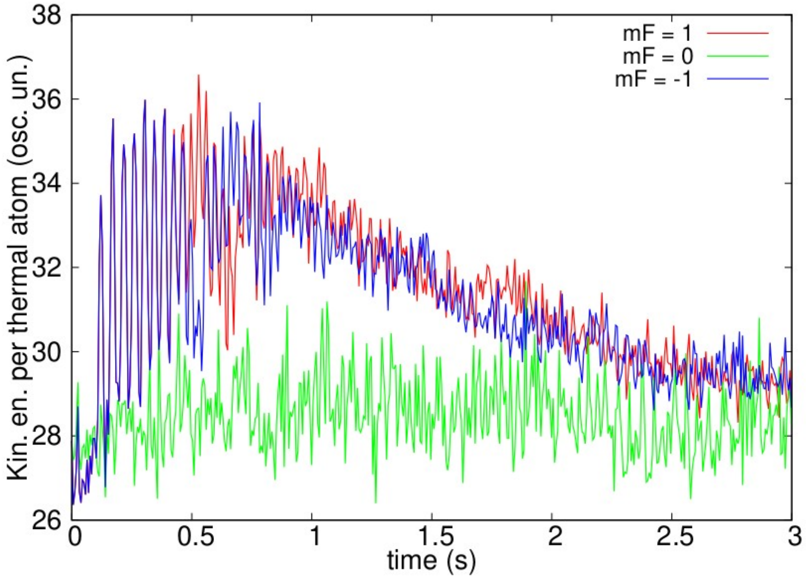
How does it cool if condensate also spreads?

popuations of thermal and condensate atoms





kinetic energy per *thermal* atom



long times:

- equal thermal populations
- equal thermal energies
- UNequal condensate populations

  (for some reason...)
- More condensate remains in mF=0

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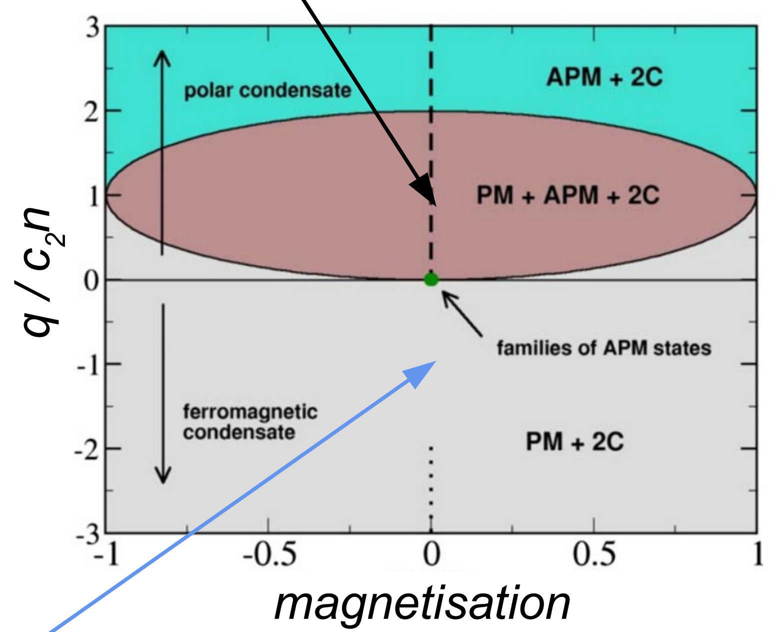
Hypothesis..

we're here



- Equal thermal population in all 3 clouds is no surprise

  (magnetic energy is negligible compared to $k_B T$)

- But – not equal in condensate, despite very many Rabi oscillations

  - This appears to rely on properties of spinor ground states?

    Matuszewski, Alexander, Kivshar, PRA **80,** 023602 (2009)

- Expect cooling when groundstate fraction of $m_F=0$ is > 1/3

  (since 1/3 of thermal cloud remains in each spin state)

  - best when groundstate fraction of $m_F=0$ is close to 1.

  - Many cooling rounds lead to zero magnetisation.

    - In that case, a groundstate fraction of $m_F=0$ close to 1 seems to desire

      opposite sign on q and c_2

      (different than what we simulated so far....) *(drat !)*

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Mechanisms

- There seem to be two variants

  * ^{52}**Cr**: T-dependent $B > B_{th}$
    - Only thermal atoms are scattered, mostly to nearest state
    - No spin changes in condensate
    - scattering due to dipolar interactions
    - magnetic threshold large compared to spin-dependent interaction energy.
    - Temperature limitation but low.

  * ^{23}**Na**: n-dependent $B < B_{max}$
    - Thermal atoms spread evenly
    - condensate atoms also spread, but unevenly
    - effectiveness presumably depends on spinor ground state
    - insensitive to temperature

- Both cool quite effectively

Conclusions

- Repeated cooling cycles check out in realistic simulations

  Best to adapt B after each cooling cycle

- Looks like two mechanisms *with common similarities*

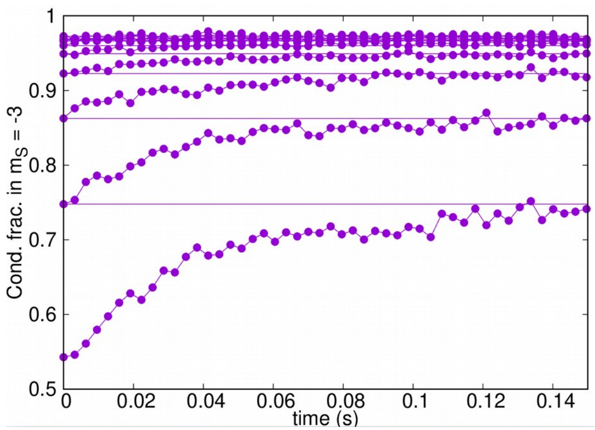
- Chromium:

  scattering of only thermal atoms into higher states

  thanks to dipolar interaction

  B(T)

  B > threshold



- Sodium:

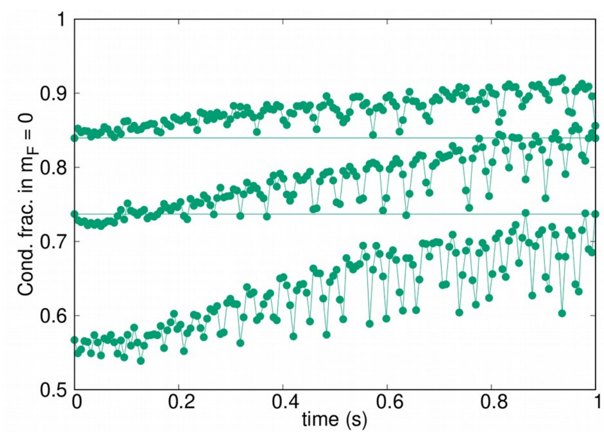
  thermal atoms scatter evenly

  condensate atoms distribute unevenly

  thanks to contact interaction

  B(n)

  B < threshold



- Some further clarifications needed:

  scattering rates in Chromium, best conditions for Sodium