Quasicondensate dynamics including both thermal and quantum fluctuations

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Thermal and quantum fluctuations

Thermal fluctuations:

$$\rho = n_0|\Psi_G\rangle\langle\Psi_G| \;+\; n_1|\Psi_1\rangle\langle\Psi_1| \;+\; n_2|\Psi_2\rangle\langle\Psi_2| + \ldots$$

(measuring components of a mixture)

Quantum fluctuations:

$$|\Psi_G\rangle = c_1|\Psi_1\rangle \;+\; c_2|\Psi_2\rangle \;+\; c_3|\Psi_3\rangle \;+ \ldots$$

(measuring in a basis mismatched to the state)

Particularly interesting case:

$$|\Psi_G\rangle = c_{11}|\Phi_1,\Phi_1\rangle \;+\; c_{12}|\Phi_1,\Phi_2\rangle \;+\; c_{21}|\Phi_2,\Phi_1\rangle \;+ \ldots$$

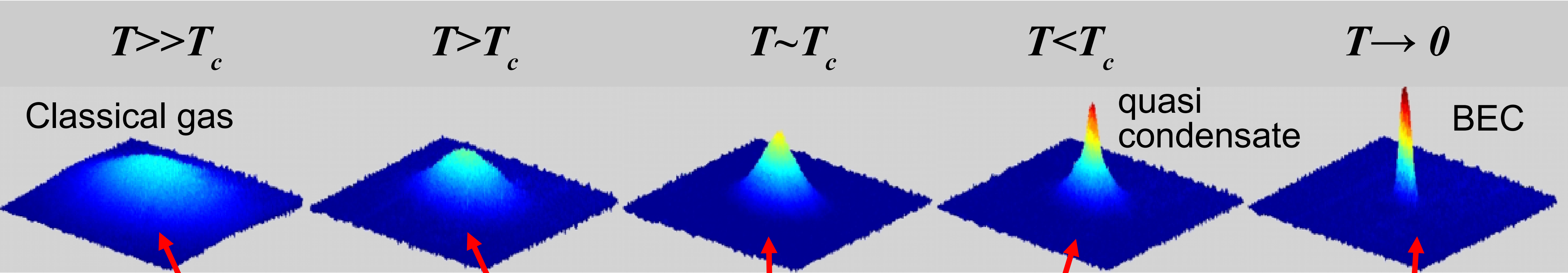
(local measurements on a correlated state)

e.g. interacting ground state

Dilute 1D Bose gas: temperature regimes

$$\hat{H} = \int d^3\mathbf{x} \left\{ \hat{\Psi}^\dagger(\mathbf{x}) \left[V(\mathbf{x}) - \frac{\hbar^2}{2m}\nabla^2 \right] \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \hat{\Psi}^\dagger(\mathbf{x})^2 \hat{\Psi}(\mathbf{x})^2 \right\}$$

Bose field $\hat{\Psi}(\mathbf{x})$



http://www-matterwave.physics.ox.ac.uk/node/1

Most relevant modes highly occupied
---> c-field description is applicable

Full quantum field

c-fields

$$\Psi(\mathbf{r}) = \sum_{\mathbf{k}} a_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}} \longrightarrow \Phi(\mathbf{r}) = \sum_{|\mathbf{k}| \leqslant \mathbf{K}_{\max}} \alpha_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}}$$

Thermal behaviour - SGPE

Thermal c-field behaviour is well described by the Stochastic GP eqn:

$$i\hbar\frac{\partial\Psi}{\partial t} = (1 - i\gamma)\underbrace{\left(-\frac{\hbar^2\nabla^2}{2m} + V - \mu + g|\Psi|^2\right)\Psi}_{\text{Gross-Pitaevskii equation (GPE)}} + \sqrt{2\hbar\gamma k_B T}\,\eta,$$

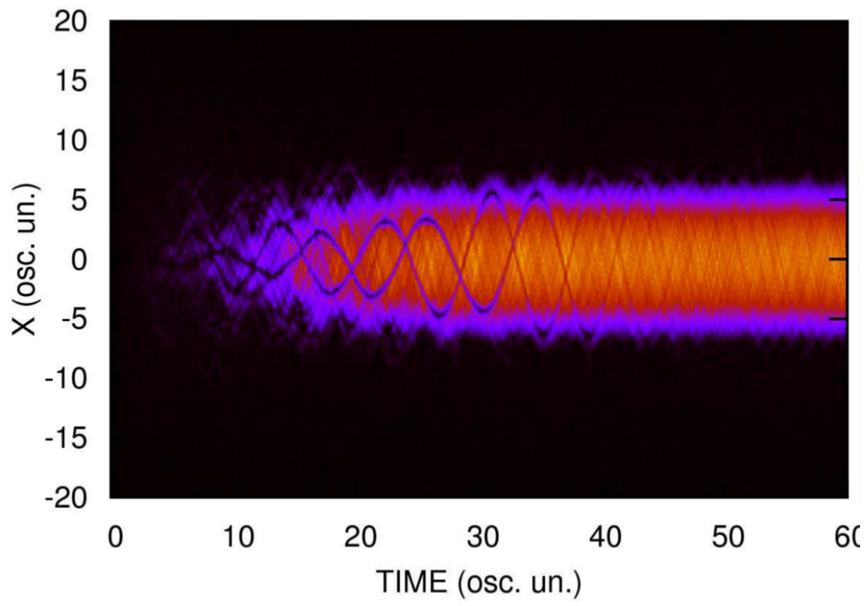
reservoir coupling
(weak $\gamma \ll 1$)

Complex white noise
(thermal fluctuations)

Applicable for a wide range of temperatures

Simulates single experimental runs

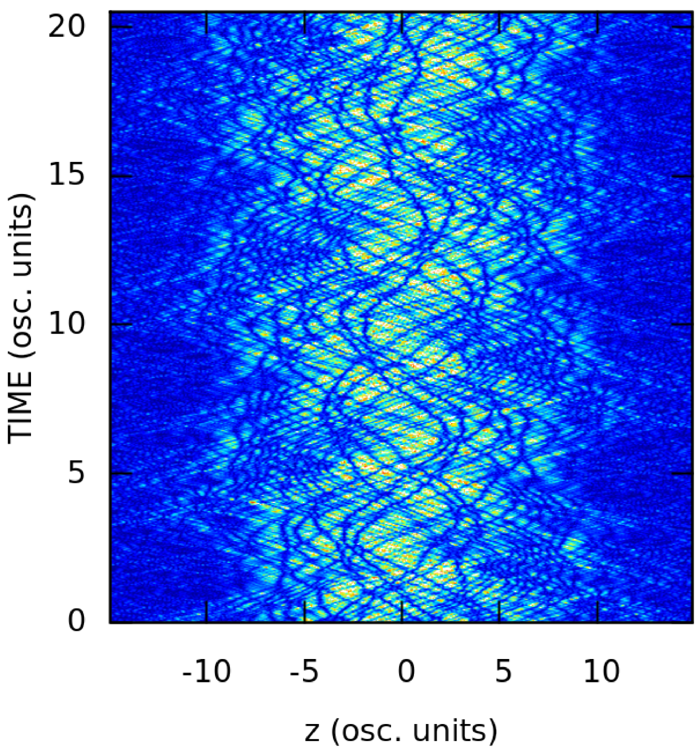
Does not include quantum fluctuations



Examples of warm spontaneous (quantum fluctuation) phenomena

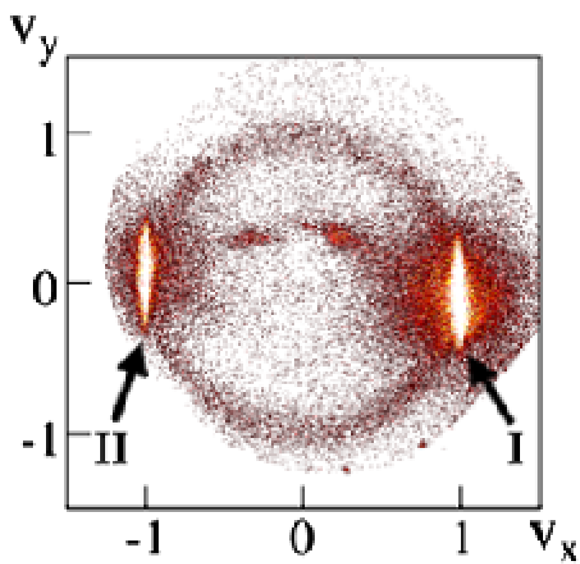
- Scattering into empty modes
- Atom-atom correlations within single shot wavefunctions

Spontaneous solitons in trapped gas



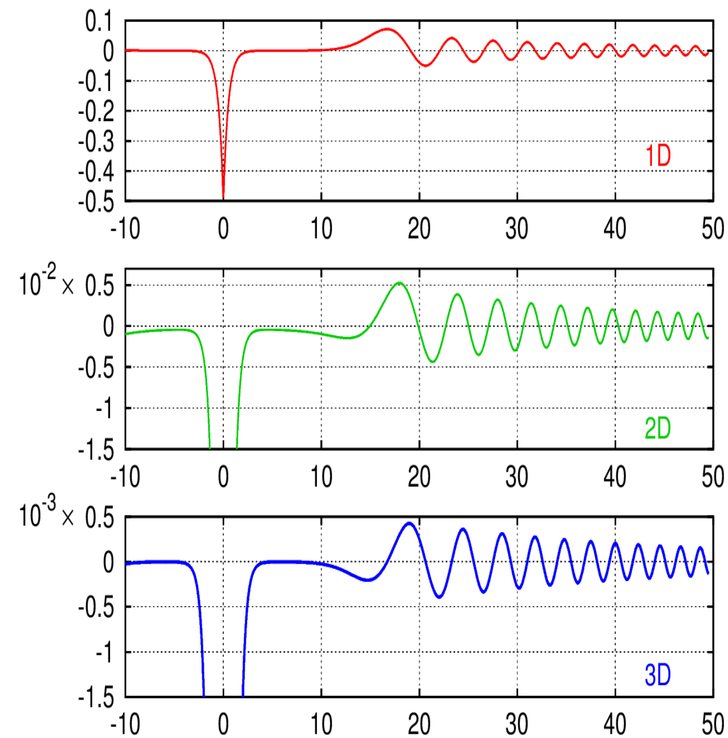
Karpiuk, PD, Bienias, Witkowska, Pawłowski, Gajda, Rzążewski, Brewczyk, PRL **109**, 205302 (2012)

Pair scattering out of colliding quasicondensates



Perrin, Chang, Krachmalnicoff, Schellekens, Boiron, Aspect, Westbrook, PRL **99**, 150405 (2007)

Pair creation in-situ after a quantum quench



PD, Stobińska, arXiv:1310.1301; Carusotto, Balbinot,Fabbri, Recati, EPJD **56**, 391 (2010)

Quantum granularity

$$i\hbar\frac{\partial\Psi}{\partial t} = (1-i\gamma)\left(-\frac{\hbar^2\nabla^2}{2m}+V-\mu+g|\Psi|^2\right)\Psi+\sqrt{2\hbar\gamma k_B T}\,\eta,$$

$$\overline{N} = \int dx\,\langle\widehat{\Psi}^\dagger(x)\widehat{\Psi}(x)\rangle.$$

- SGPE is unchanged under the transformation

$$g \to \lambda g$$
$$\Psi(x) \to \Psi(x)/\sqrt{\lambda}$$
$$\overline{N} \to \overline{N}/\lambda$$
$$T \to T/\lambda$$

- However as $\overline{N} \to 0$, c-fields progressively lose their validity because the mode occupation falls.

An observation

Positive-P equations (<u>full quantum mechanics</u>)
for a Bose field coupled to a naïve Markovian thermal reservoir

$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = \left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} + g\widetilde{\psi}^*\psi - i\overline{\gamma} + \sqrt{i\hbar g}\,\xi(\mathbf{x},t)\right\}\psi + \sqrt{\overline{\gamma}T}\,\eta(\mathbf{x},t)$$

$$i\hbar\frac{d\widetilde{\psi}(\mathbf{x})}{dt} = \left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} + g\psi^*\widetilde{\psi} - i\overline{\gamma} + \sqrt{i\hbar g}\,\widetilde{\xi}(\mathbf{x},t)\right\}\widetilde{\psi} + \sqrt{\overline{\gamma}T}\,\eta(\mathbf{x},t)$$

Real white noises
(quantum fluctuations)

Complex white noise
(thermal fluctuations)

<u>Does include quantum fluctuations</u>

<u>Simulates single runs</u>

<u>Noide amplification severely limits time</u>

SGPE equation (with a less naïve reservoir):

$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = (1-i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g|\psi|^2\right\}\psi + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$

Hybrid: thermal Initial conditions, full evolution

arXiv: 1409.0146

The similarity suggests a hybrid equation that would include:

- *both thermal and quantum fluctuations:*
- *equilibrated thermal cloud from SGPE*
- *quantum fluctuations, atom pair production, etc. from positive-P*
- *probably the positive-P time limitations*

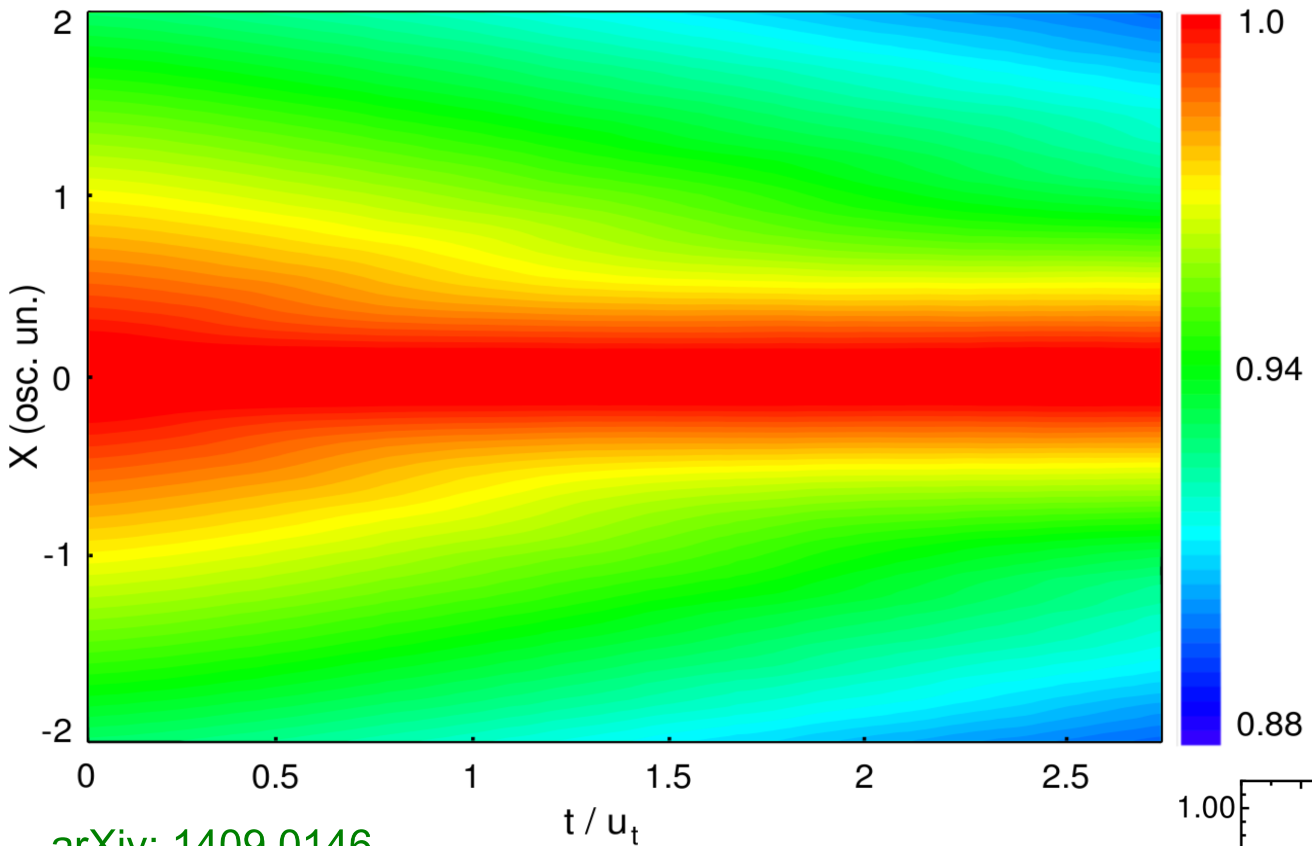
$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = (1-i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g\widetilde{\psi}^*\psi + \sqrt{i\hbar g}\,\xi(\mathbf{x},t)\right\}\psi + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$

$$i\hbar\frac{d\widetilde{\psi}(\mathbf{x})}{dt} = (1-i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g\psi^*\widetilde{\psi} + \sqrt{i\hbar g}\,\widetilde{\xi}(\mathbf{x},t)\right\}\widetilde{\psi} + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$

Quantum fluctuations

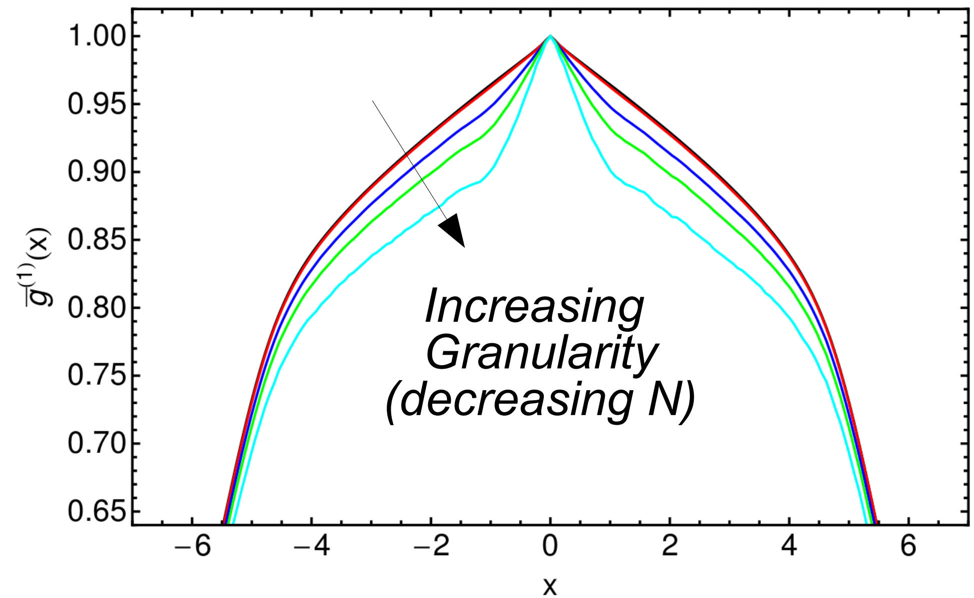
Thermal fluctuations

Phase decoherence

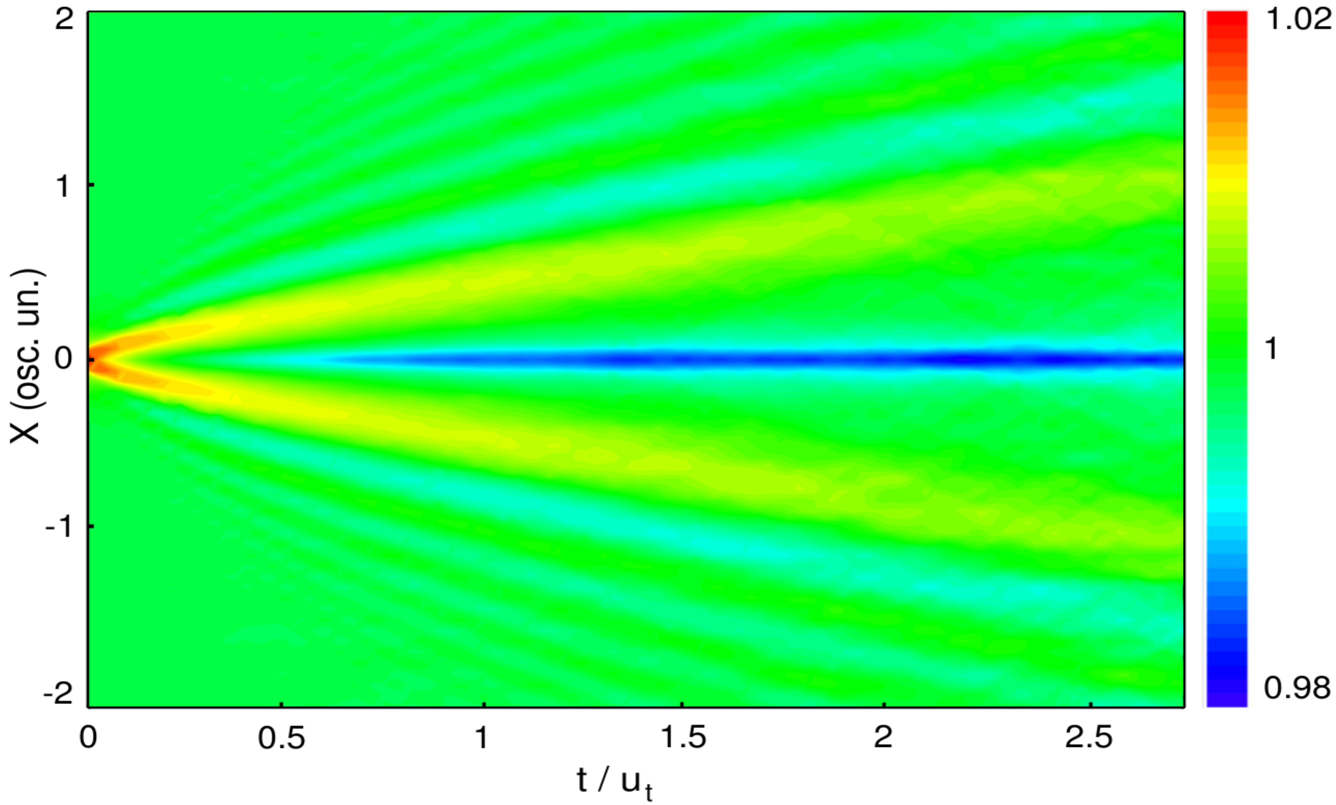


arXiv: 1409.0146

$$g^{(1)}(x) \equiv g^{(1)}(0,x) = \frac{\langle \widehat{\Psi}^\dagger(0)\widehat{\Psi}(x)\rangle}{\sqrt{n(0)\,n(x)}}.$$

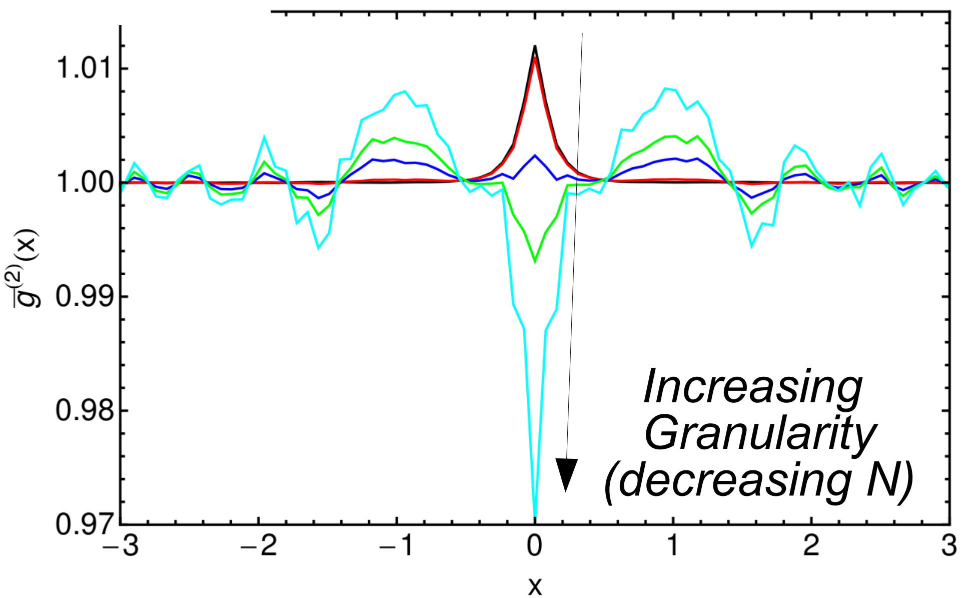


Density correlations



arXiv: 1409.0146

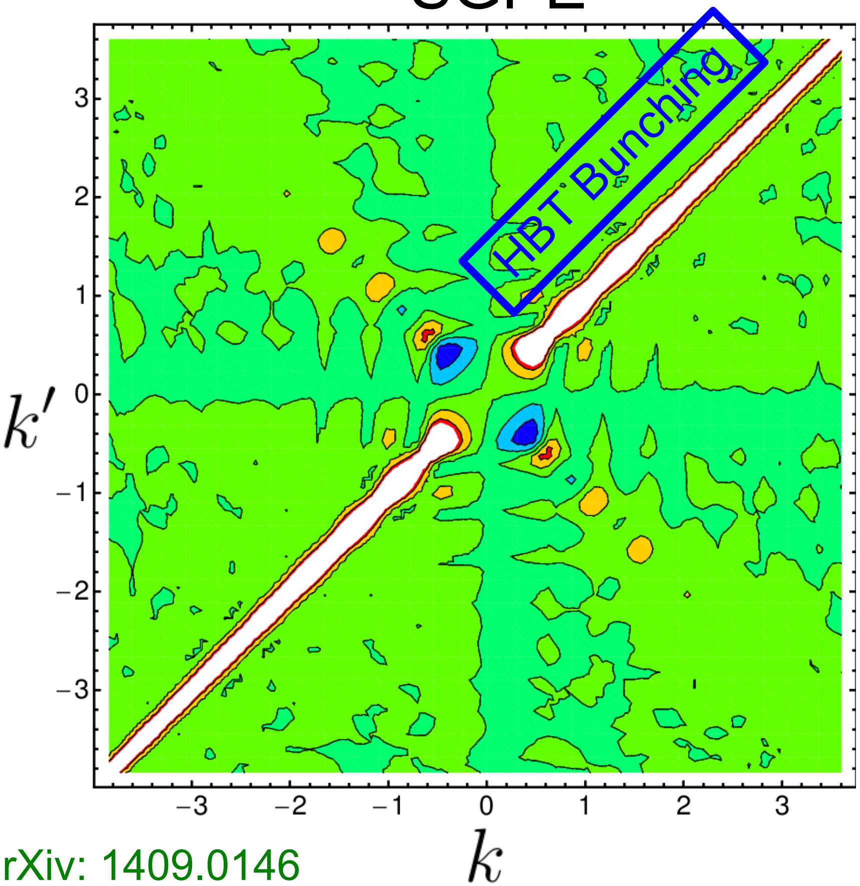
$$g^{(2)}(x) \equiv g^{(2)}(0,x) = \frac{\langle \widehat{\Psi}^\dagger(0)\widehat{\Psi}^\dagger(x)\widehat{\Psi}(0)\widehat{\Psi}(x)\rangle}{n(0)n(x)}$$



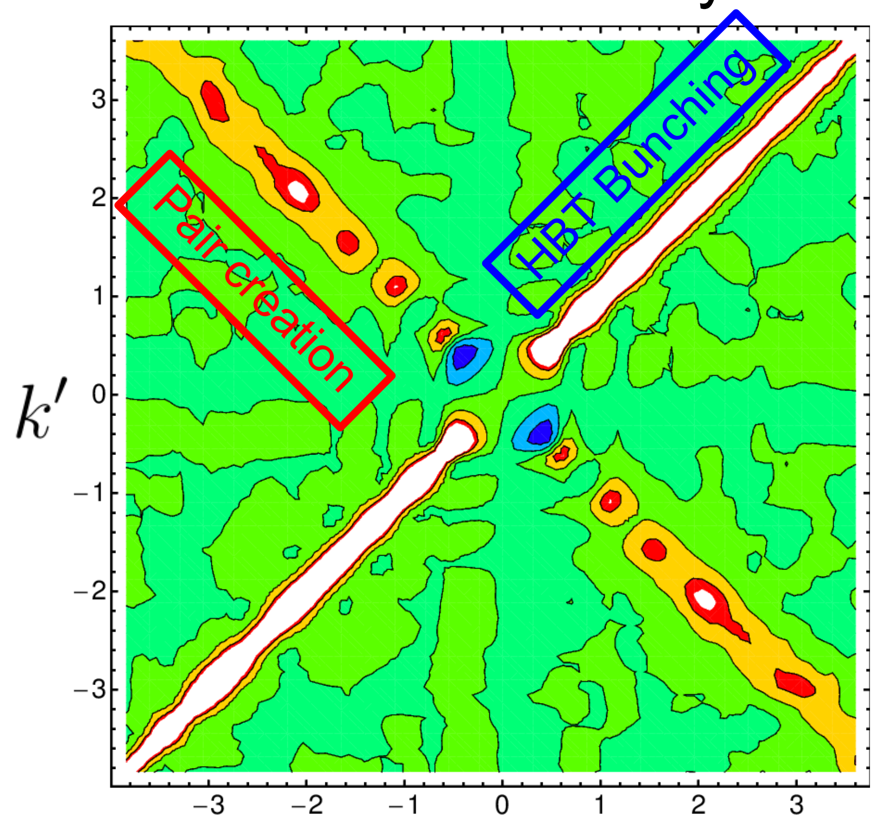
Atom pairs in situ

Momentum correlations $g^{(2)}(k,k')$

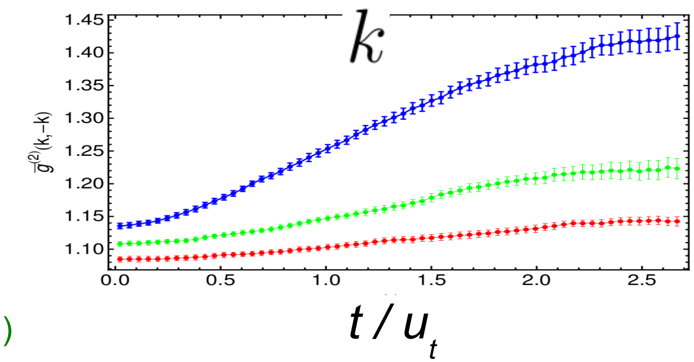
SGPE



Quantum-thermal hybrid



arXiv: 1409.0146



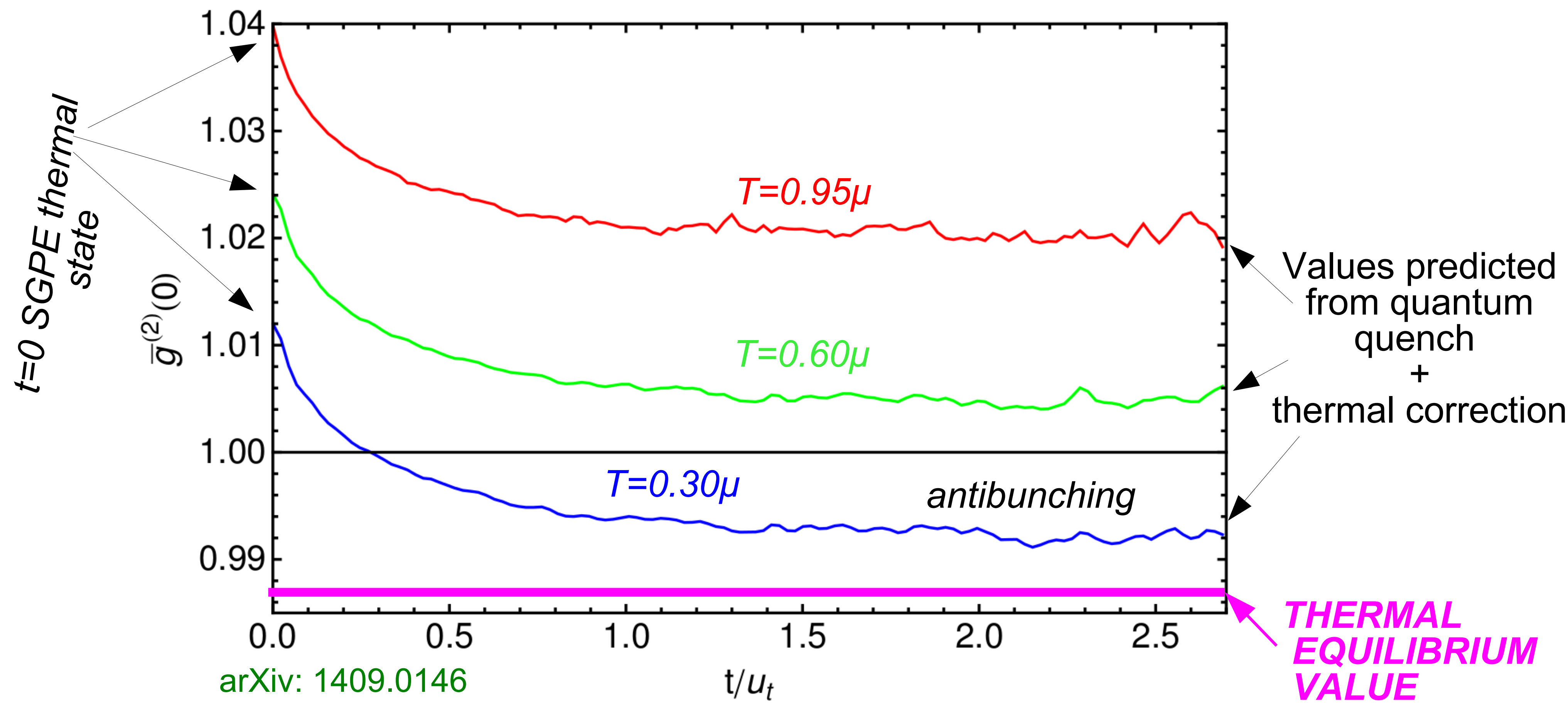
N=1000 $\mu = 22.41\,\hbar\omega$ $k_BT = 140\,\hbar\omega$

$T_\Phi \approx 900\,\hbar\omega$ $T_c \approx 1900\,\hbar\omega$

As per example in Cockburn, Negretti, Proukakis, Henkel, PRA 83, 043619 (2011)

Stationarity / prethermalization

- We do not reach thermal equilibrium
- However, stationary values for observables.
- “Prethermalization” again



arXiv: 1409.0146

Conclusions

- Hybrid equations generate dynamical evolution that:
  - Includes quantum and thermal fluctuations
  - Lasts long enough to see prethermalized observables
  - In general not long enough to see full equilibration

- Outlook
  - Obtain equilibration for other parameters
    (greater bath coupling increases stability)
  - More formal derivation
    (determine factor for quantum noise terms?)
  - Physical phenomena to study:
    (soliton filling, supersonic collisions, ...)