

Liquid thread breakup and the formation of satellite droplets

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Luís H. Carnevale,¹  Piotr Deuar,¹  Zhizhao Che,²  and Panagiotis E. Theodorakis^{1,a)} 

AFFILIATIONS

¹Institute of Physics, Polish Academy of Sciences, Al. Lotników 32/46, 02-668 Warsaw, Poland

²State Key Laboratory of Engines, Tianjin University, 300350 Tianjin, China

^{a)}Author to whom correspondence should be addressed: panos@ifpan.edu.pl

ABSTRACT

The breakup of liquid threads into smaller droplets is a fundamental problem in fluid dynamics. In this study, we estimate the characteristic wavelength of the breakup process by means of many-body dissipative particle dynamics. This wavelength shows a power-law dependence on the Ohnesorge number in line with results from stability analysis. We also discover that the number of satellite droplets exhibits a power-law decay with exponent 0.72 ± 0.04 in the product of the Ohnesorge and thermal capillary numbers, while the overall size of main droplets is larger than that based on the characteristic wavelength thanks to the asynchronous breakup of the thread. Finally, we show that the formation of satellite droplets is the result of the advection of pinching points toward the main droplets in a remaining thinning neck, when the velocity gradient of the fluid exhibits two symmetric maxima.

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I. INTRODUCTION

The breakup of liquid threads is not only a natural process observed on various occasions in everyday life (e.g., breakup of a liquid thread when water falls from a tap) but also relevant for various applications,¹ such as nanoprinting,² nanoscale manufacturing and chemical processing,³ spraying,⁴ and inkjet printing.⁵ The Rayleigh–Plateau instability,^{6,7} encountered during the pinching of liquid threads,^{8,9} remains a fascinating phenomenon. Many of its aspects require further investigations to reach a better understanding, especially those related to its molecular origin, which significantly affects the singular behavior in the vicinity of the breakup point. According to stability analysis,^{6,7,10} the liquid thread becomes unstable and pinches off for any perturbation with wavelength larger than the unperturbed cylinder circumference, $2\pi R_0$ (Fig. 1).¹¹ Moreover, linear stability analysis predicts that the most unstable mode for an inviscid fluid occurs at reduced wave-number $\chi = 2\pi R_0/\lambda = 0.697$ (the famous Rayleigh mode),¹² which corresponds to a wavelength $\lambda \approx 9.01R_0$ [Fig. 1(a)]. Plateau has estimated a wavelength of $4.38 \times (2R_0)$.¹¹ However, the exact value of this perturbation has not been properly quantified by molecular-scale *in silico* experiments.^{13,14} While the process is fundamentally driven by surface tension, various parameters are expected to affect this phenomenon, such as inertial and viscous forces and thermal fluctuations.^{15–17} Another aspect relates to the formation of satellite drops, generally unfavorable for applications (e.g., inkjet printing). These

cannot be captured by a linear theory that would predict a homogeneous breakup of the thread into equal parts [c.f. snapshots of Fig. 1(b)]. To properly describe the thread breakup and understand the mechanisms of the formation of satellite droplets, a model that can take into account the thermal fluctuations of the system needs to be employed.

In this study, we use a particle-based mesoscale model to uncover several important properties of the liquid-thread breakup phenomenon. The model properly accounts for fluid properties,¹⁸ such as surface tension and viscosity, and, also, includes thermal fluctuations (see Sec. II for details of the model and simulation method). We determine the characteristic wavelength of the formed structures during the breakup of liquid threads of different Ohnesorge (Oh) numbers and juxtapose our predictions with theoretical predictions based on stability analysis.^{10,19} We have also investigated the formation of main and satellite droplets that occurs at longer times. It is found that the number of satellite droplets follows a clear power-law dependence on the Oh and thermal capillary (Th) numbers that has not previously drawn attention, and that the size of the main droplets is larger than what would be predicted based on the characteristic wavelength at larger Oh numbers, due to the asynchronous breakup of the thread. Finally, we show that the formation of satellite droplets is the result of the advection of pinching points in the thin neck that remains between forming main droplets. The advection is due to the large velocity

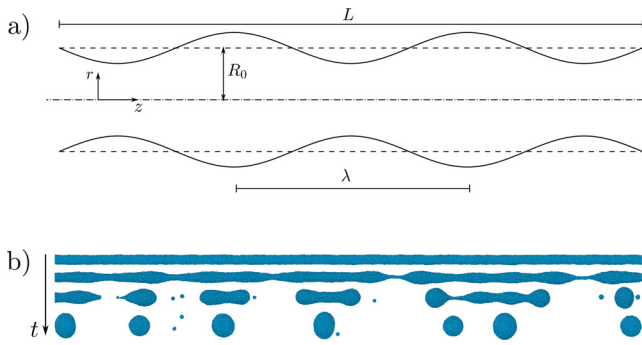


FIG. 1. (a) Schematic of the initial configuration of the system (dashed line) and the evolution of the breakup of a liquid thread of an initial radius, R_0 . (b) Characteristic snapshots from the simulations during the time evolution of the breakup from the liquid thread to individual droplets. Here, only a part of the whole thread is shown for clarity, while $R_0 = 6$ (in reduced units throughout the paper).

gradients at the pinching points. Thus, we anticipate that our study sheds more light into the underlying mechanisms of thread breakup and its fundamental aspects.

II. MODEL AND METHODOLOGY

The many-body dissipative particle dynamics (MDPD) method^{20–25} was used to carry out the simulations. This method can be used to describe flows at length and time scales relevant for describing topological changes in flows, but at the same time naturally handles singularities, such as the one developing at the pinching point during the breakup process, and provides molecular-scale resolution. Fluids with different properties are studied, i.e., different Oh numbers, while the density, surface tension, and viscosity of these fluids are the key properties and are reported in Table I. The initial configuration of the system consists of a cylindrical liquid thread with radius, R_0 , and length, L , with periodic boundary conditions applied at each end of the cylinder in the z direction (Fig. 1). Characteristic snapshots obtained by the MDPD simulations are also presented in Fig. 1(b). A range of relevant lengths, L , and radii, R_0 , have been considered to examine finite-size effects and gain better resolution on the properties studied, for fluids characterized by different Oh numbers, defined as $\text{Oh} = \mu / \sqrt{\rho\sigma R_0}$. Here, μ is the fluid's viscosity, ρ its density, and σ its surface tension. The thermal capillary number, which is also relevant

TABLE I. Fluid properties obtained for different A values in the MDPD model. The density, ρ , was obtained from the simulation, the surface tension, σ^{fit} , and the viscosity, μ , were calculated from fitting equations from Refs. 26 and 21, respectively. Here, $R_0 = 6$ was used to calculate Oh numbers.

| A | ρ | σ^{fit} | μ | Oh |
|-----|--------|-----------------------|-------|-------|
| –40 | 6.75 | 9.95 | 4.06 | 0.199 |
| –50 | 7.65 | 15.98 | 7.22 | 0.266 |
| –60 | 8.30 | 22.60 | 10.76 | 0.321 |
| –70 | 8.95 | 30.66 | 18.31 | 0.451 |
| –80 | 9.60 | 40.29 | 33.90 | 0.704 |
| –85 | 9.92 | 45.73 | 47.02 | 0.901 |
| –90 | 10.24 | 51.62 | 64.01 | 1.137 |

here, is defined as $\text{Th} = l_T / R_0$, where l_T is the thermal capillary length $l_T = \sqrt{k_B T / \sigma}$, k_B is Boltzmann's constant and T is the temperature of the system, which are set to unity and define the energy scale of the model. The analysis of the results is based on an ensemble of *in silico* experiments for each set of parameters, in order to obtain reliable statistics.

The many-body dissipative particle dynamics (MDPD) model consists in solving the Langevin equation of motion [Eq. (1)] for each particle, i , that interacts with its neighbors, j , through a conservative force F^C , i.e.,

$$m \frac{dv_i}{dt} = \sum_{j \neq i} F_{ij}^C + F_{ij}^R + F_{ij}^D, \quad (1)$$

where F^R is a random force and F^D a dissipative force acting on each particle, i . The main difference between the MDPD and the standard DPD model in its most common formulation is the expression for the conservative force, which in the MDPD model reads as

$$F_{ij}^C = A \omega^C(r_{ij}) \mathbf{e}_{ij} + B (\bar{\rho}_i + \bar{\rho}_j) \omega^d(r_{ij}) \mathbf{e}_{ij}, \quad (2)$$

where $A < 0$ and $B > 0$ are the attractive and repulsive parameters, respectively, r_{ij} is the distance between particles, \mathbf{e}_{ij} is the unit vector in the direction from particle i to particle j , while $\omega^C(r_{ij})$ and $\omega^d(r_{ij})$ are linear weight functions, which are defined as follows:

$$\omega^C(r_{ij}) = \begin{cases} 1 - \frac{r_{ij}}{r_c}, & r_{ij} \leq r_c \\ 0, & r_{ij} > r_c, \end{cases} \quad (3)$$

with r_c being a cutoff distance for the interactions, usually set to unity. $\omega^d(r_{ij})$ has the same form, however, its cutoff distance $r_d = 0.75$, which is smaller than r_c .

The repulsive term contains the many-body contributions through its dependence on local neighborhood densities, $\bar{\rho}_i$ and $\bar{\rho}_j$, which are calculated as follows:

$$\bar{\rho}_i = \sum_{0 < r_{ij} \leq r_d} \frac{105}{16\pi r_d^3} \left(1 + 3 \frac{r_{ij}}{r_d}\right) \left(1 - \frac{r_{ij}}{r_d}\right)^3. \quad (4)$$

The random and dissipative forces act as a thermostat in this model and are given by

$$F_{ij}^D = -\gamma \omega^D(r_{ij}) (\mathbf{e}_{ij} \cdot \mathbf{v}_{ij}) \mathbf{e}_{ij}, \quad (5)$$

$$F_{ij}^R = \zeta \omega^R(r_{ij}) \theta_{ij} \mathbf{e}_{ij}, \quad (6)$$

where γ is the dissipative strength, ζ is the strength of the random force, \mathbf{v}_{ij} is the relative velocity between particles, and θ_{ij} is a random variable from a Gaussian distribution with unit variance. According to the fluctuation–dissipation theorem, γ and ζ are related to each other by

$$\gamma = \frac{\zeta^2}{2k_B T}, \quad (7)$$

where the temperature of the system is T (set to 1 in our units), and the weight functions for the forces are

$$\omega^D(r_{ij}) = [\omega^R(r_{ij})]^2 = \begin{cases} \left(1 - \frac{r_{ij}}{r_c}\right)^2, & r_{ij} \leq r_c \\ 0, & r_{ij} > r_c. \end{cases} \quad (8)$$

The time integration of the equation of motion is performed by using the modified velocity-Verlet algorithm with a time step $\Delta t = 0.01$.

Different fluid properties were obtained in the simulations by means of the attractive parameter, A , while keeping constant $B = 25$, $r_c = 1$, $r_d = 0.75$, and $\gamma = 18$. They are reported in Table I. The density ρ is the mean bulk bead density in simulation units and should be distinguished from the local values $\bar{\rho}_i$ in Eq. (4). The surface tension was obtained from the fit proposed by Ref. 26 as

$$\sigma^{\text{fit}} = -\frac{\pi}{240} (0.42Ar_c^5\rho^2 + 0.003Br_d^5\rho^3) \quad (9)$$

and the viscosity values were taken from the measurements in Ref. 21. The density values were directly measured during the initial stages of the simulation and they show slightly higher values than what is typically found in the literature by using the equation of state proposed by Refs. 20 or 27, due to the difference in local density weight function used.

Simulation units can be related to physical units if desired, by matching density, temperature and further choosing appropriate values of the constants in the evolution equation to achieve the appropriate surface tension or other physical quantities for the substance under consideration—as studied, for example, in Ref. 18. Moreover, since simulation units are related to physical units by a matching procedure, it is worth keeping in mind that changing one evolution parameter, such as A , is equivalent to corresponding changes of dimensionless numbers in the fluid. For example, increasing attraction $|A|$ (as we have done in this study) increases density ρ since the potential becomes shorter range and also increases the surface tension σ^{fit} . Therefore, for example, the thermal capillary number Th , which quantifies the relative strength of thermal fluctuations, is reduced. Also, the shorter range of the inter-bead potential decreases the typical fluctuation range.

III. RESULTS AND DISCUSSION

We have first determined the characteristic wavelength of the breakup process along the thread direction. To achieve this, we take advantage of the system symmetry and use the correlation of the density fluctuations along the z direction at radial distance, r , from the cylindrical axis of the thread, which is expressed as

$$G(r, \delta z) = \frac{\langle \rho(r, \phi, z)\rho(r, \phi, z + \delta z) \rangle_{z, \phi, T}}{\langle \rho^2(r) \rangle_{z, \phi, T}}, \quad (10)$$

with $\rho(r, \phi, z)$ being the local density in a volume element at radial distance, r , from the cylindrical axis, while $\delta z \leq L/2$, due to the presence of periodic boundary conditions in the z direction.²⁸ It is also indicated that one takes the average over an ensemble of configurations at temperature, T , over the angle, ϕ , oriented normal to the cylindrical axis, and over z . The result of the calculation provides the estimation of the characteristic length scale, λ , that develops during the thread breakup. This length scale is more visible at larger distances from the axis, r , but persists over a large range of r [inset of Fig. 2(a)]. It can be determined from the peak position q_{max} of the discrete Fourier transform of $G(r, \delta z)$, as $\lambda = 2\pi/q_{\text{max}}$. Data for a particular case are shown in Fig. 2(a). A robust value of q_{max} is determined by a Gaussian fit to the points around the peak. Reliable statistics are obtained by realizing an adequate ensemble of simulations for each case, while possible side effects due to the

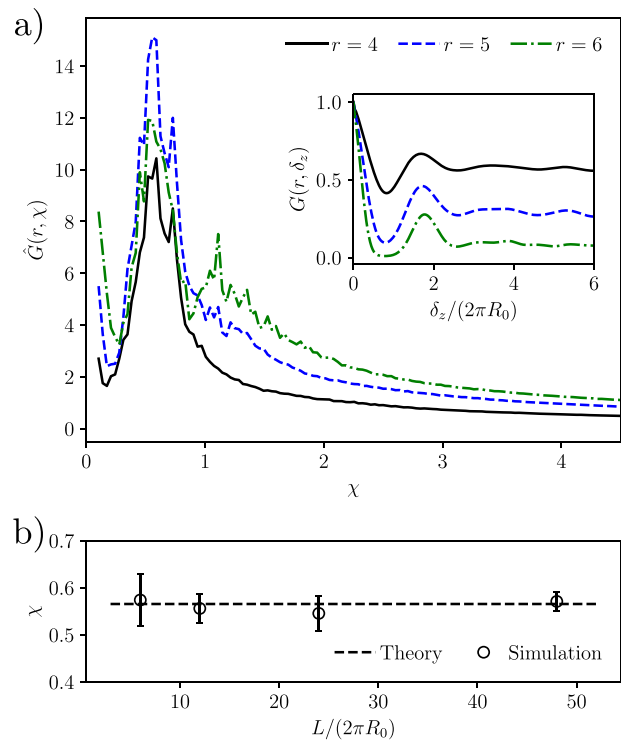


FIG. 2. (a) Fourier transform, $\hat{G}(r, q)$, of the correlation of the density fluctuations, $G(r, \delta z)$ (inset), at various radial distances, r , from the thread axis, as indicated. Example for $\text{Oh} = 0.266$; (b) The characteristic wavelength for threads of different lengths, L . The dashed line indicates the theoretical prediction, $\lambda = 0.565$, as obtained from Eq. (11).

presence of periodic boundary conditions have been investigated by considering threads of different lengths, L . Indeed, we have found that possible finite size effects quickly disappear as the length, L , becomes larger than the circumference of the thread, $2\pi R_0$. We henceforth consider long threads to obtain the highest possible accuracy on the characteristic wavelength through the Fourier transform. Moreover, a larger number of droplets formed in the case of longer threads allows for better statistics on main and satellite droplet properties.

Figure 2(b) summarizes our results for the reduced characteristic (most unstable and smallest) wavenumber $\chi = 2\pi R_0 / \lambda$. We find that $\chi = 0.57 \pm 0.05$ independently of the chosen length of the liquid thread, which is in agreement with previous predictions of stability analysis of the Navier–Stokes equations [Eq. (11)].^{10,19} Moreover, breakup has not been observed in our simulations when the length of the thread, L , is smaller than the circumference of the thread, in line with previous theoretical arguments.¹¹

Fluids with different properties have been studied and the wavenumber, χ , has been determined for each case by using the same path as described above (Fig. 2). Then, Fig. 3 presents the dependence of χ on Oh (note that Oh depends not only on the fluid’s properties but also on R_0), which shows a very good agreement with the predictions of stability analysis.^{10,19} According to the theory, the reduced wavelength follows the relation:

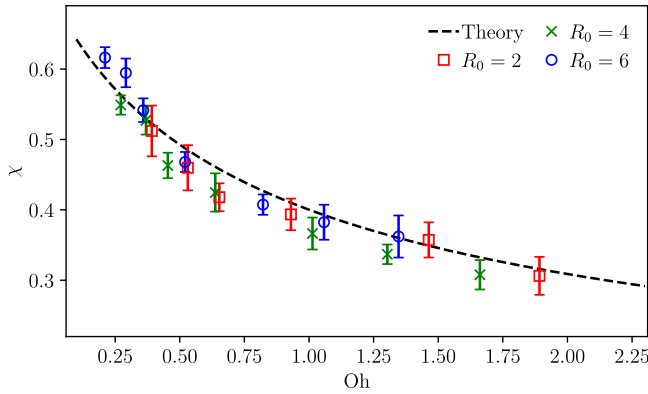


FIG. 3. Characteristic wavenumber, χ , vs Oh number. The dashed line shows the theoretical prediction [Eq. (11)].

$$\chi = \sqrt{\frac{1}{2 + \sqrt{18\text{Oh}}}} \quad (11)$$

Our results indicate that Eq. (11) is generally valid for threads of different Oh in the range that can be captured by the MDPD model, which might suggest that χ has a universal aspect and the stochastic nature of

breakup at different points along the thread due to the thermal fluctuations do not seem to affect the wavenumber, χ . In this context, note how decreasing R_0 , as we have done while keeping fluid properties constant, increases the thermal capillary number Th and can be mapped to a system with higher T and relatively more important thermal fluctuations.

Since the individual breakup events take place at different points along the liquid thread and times, the total number of formed droplets varies during the simulation, and there is a point in time when there is a maximum number of droplets (Fig. 4 shows examples for higher and lower Oh-number cases). To investigate their sizes and distribution, we have used cluster analysis to identify the droplets, where a distance 0.8 between neighboring particles has been used as the acceptance criterion to a cluster. To consider a cluster as a fully formed droplet (rather than remains of the thread), we require that its relative shape anisotropy²⁹

$$\kappa^2 = \frac{3}{2} \frac{\lambda_x^4 + \lambda_y^4 + \lambda_z^4}{(\lambda_x^2 + \lambda_y^2 + \lambda_z^2)^2} - \frac{1}{2} \quad (12)$$

should satisfy the criterion that $\kappa^2 < 0.2$. Here, λ_x , λ_y , and λ_z are the principal moments of the gyration tensor. Values of relative shape anisotropy closer to 0 indicate that a cluster has a stronger spherical

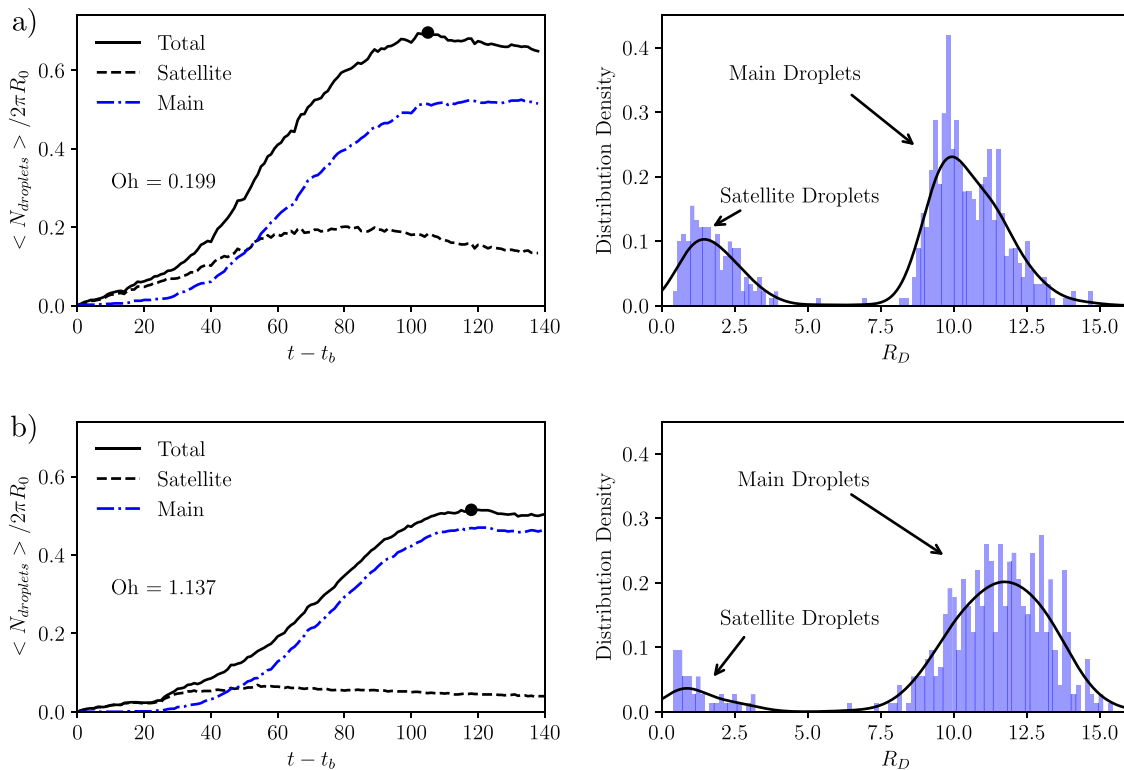


FIG. 4. The left panels illustrate the evolution of the linear number density of droplets vs time (t_b is the time of the first breakup event) for the main and satellite droplets, as well as the total sum of the two, as indicated, for (a) $\text{Oh} = 0.199$ and (b) $\text{Oh} = 1.137$. The right panels show the distribution of the size of droplets (calculated from the radius of gyration, R_D) at the time of maximum droplet count, indicated by the black dot in the left panels. The two distinct populations are the satellite (left peak at smaller R_D) and the main (right peak at larger R_D) droplets. Both cases had initial cylinder radius $R_0 = 6$.

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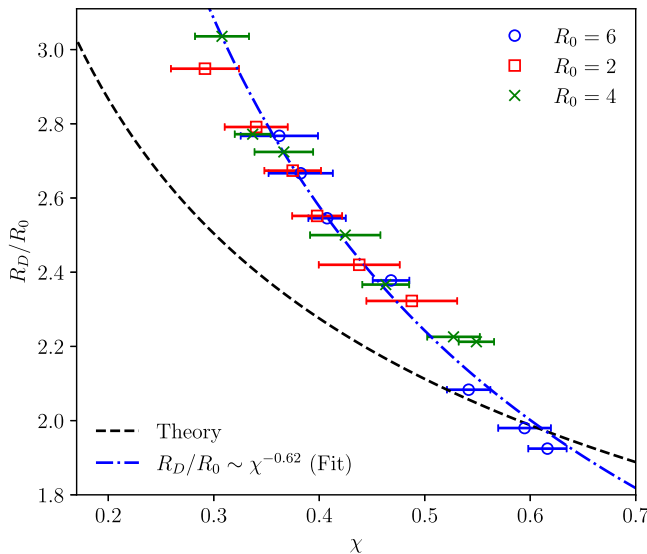


FIG. 5. Dependence of the average expected radius, R_D , of the main droplets vs the characteristic wavenumber, χ . The dashed line shows the expected value [Eq. (13)] based on the volume balance with respect to χ . The blue dashed line shows a power-law fit of the simulation data, i.e., $R_D/R_0 \sim \chi^{-0.62}$.

symmetry while values closer to 1 rather indicate that all points lie on a line.

The time evolution of the linear density of the number of droplets is shown in Fig. 4, along with the distribution of droplet sizes at the time of maximum droplet number. We observe a majority population of main droplets, characterized by a large radius of gyration (obtained from $\lambda_{x,y,z}$), and a smaller, very well separated, population of satellite droplets with much smaller radii of gyration. The reported average properties that are related to the number of droplets are calculated at this time of maximum linear density. The slow reduction in the number of droplets at later times is due to subsequent coalescence events, which after a long time should lead to the formation of a single droplet encompassing all particles.

Given that the characteristic wavelength χ naturally characterizes the breakup process, one may estimate a theoretically expected droplet radius by assuming that the transformation of a part of the thread with the initial cylindrical geometry of length λ and radius R_0 to a

spherical droplet of radius R_D would take place without any loss of material. Hence, considering that the density of the liquid phase does not change during the transformation from the cylinder to the sphere, one can assume that $V_{cyl} = V_d$, where $V_{cyl} = \pi R_0^2 \lambda = 2\pi^2 R_0^3 / \chi$ is the initial volume of the cylindrical part of the thread and $V_d = 4\pi R_D^3 / 3$ is the volume of the formed droplet. Then, one obtains a theoretical expectation of the relation between R_D/R_0 and χ ,

$$R_D/R_0 = \sqrt[3]{3\pi/2\chi}. \tag{13}$$

For all fluids studied, we find that the average radius of the fully formed main droplets (Fig. 5), namely, R_D follows a downward trend qualitatively similar to the volume conserving expectation (13) but with a steeper ascent at small χ . Thus, R_D does indeed depend on the wavenumber, χ , but a deviation from the simple expectation to larger drop sizes is seen for threads characterized by a larger Oh (smaller χ). We interpret this as being related to the asynchronous breakup of the thread combined with increased viscosity, which leads to the merging of isolated clusters of remnant neighboring droplets into larger ones than indicated by Eq. (13), instead of their breakup. Freed of the pull of neighbors at the edges of a cluster, remnant precursors gain momentum toward the center and merge. A clear example of the mechanism is shown in Fig. 6. Moreover, the same momentum gaining mechanism can inhibit the formation of satellite droplets. Due to the longer times required for breakup in the case of larger wavelengths (Oh numbers, Fig. 7), the suppression of pinching points is facilitated, and a larger deviation of the mean droplet size from the theoretical expectation [Eq. (13)] is seen.

In Fig. 6, we show the time evolution of the breakup of a section of a longer liquid thread in order to highlight the reason that the average size of the droplets is larger than the expected theoretical value [Eq. (13)] that is directly linked with χ . Let us examine the formation of droplets 3 and 4 (Fig. 6). In this case, the initial shape of the liquid thread is characterized by three bulges of approximate wavelength $\lambda = 2\pi R_0 / \chi$. These would appear to be precursors of three droplets. However, we can see that some droplets separate earlier and these may form smaller main droplets (1, 3, 5, and 6), while other precursors coalesce with neighbors before they can fully separate. Thus, two domains in the same cluster that were initially heading to be droplets of size approximately according to χ end up as one larger amalgamated droplet. Such amalgamates form the larger droplets like 2 and 4. A more viscous fluid can be expected to give more amalgamated cases, explaining the discrepancy seen in Fig. 5 for higher Oh-number fluids. Early

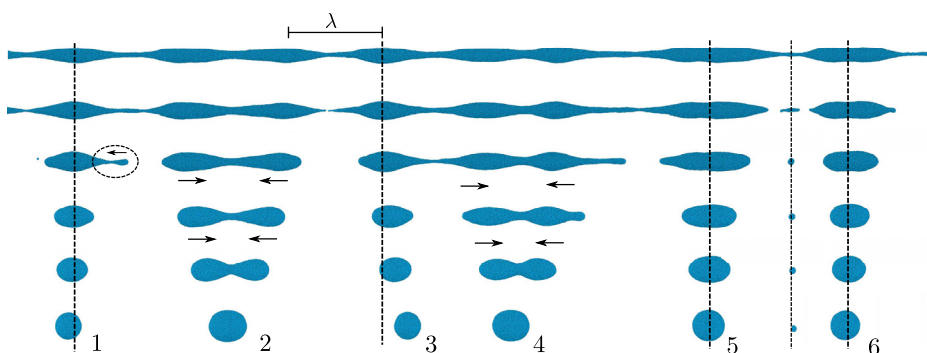


FIG. 6. Time evolution of a breakup simulation showing the mechanism of breakup suppression due to the asymmetry in the pinching events, which can also suppress the formation of satellite droplets, and the mechanism by which droplets acquire lateral momentum that will lead to coalescence later on. Numbers indicate droplets discussed in the text. This simulation corresponds to the case where $Oh = 1.137$ and $R_0 = 6$.

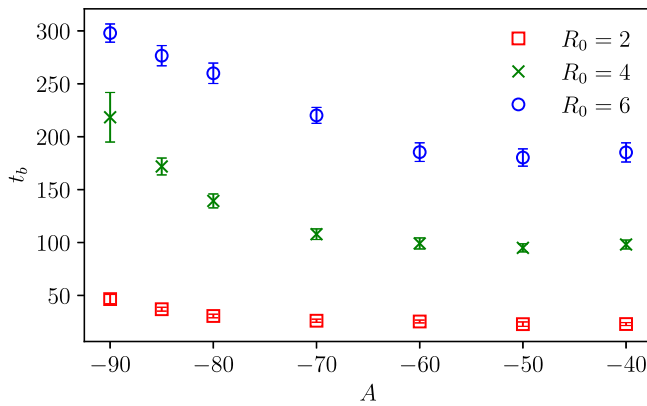


FIG. 7. Dependence of the breakup time t_b on the attractive parameter A for threads with different initial radius R_0 .

separation of droplets from the rest of the thread also frees the precursors in-between from stretching forces. For example, separation of droplet 3 from the liquid thread that spans two characteristic wavelengths to its right, frees the precursors of a force pulling left, which leads from this moment to a visible acceleration inward within the cluster 4 as shown by the arrows, and facilitates the formation of just one droplet 4. The same mechanism can inhibit the formation of satellite droplets. For example, in the region indicated by a dashed-line circle in Fig. 6, near droplet 1, a satellite droplet precursor is reabsorbed into droplet 1 after earlier separation of 2.

The deviation between the actual mean size of the droplets and the theoretical expectation based on the characteristic reduced wavenumber, χ , is greater for lower values of χ , as can be seen in Fig. 5. Moreover, Fig. 3 clearly shows that smaller χ occurs for the higher Oh values. Hence, in Fig. 7, we plot the time required for the first breakup to occur depending on the attraction strength, and it is seen to be longer at strong attraction. The higher Oh and viscosity here, as evidenced by Table I, lead to the longer times required for the breakup to occur under these conditions. Therefore, the increased deviation of droplet size can be attributed to the longer breakup timescales at low χ , high Oh, which allows more time for the amalgamation process discussed above to occur and facilitates the suppression of pinching points.

We have also counted the number of satellite droplets, whose proportion exhibits a power-law decrease with increasing Oh and Th numbers with exponent -0.72 ± 0.04 as presented in Fig. 8. While previous experimental results have suggested a linear dependence of $N_{\text{satellite}}/(N_{\text{total}} - N_{\text{satellite}})$ on the Th number,¹⁶ those have not taken into account the additional dependence on the Oh number. A significantly different relationship has been found here. Furthermore, after a certain limit, $\text{OhTh} \geq 0.15$, the thread breakup does not yield satellite droplets anymore. Unfortunately, we are not able to further probe this limit with additional data for $\text{OhTh} > 0.15$, since in this case, the MDPD model will yield a solid phase, instead of the liquid one.

It is important to try to understand the mechanism of satellite droplet formation in more detail. For this reason, we conducted simulations of a single breakup point under the perturbation of the most unstable wavenumber by setting the cylinder length as $L = 2\pi R/\chi$ and considering a case with a lower chance of satellite droplet formation ($\text{Oh} = 0.461$) and a case with higher chance ($\text{Oh} = 0.174$). In

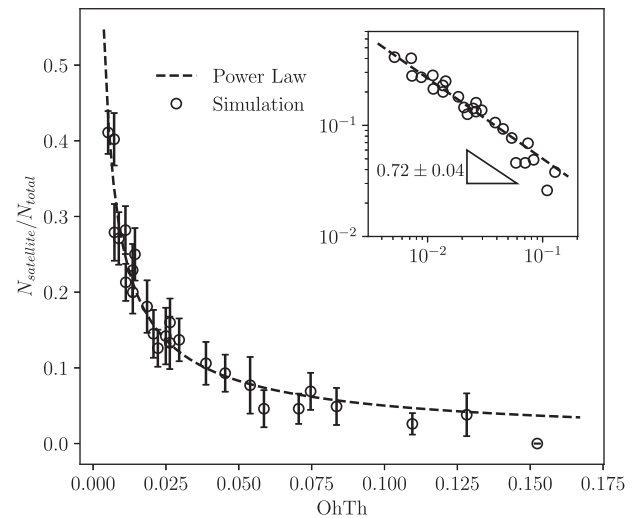


FIG. 8. Dependence of the proportion of satellite droplets, $N_{\text{satellite}}$, compared to the total number of droplets N_{total} , vs the product of the Ohnesorge and thermal capillary numbers.

turn, we analyzed the velocity field, v_z , and its gradient, $\partial v_z/\partial z$, at different times, t , during breakup.^{30–33} Figure 9 shows the satellite-droplet formation sequence and the corresponding velocity field and its gradient. Tracking the evolution, we observe that satellite-droplet formation occurs when two strong pinch points advect toward the main droplets, as is seen on the right. The figure also shows that relevant pinch points are associated with maxima of the velocity and of the velocity gradient, both of which shift in their position. For the satellite forming case (right), we see that the velocity gradient profile is characterized by two maxima at the pinching points. In this case, the neck, which is the main part of the forming satellite droplet, cannot join either of the two main droplets as the thinning process proceeds at the pinch points. In contrast, in the case of larger Oh numbers, a single pinching point will split the body of the neck in two parts, which gradually join the main droplets.

IV. CONCLUSIONS

In this study, we have provided accurate values for the characteristic wavenumber, χ , for various fluids of different Oh numbers by means of a particle-based mesoscale model. We find that the dependence of χ on Oh is in good agreement with the theoretical predictions based on stability analysis. Moreover, we have found that the number of satellite droplets follows a very clear power-law decay with the product OhTh with exponent 0.72 ± 0.04 , while the average main droplet size is related to the characteristic wavelength characterizing the breakup. However, also, certain deviations occur at larger Oh due to the asynchronous breakup and increased timescales and viscosity that suppress the pinching points. We have also found that satellite droplets form when two approximately symmetric peaks of the velocity distribution and its gradient, which can correspond to a higher capillary pressure at the pinch points, move toward the ends of the thinning neck between the main droplets. In such a scenario, the neck is not able to join either of the two main droplets, thus forming a separate satellite droplet. We anticipate that our results will motivate further

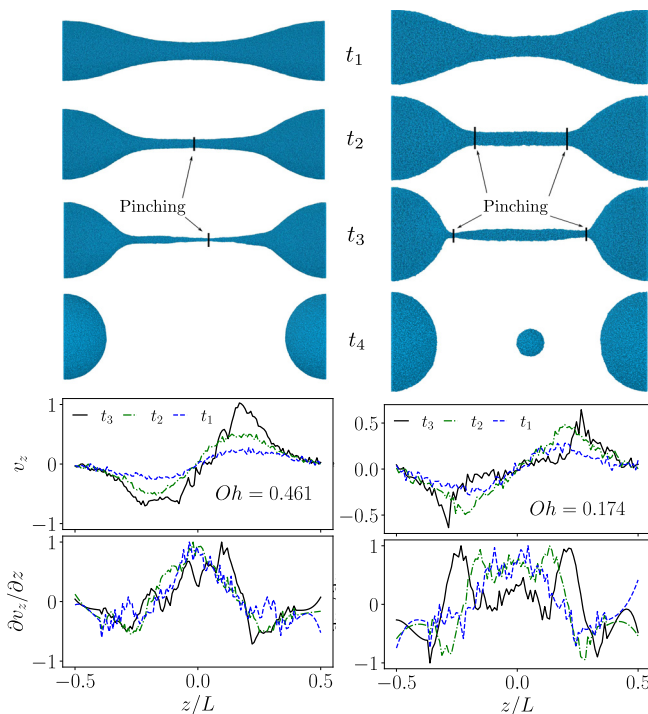


FIG. 9. Breakup of liquid threads with different Oh numbers. The formation of satellite droplets in the case of lower Oh is seen and tracked in the right panel. We observe the advection of the pinching points toward the region that connects the thinning neck to the main droplet. The advection coincides with maxima of the axial velocity gradient.

research in breakup phenomena, especially in the context of complex fluids, where MDPD, as a mesoscale method, can offer significant advantages in describing macroscopic flows and at the same time providing the detail required to capture relevant molecular mechanisms in the presence of various additives.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Luís Henrique Carnevale: Conceptualization (lead); Data curation (lead); Formal analysis (lead); Investigation (lead); Methodology (lead); Software (lead); Validation (lead); Visualization (lead); Writing – original draft (equal); Writing – review & editing (lead). **Piotr**

Deuar: Conceptualization (supporting); Methodology (supporting); Supervision (supporting); Writing – review & editing (lead). **Zhizhao Che:** Conceptualization (equal); Methodology (supporting); Supervision (supporting); Writing – review & editing (supporting). **Panagiotis Theodorakis:** Conceptualization (lead); Formal analysis (supporting); Funding acquisition (lead); Investigation (supporting); Methodology (supporting); Project administration (lead); Resources (lead); Supervision (lead); Writing – original draft (lead); Writing – review & editing (lead).

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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