Annals of Physics 265, 52–79 (1998)
Article No. PH975777

Macroscopic Local Realism: How Do We Define It and Is It Compatible with Quantum Mechanics?

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Received May 12, 1997; revised December 2, 1997

We introduce the concept of different orders (micro- through to macro-) of local realism. "Macroscopic local realism" states that events occurring at a location B cannot induce (immediate) macroscopic changes to a system at a location A spatially separated from B. "Local realism" in its entirety excludes all sizes of change. "Local realism" in its entirety is used by Einstein, Podolsky, and Rosen to deduce that results of position measurements (for certain correlated systems) are predetermined. The value for the predetermined position is specified with an uncertainty which is microscopic. "Macroscopic local realism" allows one to deduce only the existence of "elements of reality" with a macroscopic uncertainty. While Bell's theorem invalidates local realism in its entirety, little is known of the validity of "macroscopic local realism." We consider macroscopic experiments where the experimental error associated with measurements is macroscopic. We formulate the Einstein–Podolsky–Rosen argument for such a macroscopic situation. We propose that violations of Bell inequalities in such macroscopic experimental situations would imply the failure of "macroscopic local realism."

1. INTRODUCTION

"Local realism" is a premise (or more accurately a combination of two premises) introduced by Einstein, Podolsky, and Rosen (EPR) [1] and Bell [2] and others in the context of the now famous EPR argument and Bell's theorem. It was shown initially by Bell that certain quantum mechanical states are not compatible with local realism. Experimental evidence [3] supports quantum mechanics indicating that local realism is to be rejected.

The premises of local realism were first thought of as common-sense premises that must hold. The premises are fundamental to classical physics. The rejection of local realism is at the heart of the difference between classical and quantum interpretations.

The physical system considered originally by Bell which revealed an incompatibility of local realism with quantum mechanics was a microscopic one. All experiments to date testing local realism against quantum mechanics have been microscopic. For example, the experiment of Aspect *et al.* [3] involves measurements performed on a single photon. Hence, the premises of local realism have been shown to be invalid

0003-4916/98 $25.00

only at this microscopic level. The validity of these premises at a more macroscopic level is an open question.

The aim of this paper is to build a framework within which this question of the validity of local realism at a more macroscopic level might one day be answered. The first step is to give a sensible quantitative meaning to the distinction between a microscopic and macroscopic experiment and to microscopic and macroscopic measurements. In essence, we classify an experiment as macroscopic if all measurements performed have associated uncertainties which are themselves macroscopic. A microscopic experiment is one in which the associated uncertainties are microscopic.

We wish to give a sensible definition of "macroscopic local realism" and to further levels of local realism pertaining to experiments between the microscopic and macroscopic. To date local realism has been defined in a way so as to apply to all levels of experiment from the most microscopic to the most macroscopic. Local realism will be rejected if it fails in a microscopic experiment, although its validity in macroscopic experiments, where much more limited information is gained, is not known. Local realism states that events at a location B cannot induce (immediate) changes to a second system at a location A spatially separated from B. Local realism, as used to date in its entirety, excludes all sizes of change to the system at A, from macroscopic right down to microscopic. We wish to define "macroscopic local realism" so that it refers to the application of the local realism premises in macroscopic experiments only. Logically then we must define "macroscopic local realism" so as to exclude macroscopic changes only, as opposed to all sizes of change, to the system at A. Macroscopic local realism is therefore a weaker, less restrictive assumption than local realism itself. Its rejection is therefore a stronger statement than the rejection of local realism.

Our definition of "macroscopic local realism" differs from that which is suggestive from some earlier works [4]. This previous work has shown a violation of Bell inequalities for higher spin states, or many-particle states, indicating a failure of local realism in some sense, for macroscopic systems. However, in these situations measurements are always assumed to have maximum resolution, so that the spin values (or photon numbers) are clearly distinguished. There is no evidence of "macroscopic local realism," as we have defined it, failing here, since one is detecting microscopic changes to results of measurements. In an experiment showing a failure of "macroscopic local realism" it would only be necessary to perform intrinsically macroscopic measurements, where the uncertainty of measurement is macroscopic.

The second purpose of this paper is to review a version [5, 6] of the EPR argument which applies to an experiment which is, by our criteria, intrinsically macroscopic. In their original argument EPR use the premise of local realism right down to the most microscopic level to conclude that quantum mechanics is incomplete. As explained above this local realism has now been invalidated by Bell's theorem and associated experiments. The implication of the new version of the EPR argument presented here is that the EPR argument may be carried through using only the weaker premise of "macroscopic local realism." Thus we either reject the validity of this weaker premise of "macroscopic local realism," or conclude that quantum mechanics is incomplete. This new version of the EPR argument gives us strong motivation for examining the validity of "macroscopic local realism."

The application of the premises of locality and realism has never been shown to fail in experiments intrinsically macroscopic. "Macroscopic local realism," as we define it in this paper, has therefore never been shown to fail experimentally. Nor indeed, to our knowledge, has the compatibility of this "macroscopic local realism" with quantum mechanics ever been established clearly in any theoretical sense. A common, but perhaps not majority, view would be that quantum mechanics must be compatible with local realism in the true macroscopic limit.

Yet as pointed out by Schrodinger [7], quantum mechanics can predict the existence of quantum superpositions of states macroscopically distinct ("Schrodinger-cat" states). Quantum mechanics also predicts the existence of entangled superpositions of states macroscopically distinct. Such states are the logical macroscopic extension of the microscopic entangled quantum superposition states which are incompatible with local realism at the microscopic level, as shown by Bell's theorem. It would seem to be a reasonable hypothesis then that such macroscopic entangled quantum superpositions would give a contradiction with "macroscopic local realism." The third step taken in this paper is to outline a procedure for examining the compatibility of "macroscopic local realism" with certain quadrature phase amplitude and photon number measurements performed on a different types of quantum superpositions of macroscopically distinct states.

2. MICROSCOPIC "ELEMENTS OF REALITY" AND THE EINSTEIN–PODOLSKY–ROSEN ARGUMENT

We begin by outlining the meaning of the premise "local realism," as introduced by EPR [1]. First, "locality" (or "no action-at-a-distance") implies that a measurement performed in a spatial region B cannot immediately influence events occurring in a second region A, spatially separated from B. A measurement performed at B cannot be the cause of, or in any way influence, the result of a measurement simultaneously performed at A. Second, the premise of "realism" is sufficient to imply the following: If one can predict with certainty the result of a measurement made on a system, without in any way disturbing that system, then the result of the measurement is a predetermined property of the system. Einstein, Podolsky, and Rosen called such predetermined properties "elements of reality."

To formulate the EPR argument one considers two correlated, spatially separated systems A and B. In the original formulation [1] given by Einstein, Podolsky, and Rosen in 1935, the correlated system consists of two spatially separated particles which are correlated both in position and in momentum. Such correlated systems are predicted by quantum mechanics. One can infer the result of the measurement of the position of the particle at A by performing a simultaneous measurement (of position) on the particle at B. Now the "locality" assumption tells us that the measurement at B could not have disturbed the system at A. Hence, EPR reason, because one can predict precisely the result of the measurement of position of particle A without disturbing the system, "realism" implies that the position of the particle A is a predetermined "element of reality." We will denote this "element of reality" (for the particle A's position) by x_A. Now x_A is a real number and gives the value of the result of a measurement of position of particle A. The x_A is a property of the system (the particle) at A and has an existence which is independent of whether or not a measurement at A actually takes place.

We emphasize that the existence of x_A as an intrinsic property (an "element of reality") of the system at A has been concluded only by use of the combined premises of "locality" and "realism." We refer to the two premises combined as "local realism." Because the momenta of the particles at A and B are also perfectly correlated, one can apply similar reasoning to conclude that "local realism" implies the existence of a predetermined momentum for particle A. This "element of reality" is again a real number which gives the result of a measurement of momentum, if it is performed. We denote this "element of reality" by p_A.

The assumption of "local realism" has allowed us to conclude, for this correlated system, the simultaneous existence of two "elements of reality," the real numbers x_A and p_A. The numbers x_A and p_A give respectively the result of a measurement of position or momentum of a single particle, if performed. This picture, of predetermined values for position and momentum, is not contained in a quantum description of the system. In quantum mechanics the ultimate specification of a system is given by the wave function. No quantum formulation specifies the existence of two classical numbers which simultaneously describe at a given time t, with absolute precision, the results of a measurement of position or momentum, if performed. The specification of the two "elements of reality" goes beyond a quantum description. The conclusion of Einstein, Podolsky, and Rosen was that, based on the real existence of the correlated system and the assumption of "local realism," quantum mechanics is incomplete. To date the quantum prediction of the existence of the appropriately correlated particles has not been demonstrated experimentally. An essentially equivalent version [8] of the EPR argument pertaining to two spatially separated fields has been experimentally achieved however by Ou *et al.* [6].

Now it is important not to misunderstand the conclusions of the EPR argument. There is no claim that quantum mechanics is wrong in its statement of the uncertainty relation or in any of its predictions. Quite the reverse. A series of measurements performed over an ensemble of such systems will give a determination of Δx and a determination of a Δp which will confirm the uncertainty principle. As we have no scheme to physically measure simultaneously both the position and momentum of a single particle to the required precision, one cannot test directly the existence of the precisely defined $x_A^{(i)}$ and $p_A^{(i)}$, where the superscript refers to an individual member of the ensemble.

In the original EPR argument as outlined above the "elements of reality" x_A and p_A represent hidden variables for the simultaneous position and momentum of a single particle. These variables are specified precisely. They represent the precise value for the result of a perfectly accurate measurement. Now in any real measurement there will be a limit to the accuracy to which the measurement can be performed, and the measured correlation between the particles A and B may not be perfect for this and other reasons. One may not be able to predict, based on results at B, the results of position and momentum measurements at A with absolute certainty, but only to precisions given by $\pm\Delta x_A$ and $\pm\Delta p_A$, respectively. Let us suppose for simplicity of argument that the errors $\pm\Delta x_A$ and $\pm\Delta p_A$ in the predictions are uniform across all values of x_A and p_A. In carrying the EPR argument through, one can only deduce the existence of "elements of reality" for position and momentum which have associated indeterminacies $\pm\Delta x_A$ and $\pm\Delta p_A$, respectively. In order to make the EPR claim (based on local realism) that quantum mechanics is incomplete in its description of this particle, one requires the values for the two "elements of reality" (describing the predetermined position and momentum of the single particle) to be specified to $x_A\pm\Delta x_A$ and $p_A\pm\Delta p_A$, where $\Delta x_A\,\Delta p_A<\hbar/2$. Because of the microscopic nature of the uncertainty limit, we make the claim that at least one of the "elements of reality" x_A, p_A needs to be specified microscopically, meaning that at least one of the Δx_A or Δp_A is microscopic. This requirement puts a condition on the accuracy needed for the measurements of position and momentum to have the situation envisaged by EPR. In essence we are considering a situation where the measurements are sufficiently accurate so as to enable the most microscopic determination of position (or momentum).

Bohm [9] presented a version of the EPR argument in which one considers two spatially separated spin-half particles, at locations A and B, respectively. A correlation between the particles means that one can instantly infer the result of the measurement of the spin component S_x^A of the particle at A by performing a measurement on the particle at B. Alternatively, one can infer the result of a measurement of the spin component S_y^A of the particle at A by performing a different measurement on the particle at B. In this case the result of the spin measurement will be either $+1/2$ or $-1/2$.

Following along the lines of EPR, the premise of "local realism" in this case allows us to conclude that the two x and y spin components of particle A are predetermined. Here each "element of reality," s_x^A and s_y^A say, takes on one of two values, $+1/2$ or $-1/2$, representing the two possible states, spin "up" and spin "down," of the particle. One can follow along the lines of the original EPR argument to deduce that, with the assumptions of local realism, quantum mechanics gives an incomplete description. The distinction between spin "up" and spin "down" for a single spin-half particle is a microscopic one. The "element of reality" s_x^A must be defined with sufficient precision, so that this distinction is made. Measurements made must be correspondingly accurate to enable the required correlation between the spin components at A and B to be established.

In the hidden variable picture, the "element of reality" s_x^A say takes on *either* $1/2$ *or* $-1/2$, this value corresponding to the result of the S_x^A measurement should it be performed. The concept of a quantum superposition state, where the system must be thought of as being in *both* states $+1/2$ and $-1/2$ until the S_x^A measurement is performed, is ruled out.

3. "MICROSCOPIC LOCAL REALISM" FAILS: BELL'S THEOREM

It is important to realise that the existence of the predetermined "elements of reality" is implied by the assumption, made in 1935 by EPR, of "local realism." The original work in 1966 of Bell, and subsequent work by Bell and others [2], acts to question this assumption, showing that "local realism" could not be (as EPR had apparently supposed) compatible with quantum mechanics.

It is important to understand the difference between the EPR argument and Bell's theorem. The EPR argument assumes "local realism" and shows that, provided certain correlated states exist, quantum mechanics is incomplete. Thus, if one can experimentally demonstrate the existence of the correlations, we conclude *either* that "local realism" is incorrect (in which case the EPR argument cannot be followed through) *or* that quantum mechanics is incomplete.

Bell's theorem, on the other hand, concludes that "local realism" cannot be compatible with the predictions of certain ideal quantum superposition states. If these states can be experimentally realised then, one must conclude that "local realism" is incorrect. Because it tests "local realism" outright, the conclusions of Bell's theorem are more powerful than those of EPR.

Bell's theorem relates to the Bohm gedanken experiment which is microscopic. It is our objective to demonstrate that this implies the rejection of local realism at a microscopic level and that more macroscopic levels of local realism have not been tested. To ensure clarity of our arguments, we revise Bell's result as applied to the bosonic state we will later use.

We consider the following entangled quantum superposition state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (a_+^\dagger b_+^\dagger + a_-^\dagger b_-^\dagger)\, |0\rangle\, |0\rangle. \tag{3.1}$$

Here $a_\pm$ and $b_\pm$ are boson operators for two pairs of photon fields; the $a_\pm$ and $b_\pm$ fields are spatially separated. In this case we have one photon generated in the a fields and one generated in the b fields. One can produce, using beamsplitters or polarisers, transformed fields $c_\pm$ and $d_\pm$ at the locations A and B, respectively (see Fig. 1), where

$$\begin{aligned} c_+ &= a_+ \cos\theta + a_- \sin\theta \\ c_- &= -a_+ \sin\theta + a_- \cos\theta \\ d_+ &= b_+ \cos\phi + b_- \sin\phi \\ d_- &= -b_+ \sin\phi + b_- \cos\phi. \end{aligned} \tag{3.2}$$

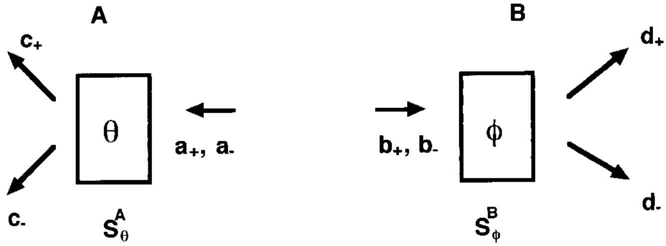

Fig. 1. Schematic diagram of the Bell experiment using photons. Measurements are performed on two spatially separated fields at A and B, respectively. The θ represents the measurement choice at A, while ϕ represents the measurement choice at B. Each measurement gives one of two outcomes, a photon at the $+$ position or at the $-$ position.

One can define the following Schwinger operators which make clear the correspondence of this bosonic state to the spin-1/2 formulation:

$$S_z^A(\theta) = (c_+^\dagger c_+ - c_-^\dagger c_-)/2$$
$$S_z^B(\phi) = (d_+^\dagger d_+ - d_-^\dagger d_-)/2. \tag{3.3}$$

These operators are measured by photon detectors at the locations A and B, placed behind the beamsplitters/polarisers used to produce the transformation to the outgoing fields $c_\pm$ and $d_\pm$. The photodetectors determine the photon numbers $c_+^\dagger c_+$ and $d_+^\dagger d_+$. The outcome for each of the photon numbers $c_+^\dagger c_+$ and $d_+^\dagger d_+$ is 0 or 1. The result 1 implies $S_z^A(\theta) = 1/2$, while the result 0 implies $S_z^A(\theta) = -1/2$.

We note that each measurement at A corresponds to a certain choice of parameter θ. Similarly a measurement at B corresponds to a certain choice of ϕ. For example if $\theta = 0$, one measures

$$S_z^A(0) = (a_+^\dagger a_+ - a_-^\dagger a_-)/2 \tag{3.4}$$

which corresponds to the z-spin component in the Schwinger formalism. We introduce an abbreviation of notation, writing $S_z^A(\theta)$ as S_θ^A, and $S_z^B(\phi)$ as S_ϕ^B.

The state $|\psi\rangle$ predicts a correlation for the outcomes of the $S_z^A(\theta)$ and $S_z^B(\phi = \theta)$ measurements. The correlation for arbitrary $\theta = \phi$ is most easily seen by rewriting the state $|\psi\rangle$ in terms of the transformed fields $c_\pm$ and $d_\pm$, where $\theta = \phi$. A correlation between the results $c_+^\dagger c_+$ and $d_+^\dagger d_+$ is apparent, a result 0 (or 1) for $c_+^\dagger c_+$ will imply 0 (or 1) for $d_+^\dagger d_+$, respectively.

The correlation given by the state $|\psi\rangle$ is that required for the EPR–Bohm situation. A measurement of $S_z^B(\theta)$ (the choice of $\phi = \theta$) allows prediction of the precise value (1/2 or $-1/2$) for the result of measurement $S_z^A(\theta)$ performed simultaneously on the second particle at A. Following the EPR argument through, the assumption of "local realism" implies that the spin components $S_z^A(\theta)$ for particle A are predetermined. We will denote these predetermined quantities (the "elements of reality") by the symbol s_θ^A. This s_θ^A always takes on one of the two values, $+1/2$ or $-1/2$. Similarly, "local realism" implies the existence of "elements of reality" s_ϕ^B for the spin components of the particle at B. Since the particle A will always be in one of the two states $s_\theta^A = 1/2$ or $s_\theta^A = -1/2$, and the particle at B will always be in one of the two states $s_\phi^B = 1/2$ or $s_\phi^B = -1/2$, we can introduce a joint probability distribution $\rho(s_\theta^A, s_\phi^B)$ that the system of two particles has "elements of reality" which take on the values s_θ^A and s_ϕ^B simultaneously. In this case simultaneous measurements of S_θ^A and S_ϕ^B will give the results s_θ^A and s_ϕ^B, respectively. The whole set of "elements of reality" s_θ^A and s_ϕ^B form a set of "hidden variables" which can be attributed to the two-particle system at a given time. Common notation symbolises the complete set of hidden variables by λ, and the underlying joint probability distribution $\rho(s_\theta^A, s_\phi^B)$, giving the probability for a certain hidden variable s_θ^A, s_ϕ^B state for the system, becomes $\rho(\lambda) = \rho(s_\theta^A, s_\phi^B)$.

Now we have been able to conclude, assuming "local realism," that the system is always in one of the hidden variable states given by a choice of values for the λ which in this case correspond to the "elements of reality" s_θ^A and s_ϕ^B. For each such state λ there is a probability $p_+^A(\theta, \lambda)$ that the result of an S_θ^A measurement will be "up." Similarly for each state λ there is a probability $p_+^B(\phi, \lambda)$ that the result of an S_ϕ^B measurement will be "up."

To establish Bell's result, one considers joint measurements (Fig. 1), where the spin component S_θ^A and the spin component S_ϕ^B are measured at the spatially separated locations A and B, respectively. A joint measurement will give one of four outcomes, $1/2$ or $-1/2$ for each particle. By performing many such measurements over an ensemble, one can experimentally determine the following: $P_{++}^{AB}(\theta, \phi)$ the probability of obtaining $+1/2$ for particle A and $+1/2$ for particle B upon simultaneous measurement of S_θ^A and S_ϕ^B: $P_+^A(\theta)$ the marginal probability for obtaining the result $+1/2$ upon measurement of S_θ^A; and $P_+^B(\phi)$ the marginal probability of obtaining the result $+1/2$ upon measurement of S_ϕ^B. In the "element of reality" picture based on the premise of local realism, the value obtained for a measurement (S_θ^A, say) is directly given by the value of the "element of reality" (s_θ^A). Thus the probability of obtaining "up" for S_θ^A is expressible as

$$P_+^A(\theta) = \int \rho(\lambda)\, p_+^A(\theta, \lambda)\, d\lambda. \tag{3.5}$$

The probability of obtaining "up" for S_ϕ^B is

$$P_+^B(\phi) = \int \rho(\lambda)\, p_+^B(\phi, \lambda)\, d\lambda. \tag{3.6}$$

The joint probability for obtaining "up" for both of two simultaneous measurements S_θ^A and S_ϕ^B is

$$P_{++}^{AB}(\theta, \phi) = \int \rho(\lambda)\, p_+^A(\theta, \lambda)\, p_+^B(\phi, \lambda)\, d\lambda. \tag{3.7}$$

Here θ and ϕ determine which component of spin is measured at the locations A and B, respectively. The "element of reality" s_θ^A, describing the spin component of the particle at A, and hence, the result of the S_θ^A measurement, has an existence and value (1/2 or $-1/2$) which is independent of the value of ϕ, the spin component the experimenter at B chooses to measure. This independence of $p_+^B(\phi, \lambda)$ on θ and $p_+^A(\theta, \lambda)$ on ϕ follows from the locality assumption. The measurement made at B (S_ϕ^B) cannot instantaneously influence the system, or the "elements of reality," at A.

It is well known that the Bell–Clauser–Horne inequality [2] can be derived from the results (3.5)–(3.7), which follow from the "element of reality" picture concluded on the basis of "local realism." We have the following which follows directly from $0 \leqslant p_+^A(\theta, \lambda), p_+^B(\phi, \lambda) \leqslant 1$:

$$B = \frac{P_{++}^{AB}(\theta, \phi) - P_{++}^{AB}(\theta, \phi') + P_{++}^{AB}(\theta', \phi) + P_{++}^{AB}(\theta', \phi')}{P_+^A(\theta') + P_+^B(\phi)} \leqslant 1. \tag{3.8}$$

The quantum prediction for the state $|\psi\rangle$ is

$$P_{++}^{AB}(\theta, \phi) = \tfrac{1}{2}\cos^2(\theta - \phi)$$

$$P_+^A(\theta') = P_+^B(\phi) = \tfrac{1}{2}.$$

The choice $\phi' - \theta' = \theta' - \phi = \phi - \theta = \pi/8$, $\phi' - \theta = 3\pi/8$ violates the inequality, indicating that the predictions of the quantum superposition are not compatible with the assumptions of "local realism" on which the inequality is based.

Experiments by Aspect *et al.* and others [3] have succeeded in demonstrating a violation of the inequality, or rather one closely related to it, which accounts for a reduced correlation due to poor detection inefficiencies. From this it is generally concluded that local realism fails.

Now we come to the point of this paper. So far rejection of local realism is in a microscopic sense only. This requires careful explanation.

The inequality (3.8) is derived using the assumption of "local realism." "Locality" states that the measurements made at B cannot instantaneously disturb the system at A at all. In the Bohm–EPR case, one uses "local realism" to assign to the system an "element of reality" which can be one of two values, representing states of the system which are microscopically distinct. The locality assumption is being used at a level sufficient to ensure that measurements made at a system B cannot cause to a second system A, spatially separated from B, even a microscopic change sufficient to change the result of measurement from 1/2 to $-1/2$. In the case where we have perfect correlation, local realism is being used to imply "elements of reality" with a spin 1/2 or spin $-1/2$ value, so that the system is in one state or the other, thus ruling out the possibility of a quantum superposition of two such states, which here are microscopically distinct. Our aim is to define a weaker level of local realism, a "macroscopic local realism," which is sufficiently strong to rule out quantum superpositions of states macroscopically distinct, but not superpositions of states microscopically distinct.

4. MACROSCOPIC LOCAL REALISM: HOW TO DEFINE IT

We begin by introducing a definition for a “macroscopic experiment.” We define a macroscopic experiment to be one in which all relevant measurements are made with an uncertainty Δ which is itself macroscopic. Our claim would be that an experiment in which position is measured to, say, an accuracy of $10^{-10}m$ is a microscopic, not a macroscopic, experiment. An experiment in which position is measured to plus or minus a few centimetres would be classified as macroscopic. Such macroscopic experiments are normally viewed as behaving according to classical laws and the premise of local realism we expect to appear to hold in these experiments.

Let us examine more closely the meaning of “local realism” in the context of such macroscopic experiments. Local realism could be used in these macroscopic situations to assign “elements of reality” to the system. In this case, however, because of the macroscopic uncertainty associated with measurements, the “elements of reality” are also intrinsically macroscopic, in the sense that they have a macroscopic uncertainty associated with them. They cannot be precisely, or microscopically, specified because the macroscopic nature of the measurement process would not permit sufficient correlation in an EPR-type situation. The “elements of reality” attributed to the system in this macroscopic situation would, being themselves intrinsically macroscopic, represent some “macroscopic property” of the system. In a classical-like theory based on “local realism” the system is specified by these properties which are predetermined, having an existence independent of a possible measurement process. In this case because the measurement, and the properties which the measurement aim to determine, are intrinsically macroscopic, it is a weaker form of locality (a “macroscopic locality”) that is assumed to hold in describing this system. Likewise it is only this weaker form of locality that could be tested experimentally in such a situation. In assuming locality here one is really making the following statement. “Macroscopic locality” states that measurements made at a location B cannot immediately influence (macroscopic) measurements made of macroscopic properties of a system at a location A, spatially separated from B. We are ruling out macroscopic changes to a system as a result of a causally separated event, but we are saying nothing about microscopic changes. Such microscopic changes would not be measurable, falling well within our experimental uncertainty. Now the general premise of “locality” is to rule out all changes, both macroscopic and microscopic, and hence, our “macroscopic locality” is a much weaker assumption.

We may consider a situation where a correlation exists between two spatially separated systems. Let us suppose the correlation is between the positions of two particles, as in the original EPR experiment. If we consider a macroscopic experiment, we will be determining the position only to an accuracy given by $\pm\Delta$, where Δ itself is macroscopic, say a centimetre. The correlation then is only measurable to this accuracy. The positions of the two particles are correlated in that position x for one implies x for the other, but x is determined at best only to a precision given by the order $\pm\Delta$. If we apply the EPR reasoning to this situation then, what can we conclude about the "elements of reality" and is the premise "local realism" needed in its entirety here? The logic of the EPR reasoning follows as in the original case, where position was defined and measurable to unlimited accuracy. One can predict the result of a measurement of the particle *A*'s position by performing a measurement on particle *B*. The position of particle *A* is determined to within a range of order plus or minus a centimetre. Locality implies that the measurement performed at location *B* could not have influenced system *A*. Here we amend the argument. We state that "macroscopic locality" implies that the measurement at *B* could not have influenced in a macroscopic way the system at *A*. We need only use the premise of "macroscopic locality," as opposed to "locality" in its entirety, because we cannot measure microscopic changes to the results of measurements. Any measurable change in the result of a measurement in this experiment must be macroscopic. We now use the EPR premise of realism. Because the system *A* has not been disturbed, we conclude the existence of a predetermined position for particle *A*, but the "element of reality" is specified only to the precision of order $\pm\Delta$ to which it can be inferred by the measurement made at *B*. We denote this "element of reality" with its associated uncertainty by $x\pm\Delta$. Here we have a view of a physical system specified by predetermined properties (call them "elements of reality" or "hidden variables"), but these elements of reality are specified only at a macroscopic level. This is a realistic view at a macroscopic level, something we would call "macroscopic realism." The combined premises of locality and realism used in this macroscopic example of the EPR situation we call "macroscopic local realism."

The above discussion attempts to give a general meaning to the term "macroscopic local realism." We also wish to construct a more specific meaning in terms of one of its consequences. Consider the original EPR argument. The result of a measurement of position of a particle at *A* can be inferred (to within an error of $\pm\Delta$ which might be of any order depending on the experiment), by performing a measurement on a second particle at *B* spatially separated from *A*. Since any real physical situation will always have an error associated with its measurements, we have included in the statement the error $\pm\Delta$ in the inferred value for position. Let us denote the inferred position by $x\pm\Delta$. "Local realism" allows us to conclude that the position of particle *A* was predetermined. It is concluded, with "local realism," that the particle *A* did actually have this position $x\pm\Delta$, at the time of the measurement at *B*. "Local realism" establishes that the particle *A* was in a state corresponding to the inferred value $x\pm\Delta$ of position, immediately prior to the measurement at *B* allowing the determination. Here we are meaning to use "local realism" in its entirety, although with Δ macroscopic this may not ever be necessary. We now claim that the following is certainly a true consequence, pertaining to the above situation, of "macroscopic local realism." "Macroscopic local realism" is sufficient to establish that the particle *A* could not have been, immediately prior to the measurement made at *B*, in a state macroscopically different to that implied by the measurement at *B*. This consequence of "macroscopic local realism" has been used recently [5] to point out the macroscopic implications of the recent experimental achievement [6] of the EPR correlations. Macroscopic local realism does not, as local realism in its entirety does, rule out the possibility that the system A was in a state microscopically close to that implied by the measurement at B. We see that we are not ruling out the possibility that the measurement at B caused a microscopic change to the description of position of particle A. Macroscopic changes however are ruled out. This is in accordance with our earlier definition of "macroscopic locality." Macroscopic local realism assigns to the system A an "element of reality" given by $x \pm \Delta_x$, where Δ_x is $\Delta + \delta$ and δ can be microscopic or mesoscopic in size, but not macroscopic. This is, in contrast to local realism in its entirety, which simply assigns to A the "element of reality" $x \pm \Delta$.

We note that where Δ, the error in our inferred value for the position of particle A, is zero and the correlation is perfect, local realism in its entirety puts the particle A in a state of definite position in the most microscopic sense. Here "local realism" rules out the possibility of the particle being in a "quantum superposition" (where the particle must be considered to be in both states until the measurement is performed) of position states, even states which are microscopically distinct. Macroscopic local realism cannot do this. This weaker premise is only sufficient to determine, by specifying "elements of reality," the particle's position to $\pm \Delta_x$, where Δ_x can be any microscopic or mesoscopic value, but it cannot be macroscopic. Macroscopic local realism is sufficient, however, to rule out the possibility that the particle was in a "quantum superposition" of position states macroscopically distinct.

Let us examine more closely situations of macroscopic experiments where Δ, the error in our inferred value for the result of the measurement at A, must itself be macroscopic. Local realism is used to establish that the particle A was in a state corresponding to the inferred value $x \pm \Delta$ of position, immediately prior to the measurement at B. "Local realism" in its entirety puts the particle in a state with an "element of reality" $x \pm \Delta$ for position. If Δ is itself macroscopic, however, our original claim was that "local realism" was not needed in its entirety to establish that particle A had predetermined position $x \pm \Delta$. In this context where Δ is macroscopic, so that $x \pm \Delta$ is an "element of reality" which is intrinsically macroscopic, we would claim that a state or description or "element of reality" is only truly distinguishable in a practical sense in a macroscopic experiment if it is macroscopically different to the "element of reality $x \pm \Delta$." In this way it suffices to replace "local realism" with "macroscopic local realism" in the above statement of this paragraph. To say the state is different to $x \pm \Delta$, where Δ is macroscopic, is to say it is "macroscopically different."

The concept of "macroscopic local realism," and the distinction between "macroscopic local realism" and "local realism" in its entirety, is perhaps most easily developed initially by considering experimental circumstances where the results of measurements are discrete. In the Bell-inequality and Bohm–EPR experiments one has two spatially separated spin-1/2 systems. Let us simplify by calling the spin 1/2 result $+1$, and the spin $-1/2$ result -1. Let us suppose there is a perfect correlation (as, for example, is the case with state (3.1)) for $\theta = \phi$. The premise of local realism in this case of perfect correlation allows us to deduce the existence of two "elements of reality," one for each subsystem A and B. We label these "elements of reality" s_θ^A and s_θ^B, where these variables always take on one of the two values $+1$ and -1, corresponding to the two possible results ($+1$ or -1) for the measurements. In this "local realism" description, therefore, both particles exist in a state corresponding either to the "element of reality" being $+1$, or to the "element of reality" being -1. The $p_+^A(\theta, \lambda)$ used in Section 3 (in the derivation of the Bell inequality implied by this "local realism" description) is the probability that the "element of reality" s_θ^A associated with measurement θ is $+1$, as opposed to -1. One is distinguishing between the two possible states, $+1$ and -1, claiming that the system must be in one or the other.

In the case where the states $+1$ and -1 are microscopically distinct, one needs to use the premise of "local realism" in its entirety, right down to a microscopic level, if we are to establish the claim that the system is in one or the other of the $+1$ and -1 states at all times. The two values for the "elements of reality" implied by the premise are microscopically distinguishable. The measurements made on the system distinguish at this microscopic level. Locality in this case is being used to exclude microscopic changes (e.g., $+1$ to -1).

Let us now consider a hypothetical situation where the states $+1$ and -1 are macroscopically distinct. We assume that the correlation between the results of measurements is still perfect, so that one can use local realism to assign "elements of reality," which have possible values $+1$ and -1, to each subsystem as before. Here, though, the two possible values for the "elements of reality" represent states of the system which are macroscopically distinct. The two possible results of the measurement are able to be distinguished, and the correlation therefore is established, in a suitable macroscopic experiment where the precision of measurement is itself macroscopic. In this case, therefore, one does not need to use the premise of "local realism" in its entirety to establish the existence of the "elements of reality." It is sufficient to use the weaker premise of "macroscopic local realism." Macroscopic local realism claims that, if a result at B implies a result of $+1$ at A, the system at A could not have been in a state macroscopically different to that implied. Since here there are only two relevant states $+1$ and -1, which are macroscopically distinct, the implication of "macroscopic local realism" is sufficient to establish that the system was in state $+1$. In this case it is implicit that the "elements of reality" themselves need only be defined to a precision giving an uncertainty which is macroscopic. In a macroscopic experiment, and with the usage of macroscopic local realism, the existence of "elements of reality" microscopically defined cannot be established. We note that the $p_+^A(\theta, \lambda)$ used in the derivation of the Bell inequality in Section 3 would now refer to the probability that the system is in $+1$, as opposed to -1, but where $+1$ and -1 are states macroscopically distinct. The premise of "macroscopic local realism" implies that the system must at all times be in one or the other of these two states, $+1$ and -1, which are macroscopically distinct. This is the premise which is therefore used to derive the Bell inequality in such a circumstance and is the premise which would be invalidated in the event that such a Bell inequality would be violated. The possibility of the system being in a quantum superposition of the two states ($+1$ and -1) which are macroscopically distinct [7] (such a state is often called a "Schrodinger-cat" state) is ruled out by the premise of macroscopic local realism. We see that the real failure of macroscopic local realism would be an astonishing fact, as astonishing as the real existence of a Schrodinger-cat superposition state.

In fact insight into the concept of "macroscopic local realism" is most readily gained by consideration of the Schrodinger-cat example. Consider two systems, at A and B, each of which can be found to be in one of two states, macroscopically distinct. Here $|A\rangle_A$ and $|D\rangle_A$ refer to two macroscopically distinct (such as a "cat alive" and a "cat dead") states of a system at a location A. Similarly $|A\rangle_B$ and $|D\rangle_B$ represent two macroscopically distinct states of a second system, spatially separated from A, at a location B. We consider a measurement which will distinguish the state $|A\rangle_A$ from $|D\rangle_A$. Suppose such measurements performed on A and B will give correlated results, so that a result "A" or "D" at location A implies the same result, "A" or "D," at location B. Such macroscopically correlated systems are not difficult to find. For example, in our Schrodinger-cat example, the dead or alive state of the cat may be correlated with the number of bullets in the gun used to kill the cat. Such macroscopically correlated systems however are (usually) described quantum mechanically by a diagonal density operator

$$\hat{\rho} = \tfrac{1}{2}\{ |A\rangle_A |A\rangle_B \, {}_A\langle A|_B \langle A| + |D\rangle_A |D\rangle_B \, {}_A\langle D|_B \langle D| \}. \tag{4.1}$$

This quantum description is equivalent to a simple classical probabilistic interpretation, where the system is always in, with equal probability, one of the two states represented quantum mechanically by $|A\rangle_A |A\rangle_B$ and $|D\rangle_A |D\rangle_B$. "Macroscopic local realism" will hold for such classical-like situations. The EPR argument applied to the system of cat-gun would be follows. We can determine the state of the cat at A, whether it is dead or alive, by observing at B the number of bullets in the gun. The act of determining the number of bullets cannot affect in a macroscopic way the state of the cat. (This is the assumption of macroscopic local realism). Therefore EPR conclude, using macroscopic local realism, that the cat is always in one of the two states (alive or dead). If we can infer from the number of bullets that the cat is dead, macroscopic local realism claims that the cat cannot be alive, since it cannot be macroscopically different to the state inferred. In this context of a truly macroscopic system, the failure of macroscopic local realism would seem ridiculous. The idea that the correlation between the measurements made on the gun and the cat is due to the measurement of the gun actually causing an observable (and therefore in the context of this experiment, macroscopic) change to the state of the cat seems absurd.

The failure of local realism in the macroscopic way we have described here is more difficult to accept than the failure of local realism at the microscopic level. Macroscopic local realism is a weaker premise. Whether there exist quantum states which do predict a failure of macroscopic local realism is not known and does not appear to have been previously investigated. An example of an extreme quantum superposition state (an "entangled Schrodinger-cat" state) is

$$|\psi\rangle=\frac{1}{\sqrt{2}}\{|A\rangle_A|A\rangle_B+|D\rangle_A|D\rangle_B\}. \tag{4.2}$$

This is a "Schrodinger-cat state" in that the two states $|A\rangle_A|A\rangle_B$ and $|D\rangle_A|D\rangle_B$ are macroscopically distinct. It is also a state which is correlated in the macroscopic way discussed in the paragraph above. The assumption of macroscopic local realism is able to be used here to deduce that the systems A and B are both always in one of two states, which we might like to call "$|A\rangle$" or "$|D\rangle$." This corresponds to the "element of reality" taking on one of two values, the "alive" value or the "dead" one. But this interpretation, that the cat and the gun are always in one of the "alive" and "dead" descriptions, would seem to contradict the quantum interpretation of the superposition nature of the state (4.2). It would seem likely then that this state is incompatible with the premise of macroscopic local realism. This will be discussed further in Sections 6 and 7.

5. MACROSCOPIC LOCAL REALISM REJECTED OR AN INCOMPLETE QUANTUM MECHANICS?

In this section we review an argument [5] which gives urgency to the question of the validity of macroscopic local realism. The EPR argument and conclusions are shown to apply to a situation where the "elements of reality" are macroscopic. In the example we consider "macroscopic local realism" is sufficient to imply that quantum mechanics is incomplete.

We being by defining two noncommuting operators which correspond to quantities measured [10] in a homodyne defection scheme (Fig. 2) commonly used in experiments detecting subshot noise ("squeezed") radiation. We define

$$\begin{aligned} S_x^A &= a_1 a_2^\dagger + a_1^\dagger a_2 \\ S_y^A &= (a_1 a_2^\dagger - a_1^\dagger a_2)/i. \end{aligned} \tag{5.1}$$

Here a_i, $a_i^\dagger$ are the usual boson operators describing the field i. We will refer to the field a_1 as the signal field, while a_2 is a very intense coherent ("local oscillator") field. The quantities S_x^A and S_y^A are measured when the local oscillator field is combined with the signal field a_1 using a beam splitter. A variable phase shift θ is introduced, so that the two outputs of the beam splitter are written

$$\begin{aligned} c_+ &= (a_2 + a_1 e^{-i\theta})/\sqrt{2} \\ c_- &= (a_2 - a_1 e^{-i\theta})/\sqrt{2}. \end{aligned} \tag{5.2}$$

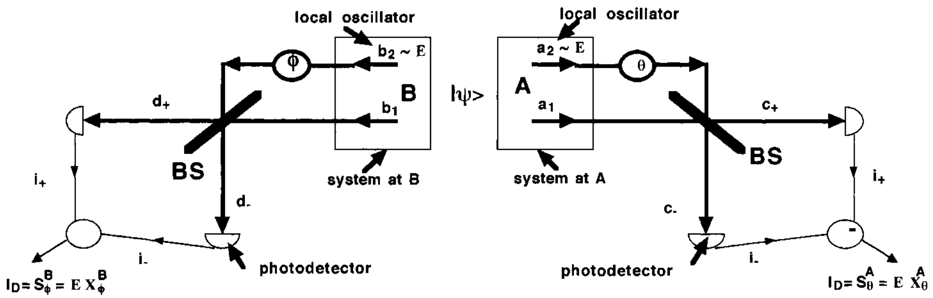

Fig. 2. Schematic diagram of the experimental arrangement used to give the new version of the EPR paradox. Here θ is a phase shift which can be chosen to give either a measurement of S_x^A or S_y^A, while ϕ is chosen to give either S_x^B or S_y^B. The a_2 and b_2 represent strong local oscillator fields. Here $|\psi\rangle$ denotes a quantum state giving correlations of the type discussed in the text.

Direct detection, using two photodetectors, gives two photocurrents proportional to the intensities $c_+^\dagger c_+$ and $c_-^\dagger c_-$, respectively. Subtraction of the two photocurrents gives a current directly proportional to $I_D = c_+^\dagger c_+ - c_-^\dagger c_-$. It is straightforward to verify that in fact this final photon number difference is

$$I_D = c_+^\dagger c_+ - c_-^\dagger c_- = S_\theta^A = a_2^\dagger a_1 e^{-i\theta} + a_2 a_1^\dagger e^{i\theta}. \tag{5.3}$$

Choice of $\theta = 0$ will give a measurement of S_x^A, while the choice of $\theta = \pi/2$ will give a measurement of S_y^A.

The commutation relations $[a_i, a_i^\dagger] = 1$ imply the uncertainty relation for the operators S_x^A and S_y^A,

$$\Delta S_x^A \, \Delta S_y^A \geqslant |<S_z^A>|, \tag{5.4}$$

where $S_z^A = a_2^\dagger a_2 - a_1^\dagger a_1$. This is recognised directly once we recognise that S_x^A and S_y^A are spin operators in the Schwinger formulation.

In the limit corresponding to the homodyne detection measurement, the field a_2 is very intense, much more intense than the signal field a_1, and is called the "local oscillator" field. In this case we may replace a_2 by the classical amplitude E (assumed real) and write $\langle a_2^\dagger a_2 \rangle = E^2$. Then we may simplify

$$\begin{aligned} S_x^A &= E(a_1 + a_1^\dagger) = EX_1^A \\ S_y^A &= E(a_1 - a_1^\dagger)/i = EX_2^A \end{aligned} \tag{5.5}$$

$$\Delta S_x^A \, \Delta S_y^A \geqslant E^2. \tag{5.6}$$

In this case the measurements of S_x^A and S_y^A corresponds to measurements of the amplified signal quadrature phase amplitudes X_1^A and X_2^A, respectively. The effect of the local oscillator E is to introduce the amplification factor E.

Now it is possible (Fig. 2) to have a second pair of fields b_1, b_2 (a second signal and second local oscillator field) which are spatially separated from but correlated with the first pair a_1, a_2 [8, 11]. We define the Schwinger operators for the pair b_1, b_2 as

$$\begin{aligned} S_x^B &= b_1 b_2^\dagger + b_1^\dagger b_2 \\ S_y^B &= (b_1 b_2^\dagger - b_1^\dagger b_2)/i. \end{aligned} \tag{5.7}$$

These operators are measurable using a second balanced homodyne detection apparatus. Importantly this corresponds to a second phase shift, which we call ϕ, which determines which measurement, S_x^B or S_y^B, is made at the location B.

The nondegenerate parametric amplifier is an example [8] of a system which generates correlated signal fields a_1 and b_1, such that S_x^A is correlated with S_x^B, and S_y^A is correlated with S_y^B. In fact an ideal parametric amplifier, with Hamiltonian

$$H = i\hbar\chi(-a_1 b_1 + a_1^\dagger b_1^\dagger) \tag{5.8}$$

is predicted by quantum mechanics to give a maximum correlation, so that measurement of S_x^B will enable precise determination of the result of a measurement of S_x^A. Similarly, measurement of S_y^B will enable precise determination of the result of a measurement of S_y^A. More likely in a real experimental situation, the correlation will not be quite perfect. This means that we cannot determine, from the measurement of S_x^B, the result of a measurement of S_x^A precisely. We have some error in our determination. We denote the magnitude of this error by Δ_1. It may be measured by performing a sequence of simultaneous measurements of S_x^A and S_x^B and by evaluating for each member of the sequence the difference δ between the inferred value for S_x^A (based on the result of S_x^B) and the real value for the result of S_x^A. By performing many measurements, a probability distribution $P(\delta)$ for the δ is built up. We assume that the inference value is chosen so that the mean of δ is zero ($\langle \delta \rangle = 0$). The distribution will have a finite standard deviation, however, which we call Δ_1. Similarly, we denote the magnitude of the error in the inference of S_y^A, given a measurement of S_y^B, by Δ_2. An experiment by Ou *et al.* [6], using a nondegenerate parametric oscillator which gives correlations similar to those described here, achieved $\Delta_1 \Delta_2 \simeq 0.7E^2$.

We are able to infer the value of either of two noncommuting observables of a system A by performing measurements on a second spatially separated system at B. We can infer the result for S_x^A by measuring S_x^B, at B. Alternatively we could infer the result for S_y^A by measuring S_y^B at B. This is precisely the situation of the EPR argument discussed in Section 2. Assuming locality, one concludes that the value for the result of the measurement S_x^A is predetermined. We denote this predetermined "element of reality" by s_x^A. Similarly, the value for S_y^A is also predetermined. Let us denote it by s_y^A. In the case where the inference for the measurements at A is not perfect, our "elements of reality" are not precise values, but are variables defined with a probability distribution $P(\delta)$. We can determine the probability that the result for a measurement of S_x^A, say, is in a certain range, without disturbing the system A. Thus, the value for S_x^A is still predetermined, but the "element of reality" has an uncertainty associated with it. We may write such an "element of reality" for S_x^A as $s_x^A \pm \Delta_1$. Similarly, the "element of reality" for S_y^A is $s_y^A \pm \Delta_2$.

The assumption of "local realism" has implied for system A the simultaneous existence of two "elements of reality," $s_x^A \pm \Delta_1$ and $s_y^A \pm \Delta_2$, which give the result of an S_x^A or S_y^A measurement respectively, if performed. The result of the S_x^A measurement is specified to an accuracy corresponding to a standard deviation Δ_1, while that of S_y^A is given to Δ_2. If

$$\Delta_1 \Delta_2 < E^2 \tag{5.9}$$

we have a description of system A which cannot be given by any quantum wavefunction, which can never specify these values to an accuracy exceeding $\Delta S_x^A \, \Delta S_y^A \geqslant E^2$. In this way the assumption of "local realism" leads one to conclude that quantum mechanics gives an incomplete description of this system.

As with the original EPR paradox and Bohm's spin version of the paradox, we might choose to disregard the possibility of drawing the conclusion that quantum mechanics is incomplete, since the assumption of "local realism" is not valid. Bell's theorem and associated experiments have shown "local realism" to be wrong. Without the validity of "local realism," the EPR arguments concluding the simultaneous existence of two "elements of reality" do not go through. However, there is an important difference between all previous examples of the EPR paradox and the version of the paradox given here. In this version, one can carry through the EPR argument and conclusions, using only macroscopic measurements and defining "elements of reality" which are intrinsically macroscopic. This means that the weaker premise of "macroscopic local realism" (as opposed to local realism in its entirety) is sufficient to imply that quantum mechanics is incomplete for this example. Now the validity or otherwise of "macroscopic local realism" has not been established or questioned. Certainly the rejection of macroscopic local realism is a much stronger statement than the rejection of local realism at the microscopic level. Yet we will show that according to this new version of the EPR argument, one draws the conclusion that quantum mechanics is incomplete based only on the validity of macroscopic local realism.

Why do we only need to assume "macroscopic local realism," as opposed to "local realism" in its entirety, in order to carry through the EPR argument for this example? A crucial point is that the quantum limit of the uncertainty relation (5.4) (or (5.6)) is macroscopic. Here E^2 is the photon number of the local oscillator field, which in this case is macroscopic. This means that we are able to define "elements of reality" for S_x^A and S_y^A which have a macroscopic uncertainty Δ_1 and Δ_2, respectively, and still obtain the EPR criterion that $\Delta_1 \Delta_2 < E^2$. One does not require to make microscopic measurements of S_x^A and S_y^A, and S_x^B and S_y^B. Recalling that the S_x^A, S_x^B, S_y^A, S_y^B measurements are really photon number measurements, a microscopic measurement would be one in which the photon number is determined precisely, down to plus or minus a very small number of photons. In the experimental situation envisaged, E is very large and acts as an amplification factor, so that the actual photon number $c_+^\dagger c_+ - c_-^\dagger c_-$, corresponding to the value of S_x^A etc., is macroscopic. The measurements made of S_x^A etc. are macroscopic, in that they determine the photon number S_x^A to a macroscopic error only, although this error may be much smaller than the actual value obtained for S_x^A. Because the measurements made are intrinsically of a macroscopic nature, we are only able to infer the result of a S_x^A measurement, by measuring S_y^B, to an error Δ_1 which is itself macroscopic. Similarly, the error Δ_2 involved in the prediction of the result of an S_y^A measurement will also be macroscopic. The "elements of reality" giving the predetermined values for S_x^A and S_y^A are defined only with macroscopic errors Δ_1 and Δ_2. But as long as $\Delta_1 \Delta_2 < E^2$, we still arrive at the conclusion that quantum mechanics is incomplete.

In order to reach the conclusion that the "element of reality" exists defined to a precision Δ_i, where Δ_i is macroscopic, it is sufficient to assume only macroscopic local realism. The Δ_i is the precision to which we can predict a result for a measurement on system A, given a result for a measurement at B. In supposing the "element of reality" we need to assume that the measurement at B did not (instantaneously) cause any change to A. But because the "element of reality" is intrinsically macroscopic, we do not need to concern ourselves with microscopic changes. We simply need to rule out macroscopic changes, and therefore we only need to assume macroscopic local realism. According to our definition of macroscopic local realism, ruling out macroscopic changes only due to measurement at B, the result of a measurement at A cannot be macroscopically different from that implied by a measurement at B. The system A could not have been, immediately prior to the measurements at A and B, in a description macroscopically different to that implied by the measurement at B.

In this situation, the "element of reality" and a result for a measurement will be defined only macroscopically, in that the quantity S_θ^A, for example, will correspond to a "bin" where the range of detected photon number I_D is a large number of photons. This will be so because experimentally one could not resolve the photon number sufficiently at the very high photon number levels impinging on the detector. In this way the measurement of S_θ^A, a discrete photon number measurement, becomes a continuous measurement. One can only say that "elements of reality" (or results of a measurement) are distinct in this context if they are macroscopically distinct. Therefore, one can only apply the use of macroscopic local realism. In fact because in order to get condition (5.9) to allow the EPR argument to go through, we need only obtain Δ_i less than E^2, a macroscopic value, we have a situation where "macroscopic local realism" need be applied only its loosest form. We can allow system A to be in a state macroscopically different to that implied by measurement at B, and thus increase the Δ_i, and still obtain the condition $\Delta_1 \Delta_2 < E^2$. This is because E can be made arbitrarily large, and there exists a whole range of macroscopically different Δ, ranging from Δ_1 through to E.

6. IS "MACROSCOPIC LOCAL REALISM" COMPATIBLE WITH QUADRATURE PHASE MEASUREMENTS MADE ON A SUPERPOSITION OF TWO MACROSCOPICALLY DISTINCT COHERENT STATES?

Compatibility of macroscopic local realism with the quantum superposition state (4.2) would seem to be an open question. Here we consider the compatibility of macroscopic local realism with quantum mechanics for measurements made on the following macroscopic superposition state

$$|\psi\rangle = N\{\exp(i\theta)\,|\alpha\rangle_A\,|\beta\rangle_B + |-\alpha\rangle_A\,|-\beta\rangle_B\}, \tag{6.1}$$

where $N = 2\{[1+\exp(-2\,|\alpha|^2 - 2\,|\beta|^2)]\}^{1/2}$ and θ is a phase factor. Here $|\alpha\rangle_A\,|\beta\rangle_B$ are coherent states for two spatially separated modes of the field. The modes are assumed to be at locations A and B and are described by the usual boson operators a, $a^\dagger$ and b, $b^\dagger$, respectively. We take $\alpha = \beta$ to be real. For α, β very large, the two states $|\alpha\rangle_A$ and $|-\alpha\rangle_A$ become orthogonal and macroscopically distinct. In this way we see that the state (6.1) is a quantum superposition of two states macroscopically distinct.

We need a measurement which will act to signature the two states $|\alpha\rangle$ and $|-\alpha\rangle$. One can perform quadrature phase measurements by the homodyne detection methods described above [10, 12]. These quadrature phase amplitudes (analogous to position and momentum measurements) are given by

$$\begin{aligned} X_1^A &= a + a^\dagger, \qquad & X_1^B &= b + b^\dagger, \\ X_2^A &= \frac{a - a^\dagger}{i}, \qquad & X_2^B &= \frac{b - b^\dagger}{i}. \end{aligned} \tag{6.2}$$

The probability, let us denote it by $P(X_1^A = x)$, of getting a result x upon a "position" measurement X_1^A is given by $|\langle x|\psi\rangle|^2$, where $|x\rangle$ is a "position" eigenstate. If the system is in the coherent state $|\alpha\rangle_A\,|\beta\rangle_B$ the probability of getting a result x upon measurement of X_1^A is a gaussian centred at $\sqrt{2}\,\alpha$. If the system is in the state $|\psi\rangle$ this probability distribution, for large α, becomes two well-separated gaussian peaks, centred at $\sqrt{2}\,\alpha$ and $-\sqrt{2}\,\alpha$, respectively (Fig. 3). The

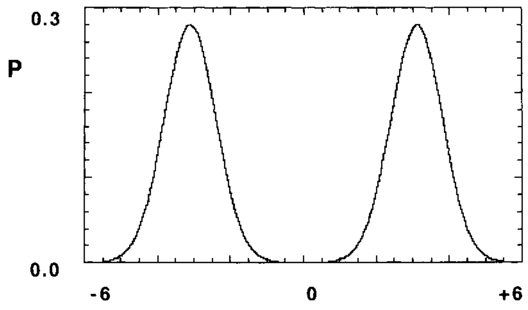

Fig. 3. Plot of $P(X_1^A = x)$ versus x. Here $\alpha = \beta = 2$.

distribution, let us call it $P(X_1^B = x)$, for a result x upon measurement of X_1^B on $|\psi\rangle$ is identical. A measurement of position on systems A and B gives, with equal probability, one of two possible sets of outcomes, corresponding to one of the gaussians. We will label the result of each measurement to be -1 if x is negative, and $+1$ if x is positive.

The results of these two measurements, let us call them S_1^A and S_1^B, determining the sign of the position for each system A and B, respectively, are, for very large α, perfectly correlated. A negative result for x at A always implies a negative result for x at B. Similarly, a positive result at A implies a positive result at B. This correlation is clear from the calculation of the joint probability distribution $P(X_1^A = x, X_1^B = y)$ for getting results x and y upon joint measurement of X_1^A and X_1^B. This distribution depicts two gaussian clusters (Fig. 4) centred at $x = \sqrt{2}\,\alpha$, $y = \sqrt{2}\,\alpha$ and $x = -\sqrt{2}\,\alpha$, $y = -\sqrt{2}\,\alpha$, implying a conditional probability given by $P(S_1^B = +1/S_1^A = +1) = 1$.

With this correlation the EPR reasoning, with the assumption of "local realism," will imply that the results of the S_1^A and S_1^B measurements are predetermined. The predetermined quantities (the "element of reality") we call s_1^A and s_1^B, where each of s_1^A and s_1^B takes on one of the two values $+1$ and -1.

We note that in this situation one need only assume "macroscopic local realism" to deduce the existence of the "elements of reality." In order to distinguish between the two outcomes (positive or negative) one does not need to use a "microscopic measurement" in which the quadrature phase amplitude is determined to a precision which is microscopic. For very large α, the amplitude x can be measured to within Δ_x (where Δ_x is macroscopic) of its true value, and one can still make the correct assignment of sign to the outcome x. The two outcomes for x (positive or negative) represent macroscopically distinct states. The correlation between the spatially separated measurements still holds perfectly to enable the EPR reasoning to be carried through.

The assumption then of "macroscopic local realism" allows us to deduce the existence of two macroscopically distinct "elements of reality" ($s_1^A = +1$ or -1, and $s_1^B = +1$ or -1) for each subsystem A and B. In this picture, each subsystem is

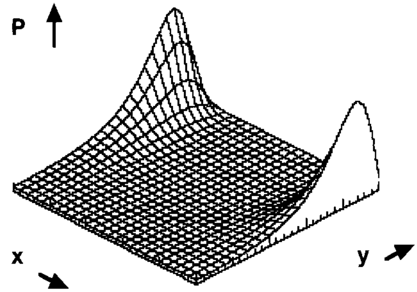

Fig. 4. Schematic drawing, depicting the two well-separated gaussian clusters, of the joint distribution $P(X_1^A = x,\ X_1^B = y)$.

always in the state $+1$ or the state -1. The correlation between measurements S_1^A and S_1^B implies that the system is actually in one of two correlated states $s_1^A = s_1^B = +1$ and $s_1^A = s_1^B = -1$ at all times. Since the probability of both outcomes is equal, "macroscopic local realism" has enabled us to deduce that the ensemble of such systems represented by (6.1) are in a 50:50 classical mixture of the two correlated states.

At first glance the classical mixture description implied by macroscopic local realism would appear incompatible with the quantum superposition state (6.1). It may be possible to find sets of quadrature phase measurements which distinguish between the two. It is known [12] that the superposition (6.1) is distinguishable from the classical mixture

$$\rho = \tfrac{1}{2}\{|\alpha\rangle\,|\beta\rangle\langle\beta|\,\langle\alpha| + |-\alpha\rangle\,|-\beta\rangle\langle-\beta|\,\langle-\alpha|\}, \tag{6.3}$$

not by measurement of X_1 (both states give two gaussian peaks), but by measurements of X_2. The joint probability distribution $P(X_2^A = x, X_2^B = y)$, for getting a result x for measurement of X_2^B and y for a simultaneous measurement of X_2^A, differs for the superposition (6.1) and the mixture (6.3). The superposition state (6.1) will give a distribution which exhibits interference fringes with a gaussian envelope [12], while the mixture (6.3) gives a joint probability distribution of gaussian shape with no fringes. Thus, we might expect to get some contradiction between macroscopic local realism and the quantum prediction for (6.1) for quadrature phase measurements.

One can deduce quite generally, however, that such a contradiction for quadrature phase amplitude measurements is not possible for situations where "macroscopically distinct" is taken to mean, as it is in the above discussion, a large separation in the quadrature phase amplitude phase space. First, we can consider situations which give results always macroscopically distinct in this sense. Results of such an experiment will be unchanged if one adds a microscopic vacuum noise term, corresponding to a variance $\Delta^2 x = 1$, to the quadrature phase amplitude measurement result x. Such a noise is added at the final stages of the measurement process, and its size is measured in terms of the error in the quadrature phase amplitude value inferred from the measurement. It has been shown that the quantum predictions for such an experiment is given by a Wigner function which is the original Wigner function $W(x^A, p^A, x^B, p^B)$ of the quantum state (6.1), convoluted with the gaussian term $(1/4\pi^2)\exp(-[x^{A^2} + p^{A^2} + x^{B^2} + p^{B^2}]/2)$. Since it is well known that this new Wigner function is always positive [13] and realising that the properties of the Wigner function allow the moments of the quadrature phase amplitude measurements to be expressed directly as integrals of phase space variables over this positive distribution, we see that the Wigner distribution can act as a local hidden variable theory which gives all quantum predictions for these quadrature phase measurements. Second, we can consider a macroscopic experiment, with continuous outcomes, by our definitions so that there is noise of macroscopic size added to the quadrature phase amplitude measurements. By the same reasoning, the incompatibility of local realism (macroscopic local realism) with quantum mechanics in such a macroscopic experiment is not possible.

While we have excluded the possibility of an incompatibility of quantum mechanics with macroscopic local realism for this situation where macroscopic is measured with respect to the separation in quadrature phase amplitude phase space, we may apply the reasoning of Section 5 to consider the homodyne measurement of quadrature phase amplitudes as a photon number measurement involving the second local oscillator field. In this case microscopic separations in phase space may correspond to macroscopic separations in photon number, giving the potential for a contradiction with macroscopic local realism even with results only microscopically separated in phase space.

7. TESTS OF MACROSCOPIC LOCAL REALISM FOR HIGHER SPIN QUANTUM STATES USING IMPRECISE PHOTON NUMBER MEASUREMENTS

Our question is this: Is "macroscopic local realism" valid? If it is, the argument outlined in Section 5 shows that we must accept a situation for which there is a physical reality not described by a quantum wavefunction. That is, the validity of macroscopic local realism would question our standard interpretation of quantum mechanics. It is more likely, many would argue, that macroscopic local realism is not correct. When then does it fail?

Here we give a proposal for Bell-inequality-type tests for different strengths, from microscopic to macroscopic, of local realism. Our initial idea is to examine the possibility of contradiction with these levels of local realism by looking at quantum states [4] which are the most obvious macroscopic generalisation of the spin states already shown to violate local realism at the microscopic level.

The higher spin state we consider is

$$|\psi\rangle = \frac{1}{N!(N+1)^{1/2}} (a_+^\dagger b_+^\dagger + a_-^\dagger b_-^\dagger)^N |0\rangle\, |0\rangle. \tag{7.1}$$

Here the $a_\pm$, $b_\pm$ are boson operators for two pairs of fields, the a's being spatially separated from the b's, as depicted in Fig. 5. One can transform the fields to produce $c_\pm$ and $d_\pm$ fields as defined in Eq. (3.2). In the commonest physical example of such a transformation, the a_+ and a_- represent orthogonally polarised modes (along y and x axes) and the transformation to get outgoing fields c_+, c_- corresponds to the use of a polariser, the c_+, c_- being orthogonal polarised modes along axes y' and x' rotated an angle θ to the original polariser axes. Such transformations can also be achieved with beam-splitters and phase-shifts using as inputs a_+, a_- fields of different k-vectors [14]. Photodetectors can be used to determine the photon numbers $c_+^\dagger c_+$, $c_-^\dagger c_-$, $d_+^\dagger d_+$, and $d_-^\dagger d_-$. The possible outcomes for each photon number measurement are 0, 1, ..., N in integer steps.

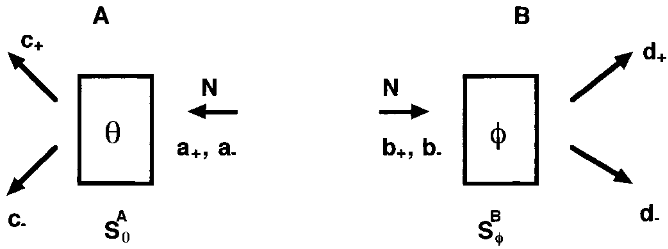

Fig. 5. Diagram of the higher spin experiment with photons. Here we have N particles incident at locations A and B, respectively. The θ determines the measurement choice at A, while ϕ determines the choice at B. The possible outcomes at each location are $n=0, 1, ..., N$. Each outcome is classified as $+1$ or -1 as indicated in the text.

The state (7.1) predicts perfect correlation between the results for photon numbers $c_+^\dagger c_+$ and $d_+^\dagger d_+$, if we choose $\theta=\phi$. This is readily seen if one rewrites the wavefunction (7.1) in terms of the transformed operators c_+, c_-, d_+, d_-. Thus, for $\theta=\phi$, a result n for measurement of $c_+^\dagger c_+$ will always correspond to a result n for measurement of $d_+^\dagger d_+$.

We denote the result of the photon number measurement as either $+1$ if $n \geqslant N/2$, or -1 if $n < N/2$. Because of the correlation in photon number, the results ($+1$ or -1) at A and B will also be correlated for $\theta=\phi$. Hence, because A and B are spatially separated, we are able to apply the EPR reasoning and, assuming local realism, we deduce the existence of "elements of reality" for each subsystem A and B. Let us denote the "element of reality" describing the result of the measurement at A as x_θ^A and the "element of reality" describing the result of the measurement at B as x_ϕ^B. The "elements of reality" always take on one of the two possible values, $+1$ or -1.

We now follow the approach of Bell and Clauser *et al.* as outlined in Section 3, to investigate the predictions based on the existence of such "elements of reality." One defines the measurable probabilities: $P_+^A(\theta)$ is the marginal probability that the outcome is $+1$ upon measurement of S_θ^A at A; $P_+^B(\phi)$ is the marginal probability that the outcome is $+1$ upon measurement of S_ϕ^B at B; and the joint probability is $P_{++}^{AB}(\theta, \phi)$ for obtaining $+1$ for both outcomes of two simultaneous measurements S_θ^A and S_ϕ^B at A and B, respectively. These are directly linked to the probabilities that the system is in a state with the appropriate "element of reality" as given by the relations (3.5) to (3.7). From these expressions one may derive the Bell–Clauser–Horne inequality of the form (3.8).

Violation by quantum mechanics of this inequality implies incompatibility of quantum mechanics with the premise of local realism on which the inequality is based. In the case where the two states $+1$ and -1 are only microscopically distinct, local realism right down to a microscopic level is being assumed in order to make the claim that the system is actually in one state or the other at a given time. If the states $+1$ and -1, however, are macroscopically distinct, then one needs only assume macroscopic local realism to make the claim that the system is always in one state or the other. Violation of the Bell inequality in this case would be a much stronger result than violation in the case where $+1$ and -1 are microscopically distinct.

The averages given in the expression (3.8) may be calculated for the state (7.1) and the quantity B computed. Violations of the Bell inequality would imply an incompatibility of quantum mechanics with local realism. To determine the degree of "separateness" of the two states represented by $+1$ and -1, we would need to examine the probability distributions for obtaining a photon number result upon measurement of S_θ^A for angles θ which give a violation of the Bell inequality. In general for states of the type (7.1) there will be a nonzero probability for obtaining a photon number result which registers as state -1, but which is also microscopically close to state -1. We conclude that the states are only microscopically separated. Therefore at this point it seems that we would only be able to deduce that the violation of the Bell inequality indicates incompatibility of quantum mechanics with local realism at the one-photon, or most microscopic level.

However, it is clear that certain pairs of results for photon number giving $+1$ and -1 results respectively are separated by of order N photons, a separation which can become macroscopic as N becomes larger. The quantum state (7.1) contains both entangled superpositions of states which are microscopically distinct and entangled superpositions of states which are macroscopically distinct for large N. It would seem possible that at least some of the contradiction with local realism is due to the macroscopic superpositions. We require a method to eliminate the effect of the microscopic superpositions from any violation shown. This is in fact automatically achieved by our requirement that a true violation of macroscopic local realism is obtained only if it can be evidenced in a macroscopic experiment which involves measurements where the errors are macroscopic.

To investigate further we require to model theoretically the process of what we have defined to be a "macroscopic measurement" of photon number. This means adding noise to the photon number results in a appropriate way. If violation of the Bell inequality still shows, with an appropriate level of noise present, we would conclude that the violation is due to incompatibility of quantum mechanics with macroscopic (rather than microscopic) local realism. We assume our readout photon number (let us call it m_A) at A to be of the form $m_A = n_A + \text{noise}(A)$, where n_A is the result of the measurement $c_+^\dagger c_+$ of photon number as predicted by quantum mechanics in the absence of noise, and noise(A) is a random noise term of suitable properties. Similarly the final readout photon number (let us call it m_B) at B is of the form $n_B + \text{noise}(B)$, where the noise(A) and noise(B) terms at the spatially separated locations are independent.

We need to re-examine the derivation of the Bell inequality for this situation of noisy number measurements. With noise added, there may be a reduction in the correlation between results for S_θ^A and S_θ^B measurements made at A and B, respectively. With a reduction in the degree of correlation the "elements of reality" are no longer precise numbers x_θ^A and x_ϕ^A, where these numbers are either $+1$ or -1. We can only infer the value for the result of the measurement of photon number at A (by the measurement made at B) to within a certain precision. The "element of reality" is a predetermined quantity which has an associated uncertainty. It is a probability distribution for certain results, rather than a well-defined number. This "element of reality" still has a predetermined existence from which we deduce the probability of obtaining a result $+1$ upon measurement. The reduction in correlation may mean that one can no longer say that the system at A is definitely in state $+1$ or state -1. Thus in this case we can no longer rule out the possibility that the system is in the quantum superposition of these two states. (If the "element of reality has a "fuzziness" of maximum range Δ we can however rule out the possibility of quantum superpositions of states which are distinct by an amount greater than Δ). We can, however, still say that there is a probability of getting a result $+1$ at A given this predetermined description of the system, and that this probability is independent of an angle ϕ chosen simultaneously at B. One can follow convention and label the predetermined specification of the system by "hidden variable" symbols λ, and define the probability $P^A_+(\theta, \lambda)$ of getting a result $+1$ at A given the particular λ. Similarly one defines a $P^B_+(\phi, \lambda)$ for a $+1$ result at B, given λ. The probability distribution for the system being specified by this hidden variable set is denoted by $\rho(\lambda)$. The measurable probabilities, assuming locality, are given by the form (3.5)–(3.7) which is sufficient to ensure the Clauser–Horne inequality (3.8) as before.

The noise added means that there is an inaccurate determination of photon number of the original field incident on the detector. We cannot distinguish between results one photon apart. Because of this we cannot hope to determine whether the measurement at B will influence the results of a measurement at A by an amount of one photon or not. Local realism at this one photon level cannot be tested here. Any violation of the Bell inequalities with the noise term present must be due to failure of local realism at a more "macroscopic" level.

VIII. CONCLUSION

We have introduced the concept of "macroscopic local realism." We claim that the sensible definition of "macroscopic local realism" is to exclude the possibility that events occurring at a system B can make immediate macroscopic changes to a second system at A, spatially separated from the first. The premise of "macroscopic local realism" is then weaker than the premise of "local realism" used previously in its entirety, which excludes the possibility of all changes to system A, from macro-, meso-, right down to micro-.

The premise of "macroscopic local realism" is only ever sufficient to deduce, for appropriately correlated systems, the existence in the EPR sense, of "elements of reality" which are intrinsically macroscopic. By this we mean that the value to which the "element of reality" corresponds has a macroscopic uncertainty associated with it. In this case, noting that the "element of reality" is a predetermined property attributed to the system, we note that "macroscopic local realism" can be sufficient to exclude superpositions of macroscopically distinct states, but can never exclude microscopic (or mesoscopic) superpositions.

With this definition of "macroscopic local realism," we have made two points. First, we have pointed out that the Einstein–Podolsky–Rosen argument of 1935, which uses the premise of "local realism" to show that quantum mechanics is incomplete, can be formulated so that the measurements need not be accurate to a microscopic level. In this case we need only use the weaker premise of "macroscopic local realism" to arrive at the conclusion that quantum mechanics is incomplete. As discussed previously, this conclusion is quite startling, since "macroscopic local realism," unlike local realism at the microscopic level which has been invalidated by the Bell inequality experiments, has not yet been questioned.

Our second major point is to provide a formulation of Bell inequality tests for situations where the measurements performed involve macroscopic uncertainties. We consider situations, such as higher spin states, where the range of possible results is macroscopic. We propose that a violation of the Bell inequality in this case would show a failure of macroscopic local realism.

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