Comment on "Quantum Entangled Dark Solitons Formed by Ultracold Atoms in Optical Lattices"

The recent Letter [1] describes full quantum simulations of a dark soliton in a Bose-Einstein condensate in a nonperturbative regime. The authors argue, based on the filling in of the two-point correlator $g^{(2)}$, that a photograph of a condensate would reveal a smooth atomic density without any localized dark soliton. This is in contrast to the perturbative regime where a photograph would show a dark soliton with a random position [2]. While we admire the quantum simulations and other results in [1], we think that their conclusion about the outcome of a single experiment is not justified by this property of $g^{(2)}$. We provide the following counterexample in the nonperturbative regime (see also [3]).

Let $\phi_q(x) \propto \tanh[(x-q)/\xi]$ be a standard condensate wave function with a dark soliton at q. With $\hat{a}_q = \int dx \phi_q^*(x)\hat{\Psi}(x)$, a state $(\hat{a}_q^\dagger)^N|0\rangle$ is a condensate with a soliton at q, and let the N-particle state be a superposition $\psi_0(q)$ of condensates with different q

$$|\psi_0\rangle \propto \int dq \psi_0(q)(\hat{a}_q^\dagger)^N|0\rangle. \quad (1)$$

After measurements of n atomic positions $x_1, \ldots, x_n$ the state (1) collapses to a conditional state

$$|\psi_n\rangle \propto \hat{\Psi}(x_n)\ldots\hat{\Psi}(x_1)|\psi_0\rangle \propto \int dq \psi_n(q)(\hat{a}_q^\dagger)^{N-n}|0\rangle,$$

where $\psi_n(q) = \phi_q(x_n)\ldots\phi_q(x_1)\psi_0(q)$. The $(n+1)$st measurement will find a particle at x_{n+1} with a probability $p_{n+1}(x_{n+1}) \propto \langle\psi_n|\hat{\Psi}^\dagger(x_{n+1})\hat{\Psi}(x_{n+1})|\psi_n\rangle$. This is equivalent to simultaneous measurement of all x_i.

Using the methods of [2], we simulated measurement of all $N = 5000$ particles on a lattice of 31 sites, where $x, q \in \{-15, 15\}$, assuming a soliton width $\xi = 1.5$, and a delocalized (uniform) superposition $\psi_0(q) \propto 1$ that is nonperturbatively wider than the soliton width. The inset in Fig. 1 shows the ensemble average particle density $p_1(x)$ and the main figure shows a generic histogram of particle positions $x_1, \ldots, x_N$ measured in a single realization. Each single realization of the experiment finds a soliton localized at some definite but random q.

What about the two-point correlator $g_2(x) = \langle\psi_0|\hat{\Psi}^\dagger(0)\hat{\Psi}^\dagger(x)\hat{\Psi}(x)\hat{\Psi}(0)|\psi_0\rangle$ that is analyzed in [1]? This turns out nearly uniform (see the inset). Hence, the "filled in g_2" → "filled in soliton" line of reasoning is clearly incorrect. Of course, the soliton simulated in [1] may still be greying, but the point here is that one cannot answer such a question by analyzing $g_2(x)$.

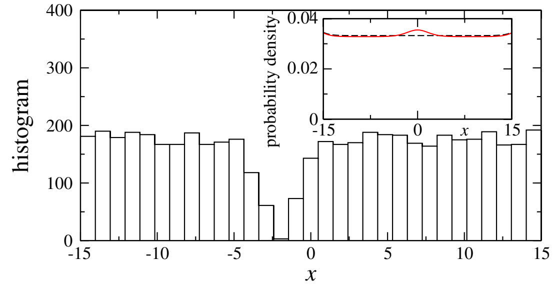

FIG. 1 (color online). Histogram of measured atom positions in a single experiment from (1). Inset: single particle density $p_1(x)/N$ (dashed black line) and $g_2(x)$ (solid red line).

Our example demonstrates that, in some cases, a low order correlator like $g_2(x)$ is insufficient to draw conclusions on the outcome of a single experiment, and that the soliton in a Bose-Einstein condensate is such a case (see also [4]). Here, g_2 is equal to our $p_2(x_2)$ after the first particle was measured at $x_1 = 0$, but if we want to infer the soliton position from a histogram of particle positions, then the number of measured particles must be large enough to provide a histogram with a well-resolved soliton notch.

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