RKKY Interaction and Nuclear Magnetism in Nanostructures

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References

2D


1D

B. Braunecker, P. Simon, and D. Loss, Phys. Rev. Lett. 102, 116403 (2009)
How to achieve 

**Nuclear magnetism in 2D and 1D conductors?**

1. **Couple nuclear spins to electrons:**
   - Hyperfine interaction;
   - **RKKY** interaction between nuclear spins

2. **Rely on electron-electron interactions:**
   - **Modification** of RKKY interaction

→ Order possible in realistic samples
Nuclear order: RKKY & electron interactions

Dimensionality matters: interactions become important through restriction of scattering phase space

3D metals: old story (Fröhlich & Nabarro, 1940)
Weiss mean-field theory often quite good

2D: RKKY interaction renormalized through electron-electron interactions can
a) overrule Mermin-Wagner Theorem
b) stabilize nuclear magnetic order


1D: Renormalization essential; electrons and nuclear spins can form a combined state of matter

Start with 2D
System: 2D electron gas (GaAs heterostructure) interacting 2D electrons

\[ H = H_{el} + \sum_i A S_{i}^{el} \cdot I_{i}^{nucl} + \sum_{ij} v_{ij}^{\alpha\beta} I_{i}^{\alpha} I_{j}^{\beta} \]

- Interacting 2D electrons
- Hyperfine interaction ~ 90 \( \mu \)eV (3D lattice)
- Dipolar interaction/quadrupolar splitting < 0.1 neV (smallest energy scales)

Not a quantum dot
From quasi-2D to 2D

- spins along $z$ couple identically to the electrons
- if they order, they order in the same way
- $I(r_{||})$ behaves like a single large spin

Reduction of the quasi-2D problem to a strictly 2D problem

$$I(r_{||}) = \int dz \ |\psi(z)|^2 I(r_{||}, z)$$

where $r = (r_{||}, z)$
Effective Hamiltonian: RKKY

\[ H = H_{el} + \sum_i A S_{el}^i \cdot I_{nucl}^{nucl} \]  \hspace{1cm} \text{(2D)}

\[ \frac{A}{E_F} \sim 1/100 \ll 1 : \text{separation of time scales between electrons & nuclear spins} \]

Schrieffer-Wolff transformation
integrate out electron degrees of freedom

\[ H_{eff} = \frac{A^2}{2n_s} \sum_{ij} \chi_s(r_i - r_j) I_i \cdot I_j \]

\[ = \frac{A^2}{2n_s} \frac{1}{N_s} \sum_q \chi_s(q) I_{-q} \cdot I_q \]

RKKY interaction \( J(q) \)

static electron spin susceptibility

Order in 2D: What about the Mermin-Wagner Theorem?

“No long-range order in 2D for sufficiently short-ranged interactions.”

RKKY interactions are long ranged

But they oscillate

\[ J(r) = \frac{A^2}{8n_s} |\chi_s(r)| \sim \frac{\cos(2k_F r)}{r^2} \]

→ Expect some extension of M-W theorem.

Indeed: Conjecture that \( T_c = 0 \) in 2D for RKKY interaction by noninteracting electrons \[ P. \ Bruno, \ PRL \ 87 \ (2001) \]

Long-range & electron-electron interactions are essential
Mean-Field Theory (naive but instructive)

Mean-field theory (Fröhlich & Nabarro 1940)

\[ H_{\text{eff}} = -\frac{1}{N_s} \sum_q J_q \mathbf{I}_{-q} \cdot \mathbf{I}_q \]

\[ E_{MF} = -\frac{1}{N_s} \sum_q J_q \langle \mathbf{I}_{-q} \rangle \cdot \langle \mathbf{I}_q \rangle \]

\[ H_{MF} = -\frac{1}{N_s} T_{MF} \langle \mathbf{I}_{-q_{\text{max}}} \rangle \cdot \mathbf{I}_{q_{\text{max}}} \]

\[ J_q = \frac{A^2}{2n_s} |\chi_s(q)| \]

Ground state determined by maximum of \( J_q \)

if at \( q = 0 \): ferromagnet

if at \( q \neq 0 \): helimagnet

Theory depends on single energy scale: \( T_{MF} = \max(J_q) \), e.g. \( J_0 \)

in particular \( T_C \sim T_{MF} \)

If a system is characterized by a **single energy scale**, this mean-field argument is valid

Braunecker, Simon, Loss,
MF inconsistent for noninteracting electrons

Electron spin susceptibility
2DEG / noninteracting electrons

$$J_q = \frac{A^2}{2n_s} |\chi_s(q)|$$

Maximum $T_{MF}$ is not well-defined

There is no order: look at fluctuations
No order for noninteracting electrons

Direct proof that there is no order:
- assume a **ferromagnetic ground state**
- fluctuations about ground state: **magnons**

\[ \omega_q = 2I(J_0 - J_q) \]

Continuum of zero energy magnons

Ferromagnetic ground state unstable
But if...

Ferromagnetic order would be stable up to some $T > 0$ if the spin wave dispersion was **linear** at $|q| \to 0$ (fluctuations diverge if dispersion $\sim q^2$)

Required: **Non-analytic** behavior

$$\delta \chi_s(q) = \chi_s(q) - \chi_s(0) \propto |q| \quad (q \to 0)$$

Many-body effect from electron-electron interactions.

Result beyond standard Fermi-liquid theory.
Electron-electron interactions

Modification of susceptibility by electron-electron interaction

- self-energy corrections:

\[ \delta \chi_s(q) = \chi_s(q) - \chi_s(0) = -|q| \frac{4|\chi_s(0)||\Gamma_s|^2}{3\pi k_F} \quad |q| \ll 2k_F \]

linear, non-analytic \(|q|\) - dependence!

\[ \Gamma_s \sim U m / 4\pi \] : backscattering amplitude

Chubukov, Maslov, PRB 68 ('03)
see also: Aleiner, Efetov PRB 74 ('06)
Renormalization

**Perturbation theory is not sufficient in 2D!**
A full renormalization is required

backscattering strongly renormalized by **Cooper channel** contribution
($p = 0$ & presence of Fermi sea)

renormalization can change the sign of the slope from the perturbation theory:
$$\delta \chi \sim - |q| \rightarrow + |q|$$

Saraga, Altshuler, Loss, Westervelt PRB 05
Simon, Braunecker, Loss PRB 08
Chesi, Żak, Simon, Loss PRB 09
Shekhter, Finkel'stein PRB 06, PNAS 06
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Posters: 292 & 276
Possible outcome

Summation over dominating Cooper channel diagrams:
- sign of slope at small $q$ nonuniversal
- depends on bare backscattering amplitude

Magnon spectrum
$$\omega_q = 2J_1(J_0 - J_q)$$

Result obtained through local field factor calculation

Required:
- experiments & numerics for $\chi(q)$ at $q \sim 2k_F$

Simon, Loss, PRL 98, 156401 (2007)
Simon, Braunecker, Loss, PRB 77, 045108 (2008)
Energy scales for the nuclear ferromagnet?

focus on possible outcome allowing for nuclear ferromagnetism
Energy scales for the nuclear ferromagnet?

Both scales are comparable. Which scale is dominating?

Simon, Loss, PRL 98, 156401 (2007)
Simon, Braunecker, Loss, PRB 77, 045108 (2008)
$T_c$ for the ferromagnet?

Calculate magnetization $m$ per nuclear spin.

\[
m = 1 - \frac{1}{I n_s} \int dq \frac{1}{2\pi} e^{\beta \omega_q} - 1 = 1 - \left( \frac{T}{T_0} \right)^2
\]

expression valid for $T < T^*$

finite because $\omega_q$ is linear at small $q$

\[
T_0 = \frac{I}{2k_F} \sqrt{\frac{3In_s}{\pi}} T^* \sim \frac{\lambda_F}{a} T^*
\]

$T_0$ provides an estimate of $T_c$
- consistent with noninteracting limit: $T^* = 0 \rightarrow T_0 = 0$
- $T_{MF}$ absent (but role not clear)

Note: For Ising spins with purely ferromagnetic interactions: $T_{MF} > T_c$
— NOT the system described here: neither Ising, nor purely FM

M. E. Fisher
Phys. Rev. 1967

Simon, Loss, PRL 98, 156401 (2007)
Simon, Braunecker, Loss, PRB 77, 045108 (2008)
Estimate of $T_0$

Estimate for GaAs 2DEG (using various calculation schemes)

$$T_0 \sim 25\mu K - 1 mK \quad \text{for} \quad r_s \sim 0.8 - 8$$

Essential point: Increases with interactions

Compare with $T_{MF} \sim 1 \mu K$

Simon, Loss, PRL 98, 156401 (2007)
Simon, Braunecker, Loss, PRB 77, 045108 (2008)

Conclusions so far:

- **nuclear ferromagnetism** in 2DEGs is possible
- **electron-electron interactions** are essential
  (important: reduced dimension 2D)
- transition temperatures can lie in **mK range**
What about 1D?
What is special in 1D?

Confinement of electrons:
massively reduced scattering phase space
even weak electron interactions lead to breakdown of Fermi-liquid paradigm
elementary excitations: collective bosonic modes

→ **Luttinger liquid** picture

- single-electron picture inappropriate (i.e. difficult)
- bosonic density waves better description
Luttinger liquid physics

Interacting electron system expressed by linear hydrodynamics of bosonic density waves

\[ H_{el} = \int \frac{dr}{2\pi} \sum_{\kappa = c,s} \left[ \frac{v_{\kappa}}{K_{\kappa}} \left( \nabla \phi_{\kappa}(r) \right)^2 + v_{\kappa} K_{\kappa} \left( \nabla \theta_{\kappa}(r) \right)^2 \right] \]

**electron-electron interactions:**
- strong renormalization of velocities: compressibility susceptibility
- charge/spin fraction of density waves

similar 2-band description for carbon nanotubes
1D Electron Conductors

Required: Real materials (avoid mere model study)

Examples:

**Single-wall carbon nanotubes**

- armchair nanotube
  - Modeling: Kane, Mele 1997; Kane, Balents, Fisher 1997; Egger Gogolin 1997

**GaAs quantum wires**

  - Cleaved edge overgrowth technique

probably have Luttinger liquid physics
Where are the nuclear spins?

Single-wall carbon nanotubes

substitution $^{12}\text{C} \rightarrow ^{13}\text{C}$
lattice of nuclear spins 1/2

very recently experimentally achieved to 99% purity of $^{13}\text{C}$

F. Simon et al. PRL (2005)

GaAs quantum wires

Ga, As have a nuclear spin 3/2

high quality samples available

Hyperfine interaction

Cartoon of the 1D conductor

Hyperfine coupling between nuclear and electron spins

\[ H = H_{el} + \sum_i A S_i^{el} \cdot I_{i}^{nuc} \]

interacting electrons (Luttinger liquid) hyperfine interaction

nanotubes: \( A \sim 0.6 \mu \text{eV} \)
GaAs wires: \( A \sim 90 \mu \text{eV} \)

will be strongly renormalized!
Long range order?

No true long-range order in these systems

However:

Long-range order for any practical purpose

For realistic samples with length $L \sim 10 \, \mu m$
- order extends over whole system
- transition temperature $T^*$ independent of $L$

Braunecker, Simon, Loss, PRL 102, 116403 (2009)
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RKKY Interaction in the Luttinger liquid

$-J_q = -C(g) \frac{A^2}{E_F} \left( \frac{n_{el}}{n_{nl}} \right)^{1-2g} \left( \frac{E_F}{k_B T} \right)^{2-2g} \Gamma \left( \frac{g}{2} - \frac{\lambda_T}{4\pi} (q - 2k_F) \right) \Gamma \left( \frac{2-g}{2} - \frac{\lambda_T}{4\pi} (q - 2k_F) \right)^2$

depth & width determined by $T$

theory depends on a single scale

$k_B T^* = J_{2k_F} (T^*)$

nuclear helimagnet

Braunecker, Simon, Loss, PRL 102, 116403 (2009)
Nuclear helimagnet

Helical order of nuclear spins

Order stable under spin-wave fluctuations due to finite length

Transition temperature set by $k_B T^* = J_2 k_F T^*$ independent of length!

$T^* \sim 1\mu K$ nanotubes

$T^* \sim 50\text{mK}$ quantum wires

strongly interacting wires with $K_c = 0.5$

Nuclear helimagnet

Helical order of nuclear spins

Transition temperature set by Luttinger theory: order of magnitude only!

Order stable under spin-wave fluctuations due to finite length

Effect of nuclear magnetic field on electrons?

strongly interacting wires with $K_c = 0.5$

Feedback is essential!

Nuclear magnetic (Overhauser) field

\[ \langle I_i \rangle = \text{Im} m_{2k_F} \left[ \cos(2k_Fr_i) e_x + \sin(2k_Fr_i) e_y \right] \]

Feedback on electrons

\[ H_{Ov} = \sum_i A \langle I_i \rangle \cdot S_i = \sum_i A \text{Im} m_{2k_F} \cos \left( \sqrt{2} [\phi_c(r_i) + \theta_s(r_i)] \right) \]

Bosonization treatment; relevant sine-Gordon interaction

Opening of a **mass gap** for \( \phi_+ \propto (\phi_c + \theta_s) \)

But a gapless field \( \phi_- \) remains!

Flow to strong coupling

\[ H_{Ov} = \sum_i A \langle I_i \rangle \cdot S_i = \sum_i AIm_{2k_F} \cos \left( \sqrt{2} [\phi_c(r_i) + \theta_s(r_i)] \right) \]

\[ \frac{dA^*(\ell)}{d\ell} = \left( 2 - \frac{K_c + K_s^{-1}}{2} \right) A^*(\ell) \]

\[ A^* = A \left( \frac{\xi}{\alpha} \right)^{1 - \frac{K_c + K_s^{-1}}{2}} \]

-effective Overhauser field flows to strong coupling limit

\[ \xi: \text{correlation length} \]

-precise form depends on material

-renormalization absent in Fermi liquids \((K_c = K_s = 1)\)

-large effective field: freezes out modes \( \phi_+ \propto (\phi_c + \theta_s) \)

→ density wave order combining charge & spin

Flow to strong coupling

\[ H_{Ov} = \sum_i A \langle I_i \rangle \cdot S_i = \sum_i A I m_{2k_F} \cos(\sqrt{2}[\phi_c(r_i) + \theta_s(r_i)]) \]

effective Overhauser field flows to strong coupling limit

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\[ A^* = A(\xi/a)^{1 - \frac{K_c + K_s^{-1}}{2}} \]

\( \xi \): correlation length
precise form depends on material
renormalization absent in Fermi liquids \((K_c = K_s = 1)\)

\[ A = 0.6 \, \mu\text{eV} \rightarrow A^* \sim 20 \, \mu\text{eV} \quad \text{nanotubes} \]
\[ A = 90 \, \mu\text{eV} \rightarrow A^* \sim 400 \, \mu\text{eV} \quad \text{quantum wires} \]

Renormalized RKKY interaction

RKKY interaction determined by remaining gapless modes

- same shape
- much deeper
  (modified exponents)
- boosts $T^*$

$$k_B T^* = J_{2k_F}(T^*)$$

<table>
<thead>
<tr>
<th>$T^*$</th>
<th>nanotubes</th>
<th>$T^*$</th>
<th>quantum wires</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 $\mu$K</td>
<td>$\sim$ 10 mK</td>
<td>50 mK</td>
<td>$\sim$ 75 mK</td>
</tr>
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</table>

Combined nuclear / electron order

- ordered phases mutually dependent below $T^*$:
  ordered phase of nuclear & electron degrees of freedom
- self-stabilizing
- independent of system size for realistic samples

Conductance

Reduction of conductance

Through feedback: Pinning of $\phi_+$ channels

- Blocking of $\frac{1}{2}$ of the conducting channels
- Universal reduction of conductance by factor 2

Example: Luttinger liquid adiabatically connected to metallic leads

Conductance: $G = \frac{e^2}{h} n$

number of conducting channels reduced to $n \rightarrow n / 2$

Important experimental signature of Luttinger liquid physics.

Maslov & Stone, Ponomarenko, Safi & Schulz (all in PRB 1995)

Anisotropy in susceptibility

Anisotropy in electron spin susceptibility

Overhauser field defines spin (x,y) plane

Anisotropy between spin (x,y) and z directions

\[
\chi^{x,y}(q) \sim \left| \frac{1}{q - 2k_F} \right|^{2-2g'} \\
\chi^{z}(q) \sim \left| \frac{1}{q - 2k_F} \right|^{2-2g''}
\]

\[
g' = 0.33 \quad g'' = 0.17 \quad \text{nanotubes (with } K_c = 0.2 \text{)}
\]

\[
g' = 0.67 \quad g'' = 0.33 \quad \text{GaAs quantum wires}
\]

(with \( K_c = 0.5 \))

Conclusions

2D: Nuclear magnetic order at $T > 0$ is possible. Electron-electron interactions play the essential role for a finite Curie temperature; mK range possible.

Open questions: Universality of susceptibility? Disorder? ...

1D: Combined order of nuclear spins and electrons; self-stabilizing through strong feedback.

Pure Luttinger liquid effect, absent in Fermi liquids.

Characteristic temperatures ~ mK even for very small hyperfine interaction.

Renormalized Overhauser field.
Reduction of conductance by factor 2.
Anisotropy in electron response functions.