Superfluid ground state of Fermi-Bose mixtures

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Abstract

We consider a mixture of fermionic (in two hyperfine states) and bosonic atoms at zero temperature. The only interactions taken into account are the contact interactions between fermions and bosons (described by the coupling constant g_{BF}) and among the bosons themselves (described by g_B). We neglect the interaction between distinguishable fermions on purpose and ask the question whether the strong enough attraction between bosons and fermions can induce pairing in fermionic component.

H-F equations

$$-\frac{\hbar}{2m_{F_j}}\nabla^2\varphi_i^{(F_j)} + V_{trap}^{(F_j)}\varphi_i^{(F_j)} + g_{BF}N_B|\varphi^{(B)}|^2\varphi_i^{(F_j)} = \varepsilon_i^{(F_j)}\varphi_i^{(F_j)} \qquad j=1,2 \quad i=1,\dots,N_F$$

G-P equation

$$-\frac{\hbar}{2m_B}\nabla^2\varphi^{(B)} + V_{trap}^{(B)}\varphi^{(B)} + g_B(N_B-1)|\varphi^B|^2\varphi^{(B)} + g_{BF}\sum_{i=1}^{N_{F_1}}|\varphi_i^{(F_1)}|^2\varphi^{(B)} + g_{BF}\sum_{i=1}^{N_{F_2}}|\varphi_i^{(F_2)}|^2\varphi^{(B)} = \varepsilon^{(B)}\varphi^{(B)}$$

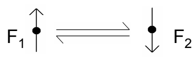
Many body weave function of 3 component fermi-bose gas $\Longrightarrow$

$$\Psi = \Psi_{F1}\Psi_{F2}\Psi_B \qquad \Psi_B = \prod_{i=1}^{N_B}\varphi^{(B)}(\mathbf{z}_i) \qquad \Psi_{F_j} = \frac{1}{\sqrt{N_{F_j}!}}\begin{vmatrix} \varphi_1^{(F_j)}(\mathbf{x}_1) & \cdots & \varphi_1^{(F_j)}(\mathbf{x}_{N_{F_j}}) \\ \vdots & \ddots & \vdots \\ \varphi_{N_{F_j}}^{(F_j)}(\mathbf{x}_1) & \cdots & \varphi_{N_{F_j}}^{(F_j)}(\mathbf{x}_{N_{F_j}}) \end{vmatrix}$$

Hartree-Fock energy E_{HF}
It is a grorung state energy $\Longrightarrow$

$$E_{HF} = \langle\Psi|\hat{H}|\Psi\rangle$$

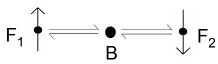
Electrons in a crystal lattice



$\Longrightarrow$ Phonon-mediated interaction leads to the effective attraction between electrons $\Longrightarrow$

$$|\Psi_{BCS}\rangle = \prod_k (u_k + v_k f_{k\uparrow}^+ f_{-k\downarrow}^+)|0\rangle$$

Bose-Fermi mixture



$\Longrightarrow$ Attractive fermion-boson interaction leads to the effective attraction between fermionic atoms $\Longrightarrow$

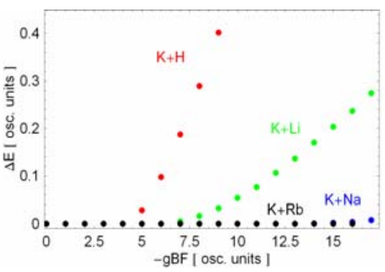
$$|\Psi_{FFB}\rangle = \frac{1}{\sqrt{R}}\prod_k (u_k + v_k f_{k\uparrow}^+ f_{-k\downarrow}^+ b_0^+)|0\rangle$$

Energy of the Bose-Fermi mixture $\Longrightarrow$

$$E_{FFB} = \langle\Psi_{FFB}|\hat{H}|\Psi_{FFB}\rangle$$

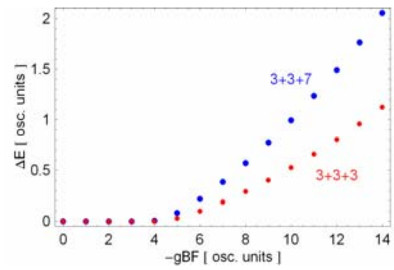
We look for situations when BCS-type pairing leads to the state of lower energy $\Longrightarrow$

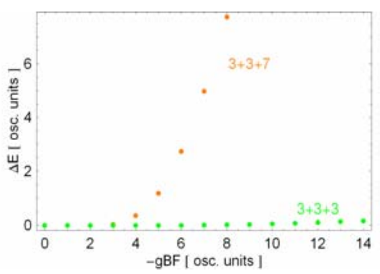
$$\Delta E = E_{HF} - E_{FFB} > 0$$



Difference ΔE as a function of $|g_{BF}|$ for various Bose-Fermi mixtures (^{40}K-^{1}H, ^{40}K-^{7}Li, ^{40}K-^{23}Na and ^{40}K-^{87}Rb) in 3+3+3 system. The figure suggest the existence of critical strength g_{BF}. The critical strength is larger for heavier bosonic atoms (a sort of isotopic effect).

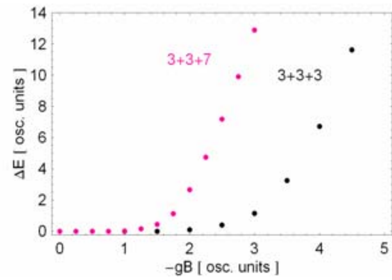
Dependence of ΔE on the number of mediating bosons for ^{40}K-^{1}H mixture for systems of 3+3+3 and 3+3+7 atoms. The pairing gap ΔE increases with number of bosons. The coupling constant g_B of hydrogen is positive.





Dependence of ΔE on the number of mediating bosons for ^{40}K-^{7}Li mixture for systems of 3+3+3 and 3+3+7 atoms. The pairing gap ΔE is dramatically sensitive with respect to the number of bosons when g_B is negative.

Dependence of ΔE on the number of mediating bosons for ^{40}K-^{87}Rb mixture for systems of 3+3+3 and 3+3+7 atoms. Here the coupling constant, g_B, of ^{87}Rb is tuned to negative values using the Feshbach resonance technique.



Conclusions:

- BCS pairing state is the ground state of the system for sufficiently strong attraction between bosons and fermions
- Isotopic effect – ΔE decreases when mass of mediating boson increases.
- Pairing gap depends both on the sign of g_B as well as on the number of mediating bosons.
- Pairing in Potasium-Rubidium mixture is possible if both boson-boson and boson-fermion interactions are attractive.