Autoreferat

1. Name and Family Name.

Tomasz Marek Rusin

2. Diploma, academic degrees.

PhD in 1999

PhD Thesis in the Institute of Physics of Polish Academy of Science, Warsaw

„Hydrogen-like states in quantum heterostructures of semimagnetic semicondutors and their effect on magnetic interactions in these structures"

Supervised by Prof. Dr hab. Jacek Kossut.

Master Degree in 1993

University of Warsaw, Faculty of Physics, supervised by Prof. Dr hab. Józef Spałek.

3. My current academic career.

In 1993-1998, I was a PhD student at the Institute of Physics, Polish Academy of Sciences in Warsaw. In 1996-1997 (6 months) I was a Fulbright Scholar at the University of Purdue, USA.

Since 1998 I am not working in academic or scientific institutions. Current employment: Orange Customer Service, sp. o.o., ul. Twarda 18, 00-105 Warszawa, as a "Head of Reporting Department" and I direct a team of 32 employees. I deal with customer data analysis, data mining, and I am responsible for reporting in the company and for definitions and reporting the key operational indicators.

4. Description of achievement underlying the habilitation

The achievement underlying the habilitation consists of a collection of eight papers from the period of 2007-2012 dealing with the phenomenon of Zitterbewegung (ZB, trembling motion) in crystalline solids (semiconductors, graphene) and relativistic quantum physics.

Introduction

The trembling motion was predicted theoretically by E. Schrödinger in 1930[1]. Describing motion of the free electron according to the Dirac equation Schrodinger noted that, even in the absence of external fields, motion of the relativistic electron is not rectilinear at a constant velocity, but oscillates around the classical trajectory with the frequency close to $2m_0c^2/\hbar$ and the amplitude close to $\lambda_c = \hbar/m_0c$. The rapid oscillations of position and velocity of the electron are called Zitterbewegung, which can be translated as "trembling motion". Since its discovery the phenomenon of ZB raised controversy. First, both the very high frequency of ZB oscillations (about 10^{21} s^{-1}) and very small amplitude (0.00386 Å) prevent its observation in a vacuum. Second, in its original derivation the ZB oscillations describe time dependence of the position *operator*, but not the velocity or the position of a particle. Therefore the ZB was treated for many years as a kind of curiosity or an artifact in the Dirac theory. Since the discovery of ZB in 1930 until 2005 there appeared annually a few works related to ZB, often as a secondary thread to the main subject of the work. In this period the majority of papers dealt with the ZB in relativistic theory for particles in a vacuum. Most important works concerning the ZB in a vacuum, whose results are used in the analysis of ZB in crystalline systems, simulations or wave phenomena, are:

- Huang[2] – analysis of ZB with the use of delta packet,
- Lock[3] – indication of disappearance of ZB in time for a wave packet as a consequence of the Riemann – Lesbegue lemma.
- Barut *et al.*[4] – papers dealt with various aspects of ZB in a vacuum.

Isolated papers dealing with the ZB in solids (Lurie and Cremer[5], Cannnata *et al.*[6], Wonsowski *et al.*[7]) did not get through the mainstream of scientific literature. A review of these works is presented in the review article (no "6" in this presentation).

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[1] E. Schrodinger, Sitzungsber. Preuss. Akad. Wiss. Phys.Math. Kl. **24**, 418 (1930).
[2] K. Huang, Am. J. Phys. **20**, 479 (1952).
[3] J. A. Lock, Am. J. Phys. **47**, 797 (1979).
[4] A. O. Barut and A. J. Bracken, Phys. Rev. D **23**, 2454 (1981); A. O. Barut and W. D. Thacker, Phys. Rev. D **31**, 2076 (1985).

The situation changed in 2005, when two papers suggesting the possibility of existence of ZB in solids were published. Zawadzki[8] demonstrated that for the simplified Kane model one can calculate $x(t)$ dependence in the same way as for the Dirac equation. The main difference between the ZB for the Dirac electron in a vacuum and the ZB for electron for the Kane model for semiconductors is the amplitude of oscillations, which is on the order of $\lambda_Z = \hbar/m^* u$, where m^* is the electron effective mass, and u is its maximum velocity. Assuming typical values of m^* and u for semiconductors it turned out that λ_Z ranges from several to tens of Angstroms, and the frequency of ZB oscillations corresponds to the frequency of the infrared light. Therefore, the ZB for electron in semiconductors can be much easier to observe than the ZB for electron in a vacuum. The second work having an important impact on the growth of interest in ZB was the paper by Schliemann *et al.*[9], in which the ZB was calculated for a wave packet for the spin Rashba Hamiltonian. The authors showed that the necessary condition for the existence of ZB is a non-zero initial velocity of wave packet. The frequency of ZB oscillations calculated by Schliemann *et al.* corresponds to typical frequencies occurring in semiconductors, and the amplitude of oscillations is on the order of inter-atomic distances.

A number of other works during 2006-2012 analyzed possibilities of occurrence of ZB in semiconductors, graphene, periodic wave systems and optical super-lattices. The properties of ZB motion were studied in various theoretical models and several experimental configurations allowing an observation of ZB were proposed. For the one-dimensional Dirac equation the ZB oscillations were simulated experimentally in 2010 by Gerritsma *et al.*[10] with the use of ion trap technique and by Dreisow *et al.*[11] in optical super-lattices, while the wave version of ZB oscillations was observed in 2008 in sonic crystals by Zhang and Liu[12]. From a formal point of view, the phenomenon of ZB in a vacuum for the Dirac electron and the spin-0 particles described by the Klein-Gordon equation is similar to the ZB in crystalline solids – narrow gap semiconductors and graphene. The collection of papers by the author and Zawadzki includes

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[5] D. Lurie and S. Cremer, *Physica* **50**, 224 (1970).
[6] F. Cannata, L Ferrari, and G Russo, *Solid State Commun.* **74,** 309 (1990); L. Ferrari and G. Russo, *Phys. Rev.* B **42**, 7454 (1990).
[7] S. V. Vonsovskii, M. S. Svirskii, and L. M. Svirskaya *Teor.Mat. Fiz.* **85**, 211 (1990) (in Russian).
[8] W. Zawadzki, Phys. Rev. B **72**, 085217 (2005).
[9] J. Schliemann, D. Loss, and R. M. Westervelt, Phys. Rev. Lett. **94**, 206801 (2005).
[10] R. Gerritsma, G. Kirchmair, F. Zahringer, E. Solano, R. Blatt, and C. F. Roos, Nature (London) **463**, 68 (2010).
[11] F. Dreisow, M. Heinrich, R. Keil, A. Tunnermann, S. Nolte, S. Longhi, and A. Szameit, Phys. Rev. Lett. **105**, 143902 (2010).
[12] X. Zhang and Z. Liu, Phys. Rev. Lett. **101**, 264303 (2008).

elements relevant to understanding and explanation of ZB phenomenon in a vacuum and in crystalline solids. These works are reviewed below.

1) T. M. Rusin i W. Zawadzki

"Zitterbewegung of nearly-free and tightly-bound electrons in semiconductors"

J. Phys.: Condens. Matter **19,** 136219 (2007) [18 pages].

DOI:10.1088/0953-8984/19/13/136219

18 citations (Web Of Science), Impact Factor = 1.886

Participation of the author: **75%.**

Author's input: participation in determining the subject, performing calculations, preparing figures, participation in discussion of results, participation in text preparation.

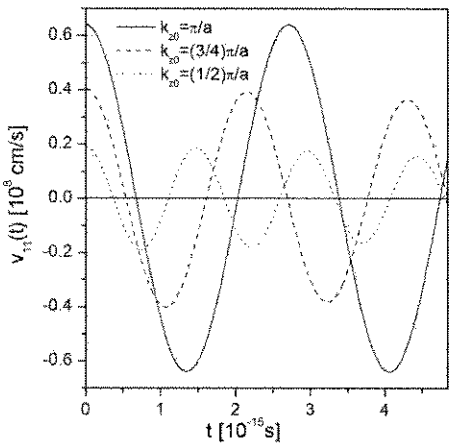

Fig. 1: ZB oscillations for a Gaussian wave packet in NFE model. Parameters correspond to GaAs band structure.

The paper analyzes influence of the periodic potential of the lattice on existence of ZB. The calculations are performed both for a weak periodic potential (nearly free electron method, NFE) and assuming a strong periodic potential in the tight binding method (TB). For the NFE model the periodic potential creates an infinite number of energy bands separated by gaps. Taking into account two lowest bands one obtains an approximate Hamiltonian H_Δ in the form of a 2×2 matrix. It is shown that the Hamiltonian does not commute with the velocity operator $v_i=\partial H_\Delta/\partial p_i$, where p_i is the momentum operator in the direction i. Calculating double commutator of the position operator and the Hamiltonian one obtains a differential equation for time derivatives of $z(t)$ with time-independent operator coefficients. Solving the differential equation one has

$$z(t) = z_0 + v_{kz0}t + u_\Delta \Delta \hat{H}_\Delta^{-1} t - \frac{1}{4}\hbar^2 A_0 \left[\exp\left(-2i\hat{H}_\Delta t/\hbar\right) - 1\right]\hat{H}_\Delta^{-2}.$$

The above equation uses the notation from the paper. The motion of electron consists of the classical linear motion at a constant velocity (the first three terms) and fast oscillations described by the fourth term. This term is very similar to the term describing the ZB oscillations in the Dirac equation. For the NFE model the ZB oscillations have the frequency on the order of the energy gap $2E_\Delta/\hbar$ and the amplitude on the order of λ_Z, which is a

combination of the lattice constant, effective mass of the electron, and the Fourier component of potential V_q. For semiconductors the energy gap $2E_\Delta$ is in the range of 0.2–3 eV, which is more than five orders of magnitude smaller than $2m_0c^2 \approx 1$MeV in a vacuum. Typical values of λ_Z for semiconductors are in the range from one to tens of Angstroms, which is more than three orders of magnitude larger than the amplitude of ZB oscillations in a vacuum λ_c=0.00386 Å. Thus, the characteristics of Zitterbewegung in semiconductors are more easily measurable than in a vacuum. The motion of the Gaussian wave packet for the NFE model is calculated numerically and oscillations of the average position are found. For the packet the ZB oscillations have frequency close to the energy gap and amplitude on the order of the lattice constant. It is shown that the frequency of ZB oscillations depends weakly on the width of the packet. However, the amplitude of ZB oscillations depends on the packet parameters and it is generally smaller than λ_Z.

The case of a strong periodic potential is analyzed for the TB model. Taking into account only interactions between the nearest neighbors and using parameters describing CdTe it is shown, that also for the TB model the velocity operator does not commute with the Hamiltonian. The packet motion is found numerically and ZB oscillations of the average packet position are obtained, having frequency close to the energy gap and amplitude on the order of one Angstrom. The results obtained for the NFE and the TB models point out the role of the periodic potential of the lattice in formation of ZB oscillations in crystalline solids. Finally, a transformation analogous to the Foldy-Wouthuysen transformation is introduced for the NFE model and it is shown that λ_Z is the characteristic length of a non-locality of the wave function after the transformation. This result shows the versatility of the length λ_Z, which replaces the Compton wavelength λ_c both in the description of Zitterbewegung and the Foldy-Wouthuysen transformation.

Key results and suggestions:

- For the simplest models of crystalline solids, both weak and strong periodic potential leads to the ZB oscillations. This suggests the commonness of ZB phenomenon in crystalline solids.
- For the NFE model the equation of motion of the electron is analogous to the equation of motion for the relativistic electron in a vacuum, which justifies the association of

oscillations of electron trajectory for the NFE model with the ZB oscillations in a vacuum.

- For a wave packet, the frequency of ZB oscillations in semiconductors is close to the gap energy (divided by $\hbar$), and the amplitude of ZB oscillations depends both on packet width and λ_Z, and it is on the order of a few Angstroms. Therefore, the observation of ZB in semiconductors can be much easier than the observation of ZB in a vacuum.

2) T. M. Rusin i W. Zawadzki

"Transient Zitterbewegung of charge carriers in mono- and bilayer graphene, and carbon nanotubes"

Physical Review B **76**, 195439 (2007) [7 pages].

DOI: 10.1103/PhysRevB.76.195439

37 citations (Web Of Science), Impact Factor = 3.172

Participation of the author: **75%.**

Author's input: participation in determining the subject, performing calculations, preparing figures, participation in discussion of results, participation in text preparation.

The problem of Zitterbewegung is analyzed for three materials: bilayer graphene, monolayer graphene and carbon nanotubes (CNT). Motivation to take up this subject was the recent discovery of graphene and the fact that the Hamiltonians for electron in monolayer graphene and CNT are similar to the ultra-relativistic Dirac Hamiltonian. Furthermore, it is expected that the ZB oscillations in these materials are more suitable for experimental observation than the ZB oscillations in a vacuum. In contrast to the original approach of Schrodinger, in the present work the operator $x(t)$ is calculated in the Heisenberg picture $x(t)=e^{iHt/\hbar}xe^{-iHt/\hbar}$, treating the expression for $x(t)$ as the product of three operators. In the momentum space the Hamiltonians for electron in graphene and CNT are 2×2 matrices of numbers, which allows one to calculate the exponent of the Hamiltonian $e^{iHt/\hbar}$, and then the product of the operators. The result is the operator $x(t)$ in form of a 2×2 matrix which, in the next step, is averaged over the Gaussian wave packet.

The most important result of the work is equation (5) which gives an *analytical* expression for the time evolution of the Gaussian wave packet in bilayer graphene. Assuming a packet of the width d and the initial velocity $\hbar k_{0y}/m^*$ one has

$$\bar{x}_Z(t) = \frac{1}{k_{0y}}\left[\exp\left(\frac{-\delta^4 d^2 k_{0y}^2}{d^4+\delta^4}\right)\cos\left(\frac{-\delta^2 d^4 k_{0y}^2}{d^4+\delta^4}\right) - \exp\left(-d^2 k_{0y}^2\right)\right],$$

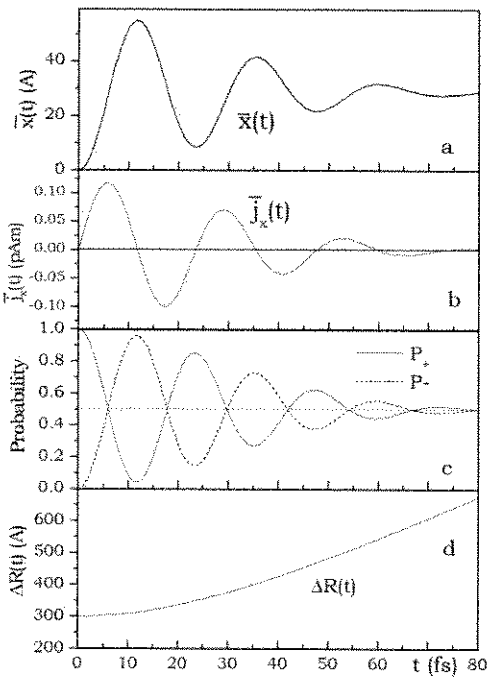

Fig. 2: Zitterbewegung of electron in bilayer graphene for a Gaussian wave packet of width d=300 Å and k_{0y}=3.5x10^8 m^{-1}: (a) position, (b) current, (c) probability of occupation of upper and lower component of wave function, (d) dispersion $\Delta R(t)$. Decay time Γ_Z^{-1}=40 fs.

where $\delta=(\hbar t/m^*)^{1/2}$ contains the time dependence. The above equation is so far *the only analytical model* describing the Zitterbewegung of the Gaussian packet. The following physical consequences for the ZB result from the above equation. First, for the wave packet the ZB disappears in time, since for high values of δ both exponents cancel. Mathematically, disappearance of ZB at large times is a consequence of the Riemann-Lebesgue theorem. Second, in graphene the ZB oscillations occur in direction perpendicular to the direction of the initial wave vector k_{0y}. For $k_{0y}=0$ there are no ZB oscillations. Third, the frequency of ZB oscillations depends weakly on packet width d but strongly on the wave vector k_{0y}. Since the Hamiltonian for electron in graphene does not contain the energy gap, which is zero, its role is played by the distance between the states with positive and negative energies for given k_{0y}. Fourth, the amplitude of oscillations strongly depends on the packet width d. Physical consequence of the above dependences is the existence of ZB only within certain range of packet parameters, namely when the product dk_{0y} is on the order of the unity. Fifth, for wide packet (large d), one obtains oscillations in the form $x_Z(t) \sim (1/k_{0y})\cos(\delta^2 k_{0y}^2)$. This corresponds to the case when the wave packet is not very different from the plane wave e^{ikr}, for which the ZB oscillations do not disappear in time. In this limit the ZB motion is described by permanent oscillations of the amplitude $(1/k_{0y})$ and the frequency $\omega_Z = \hbar k_{0y}^2/m^*$. Sixth, in the initial phase of motion the ZB disappears exponentially with the decay constant $\Gamma_Z = \hbar k_{0y}/(m^* d)$. For bilayer graphene the decay time of ZB oscillations is on the order of

$t_Z=1/\Gamma_Z$ and it amounts to tens of femtoseconds. The above properties of ZB motion are of a general nature and similar conclusions can be drawn from numerical calculations of ZB motion for other systems.

The present calculations show that the decay time of the wave packet is much longer than the decay time of the ZB oscillations. Thus, the spreading of the wave packet does not explain the rapid decay of ZB oscillations. The physical mechanism causing rapid disappearance of ZB is a growing distance between two sub-packets of positive and negative energies. Since the initial Gaussian packet is not actually an eigenstate of the Hamiltonian, it can be decomposed into two sub-packets, respectively, consisting of states of positive and negative energies, and an average velocity of the two sub-packets can be calculated. It is shown that the sub-packets are moving in opposite directions with the relative velocity $\hbar k_{0y}/m^*$. The ZB oscillations occur when the sub-packets overlap with each other, that is when their average positions are spaced apart by less than the packet width d. Therefore the ZB should disappear after time on the order of $t_Z = m^* d/(\hbar k_{0y}) = 1/\Gamma_Z$ obtained previously. For typical values of the packet parameters, this time is much shorter than the decay time of each sub-packets. This result confirms *quantitatively* the previously known interpretation stating that ZB is a consequence of interference between sub-packets of positive and negative energies.

The analysis of ZB motion in monolayer graphene gives similar results as for bilayer graphene. Calculations are performed for carbon nanotubes (CNT), where the packet motion is confined to one dimension. The curvature of the nanotube leads to quantization of motion in the direction perpendicular to tube's axis, resulting in the "freezing" of the non-zero value of the momentum in this direction. Therefore, the ZB oscillations in CNT are possible even when the initial packet velocity is zero. The ZB oscillations in nanotubes are qualitatively different than the ZB oscillations in graphene. Because there is only one direction of the motion (along the tube), it is not possible for the sub-packets to move away in the direction perpendicular to the direction of the motion. Therefore the sub-packets of positive and negative energies overlap until the complete decay of the packet. For this reason the ZB oscillations in CNT disappear very slowly in time, and the envelope function of oscillations decays as $t^{1/2}$. As a consequence, the ZB oscillations in CNT can be observed even after a few picoseconds.

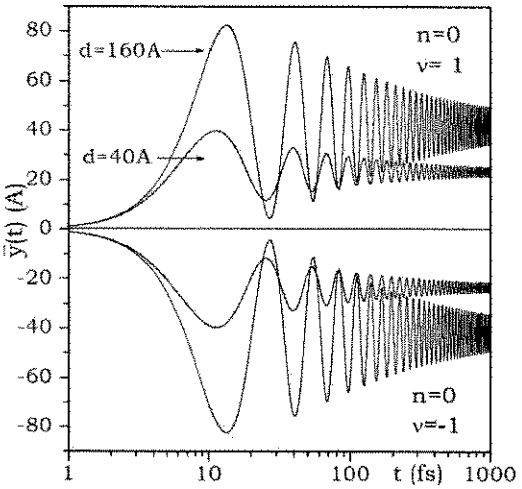

Fig. 3: Zitterbewegung of electron in two lowest bands of carbon nanotube of diameter L=200 Å for two Gaussian packets of different widths d and k_{0y}=0.

Key results and suggestions:

- The ZB oscillations of a wave packet disappear in time, which is a consequence of the Riemann-Lebesgue theorem.
- The disappearance of ZB oscillations is related to moving away by the two sub-packets having positive and negative energies. The ZB exists only when the sub-packets overlap in space.
- The ZB oscillations exist only when the product dk_{0y} is on the order of unity.
- In graphene the ZB disappears after several femtoseconds, while in CNT the ZB oscillations can exist even for a few picoseconds.

3) ***T. M. Rusin i W. Zawadzki***

„ Zitterbewegung of electrons in graphene in a magnetic field"

Physical Review B **78**, 125419 (2008) [9 pages].

DOI: 10.1103/PhysRevB.78.125419

33 citations (Web Of Science), Impact Factor = 3.322

Participation of the author: **75%.**

Author's input: participation in determining the subject, performing calculations, preparing figures, participation in discussion of results, participation in text preparation.

The paper is concerned with the Zitterbewegung in monolayer graphene in the presence of a uniform magnetic field perpendicular to the layer. The presence of a magnetic field leads to complete quantization of the electron motion. Preparing the wave packet as a combination of states having positive and negative energies (e.g. in form of a Gaussian packet), one obtains the motion consisting of oscillations with intraband and interband frequencies, interpreted as the ZB. In the paper the oscillations of position are calculated in the Heisenberg picture by averaging the operators $x(t)$ and $y(t)$ over the wave packet $|f\rangle$. To calculate average value of the position operator $\langle x(t)\rangle = \langle f| e^{iHt/\hbar} x e^{-iHt/\hbar} |f\rangle$ the unity operator $1=\Sigma_{\underline{n}}|\underline{n}\rangle\langle\underline{n}|$ is inserted two times. Here $|\underline{n}\rangle=|n,k_x,s\rangle$ is the eigenstate of the Hamiltonian for electron in graphene in a magnetic field. The calculation of $\langle x(t)\rangle$ requires an integration over the wave vectors k_x and a summation over the Landau levels in two energy bands. Calculation of the average value of $y(t)$ proceeds in the same way. For the Gaussian wave packet in graphene the amplitudes of motion components are analytical functions of the packet parameters and the magnetic length $L=(\hbar/eB)^{1/2}$.

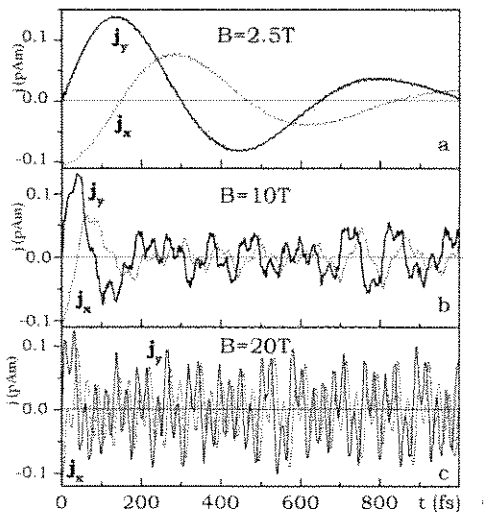

Fig. 4: Zitterbewegung of average current in graphene calculated for a spherical Gaussian packet of width $d_x=d_y=81$ Å and wave vector $k_{0x}=0.035$ Å^{-1} for three values of a magnetic field.

Numerical calculations lead to the following conclusions. First, for non-zero magnetic field there are infinite number of oscillations having intraband frequencies $\omega_n^C=\omega_{n+1}-\omega_n$ corresponding to the cyclotron motion, and the interband frequencies $\omega_n^Z=\omega_{n+1}+\omega_n$ corresponding the ZB. We recall that in graphene in the absence of fields there is only one ZB frequency. Second, the motion of the packet is sustained and it does not disappear in time. Third, the structure of oscillations represents an infinite sequence of events „collapse and revival", as in the case of oscillations of atomic levels populations in

quantum optics. The physical reason of decay and rebirth of packet oscillations is the confinement of sub-packets of positive and negative energies by a magnetic field, which does not allow the sub-packets to depart too far from the center of motion (which is the center of the classical cyclotron orbit). When the sub-packets move away the oscillations disappear, while when they come close the ZB oscillations re-emerge. Fourth, for a given Landau level n the amplitude of components of frequencies ω_n^Z is always smaller than the amplitude of frequencies ω_n^c, which means that the ZB oscillations are always smaller than the oscillations due to the cyclotron motion. Fifth, in practice, the Fourier sums are well approximated by finite sums consisting of several to tens terms having the largest amplitudes. Sixth, the magnetic field changes qualitatively motion of the packet and the ZB oscillations. For high fields the motion may appear chaotic (but with well-defined frequencies), while for low fields there are slowly vanishing oscillations with superimposed small ZB oscillations. Depending on the intensity of a magnetic field, trajectory of the packet can take the shape of vanishing and growing spiral or seemingly chaotic orbit. Technical difficulty in a calculation of the packet motion in a magnetic field is the necessity of numerical calculation of the Hermite polynomials for large n (n> 100). This requires the use of special numerical methods allowing one to carry calculations with the accuracy of a few hundreds significant digits.

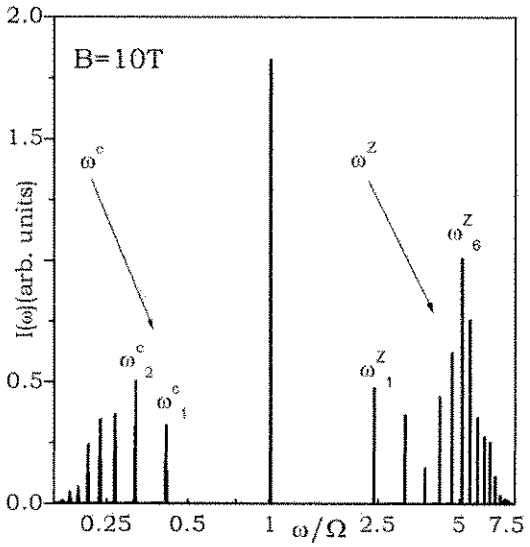

Fig. 5: Power spectrum of radiation emitted by electron Gaussian packet of width d=81 Å and k_{0x}=0.035 Å^{-1} during first 20 ps of motion for a magnetic field B=10 T.

In the remainder of paper, a radiation generated by the oscillating wave packet is analyzed. The initial electron packet has a non-zero dipole moment. Since the packet is not actually the eigenstate of the system (because it includes states with different energies), then during the motion it can emit the electromagnetic radiation. Thus, the presence of radiation with interband frequencies ω_n^Z in the emission spectrum of the packet may be the evidence of existence of ZB motion with frequency ω_n^Z. The ZB differs from the usual interband luminescence by specific dependence of individual spectral lines intensities with frequencies ω_n^Z for different values of the packet parameters and magnetic field strengths.

The paper also examines a contribution of second point K' in the Brillouin zone on the ZB oscillations of the packet. An influence of change of the gauge for a magnetic field on the results is discussed. An analytical approximation for Fourier sums having large number of components is proposed.

Key results and suggestions:

- The motion of a Gaussian wave packet in a magnetic field consists on infinite number of components oscillating with intraband frequencies $\omega_n^c = \omega_{n+1} - \omega_n$, and interband frequencies $\omega_n^Z = \omega_{n+1} + \omega_n$, interpreted as the ZB. In the absence of a magnetic field there appears only one interband frequency.
- For graphene in a magnetic field the ZB oscillations do not disappear in time. Infinite sequences of "collapse and revival" events appear.
- The character of the packet motion depends strongly on the magnetic field.
- An oscillating packet emits dipole radiation whose intensity and frequency depends on packet parameters. Observation of the interband transitions and their dependence on the packet parameters may confirm existence of ZB motion.

4) T. M. Rusin i W. Zawadzki

„Theory of electron Zitterbewegung in graphene probed by femtosecond laser pulses"

Physical Review B **80**, 045416 (2009) [9 pages].

DOI: 10.1103/PhysRevB.80.045416

26 citations (Web Of Science), Impact Factor = 3.475

Participation of the author: **75%.**

Author's input: participation in determining the subject, performing calculations, preparing figures, participation in discussion of results, participation in text preparation.

The paper considers a possibility of observation of ZB in monolayer graphene in the presence of a magnetic field illuminated by a short laser pulse. The p-type doped monolayer graphene sample is placed in a magnetic field $B < 40$ T perpendicular to the surface. It is assumed that the Fermi level is in the valence band at $n = -3$ or lower Landau level and the initial state of the electron is $n = -1$ (e.g. pumped earlier by cw laser or excited thermally). The pulse (of duration on the order of few femtoseconds) creates the wave packet containing states of positive and negative energies. As a result of the excitation, the final state of the electron is a

combination of states with $n = \pm 2$, $n = -1$ and $n = 0$, forming the packet containing components with energies $-2^{1/2}\hbar\omega$, $-\hbar\omega$, 0, and $+2^{1/2}\hbar\omega$. The amplitudes of packet components are calculated approximately, treating the laser electric field as a perturbation. The resulting packet is not a Gaussian packet considered in previous works; its shape depends on pulse parameters and strength of a magnetic field. After a time interval longer than the pulse duration the packet evolves in time, and its motion includes components of the intraband frequencies (e.g. $\omega_1^C = \omega_2 - \omega_1$), and the interband frequencies (e.g. $\omega_1^Z = \omega_2 + \omega_1$) corresponding to the ZB oscillations. Because the electron wave packet has a non-zero oscillating dipole moment, the trembling motion of the electron can be observed indirectly by measuring radiation emitted by the packet. In the paper it is assumed that the radiation is emitted by a single electron and effects of radiation by many electrons are not considered.

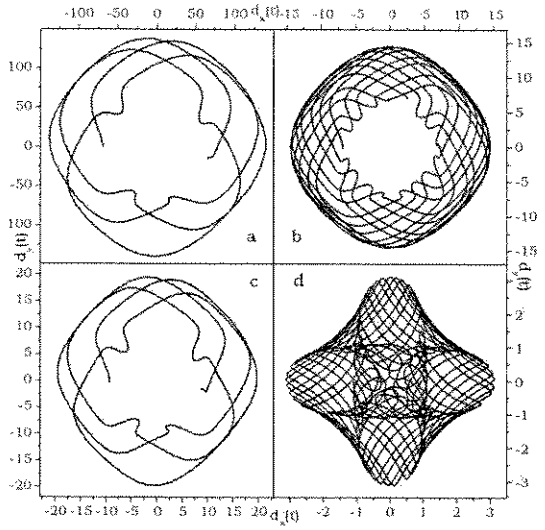

Fig. 6: Oscillations of dipole moment of wave packet created by a short laser pulse in graphene in a magnetic field. Pulse parameters according to paper "4".

Two methods of measuring the dipole radiation emitted by the packet are proposed. The first method proposes to measure the power of emitted radiation as a function of two model parameters: the strength of a magnetic field and the duration of laser pulse. Doing an experiment with two varying parameters one can isolate the ZB from other effects in real graphene samples. The second method is to measure the time dependence of luminescence (Time-Resolved Luminescence). In this method the light emitted by the sample passes through two filters: the time gate opened during time T and the frequency filter that selects frequencies around a central frequency ω_0. Then the intensity of radiation passing through the two filters is measured as a function of the gate opening time T. This dependence is calculated theoretically for typical filter parameters and it is shown that the described method allows one to identify the ZB oscillations in graphene samples in a magnetic field.

Attention is paid to the influence of scattering effects in real graphene samples on the character of ZB oscillations. In particular, the scattering between electrons in graphene gives rise to a small (several meV) energy gap in the band structure and to a broadening of the

Landau levels. The appearance of a non-zero energy gap does not affect significantly the ZB oscillations. On the other hand, the broadening of energy levels leads to complex frequencies of oscillations: $\omega_n \rightarrow \omega_n - i\,\Gamma_n$, where Γ_n is the decay constant for a given n. As a result, under certain simplifying assumptions, the amplitude of ZB oscillations decays exponentially in time and after about 400 fs the oscillations practically disappear. In this case, the decay time of oscillations depends only on sample parameters.

Key results and suggestions:

- Ultra-short pulse laser illuminating the monolayer graphene in a magnetic field creates the electronic packet, which includes components oscillating with cyclotron and interband frequencies. The latter are interpreted as the ZB.

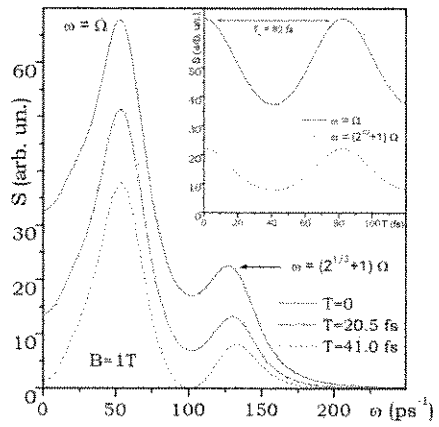

Fig. 7: Calculated power spectrum of radiation emitted by a wave packet created by short laser pulse, for three opening gate times *T* in Time Resolved Luminescence method.

- Oscillating electron packet has a non-zero dipole moment varying in time, which emits the electromagnetic radiation.
- The ZB oscillations can be measured, in principle, with the use of "Time-Resolved Luminescence" technique.
- Electron scattering in real graphene samples causes broadening of the Landau levels, leading to an exponential decay of ZB oscillations.
- In the paper, the radiation from a single electron is calculated and effects of radiation by many electrons are not considered.

5) W. Zawadzki i T. M. Rusin

„Nature of electron Zitterbewegung in crystalline solids“

Physics Letters A **374**,3533-3537 (2010).

DOI: 10.1016/j.physleta.2010.06.028

6 citations (Web Of Science), Impact Factor = 1.963

Participation of the author: **50%.**

Author's input: performing calculations for the quantum model, preparing figures, participation in discussion of results, participation in text preparation.

For many models of the energy band structure of solids, such as the two-band **k·p** model, the simplified Kane model, the Dimmock model for lead salts, graphene, the spin Rashba Hamiltonian, the ZB oscillations are obtained using the formalism similar to that used for the calculation of ZB of the relativistic electrons in a vacuum. The similarity between the two formalisms, however, does not necessary means that one deals with the same physical reason of ZB in crystalline solids and in a vacuum. It is shown that the ZB oscillations in solids occur as a result of the periodic potential of the lattice. The paper shows that the ZB oscillations are related to the varying electron velocity in strong and weak areas of the periodic potential within the unit cell. For models of the band structure of solids, the periodic potential does not appear explicitly and its presence is in some sense "hidden" in the matrix elements of the momentum operator and the energy gap. On the other hand, for the relativistic electron in a vacuum there is no periodic potential and the ZB oscillations are a result of the two-band structure of the Dirac Hamiltonian. Therefore, despite of formal similarities, the ZB oscillations in crystalline solids have a different nature than the ZB in a vacuum.

The analysis of effect of the periodic potential on ZB begins with a simple classical model. The potential is assumed in shape of a saw with triangular teeth. If the total electron energy is greater than the maximum value of potential, the electron moves along the saw and its velocity periodically increases and decreases. This motion can be described by assuming that the electron is moving at a constant average velocity with the instantaneous velocity oscillating around the mean value. Rapid oscillations of the electron velocity correspond to the ZB oscillations in the crystal lattice.

The model described above qualitatively explains the nature of ZB motion in crystalline solids. However, one should also *quantitatively* compare the description of ZB oscillations caused by the periodic potential with the velocity oscillations obtained in the matrix **k·p** method. Such comparison could be made for the Kronig-Penney Hamiltonian. For this model, one can directly calculate oscillations of the packet velocity. One can also expand the Kronig-Penney Hamiltonian in terms of the Luttingera-Kohn (LK) functions and obtain an approximate **k·p** Hamiltonian in the form of a finite matrix describing n energy bands. Then one can calculate an average value of the velocity operator for the **k·p** method and compare the results with those obtained directly from the Kronig-Penney model.

The scheme described above is applied to two lowest energy bands for one-dimensional Kronig-Penney model in the vicinity of the edge of the first Brillouin zone. The resulting **k·p** Hamiltonian, which in this case is a 2×2 matrix, does not commute with the velocity operator, which leads to the ZB oscillations. The initial wave packet is expanded in the LK functions; contributions from higher energy bands are neglected. Since the eigenfunctions of the Kronig-Penney Hamiltonian are given analytically, one can calculate matrix elements of the momentum operator between the LK states, value of the energy gap and all elements of the 2×2 Hamiltonian matrix. It is shown that for the Gaussian packet centered close to the edge of the Brillouin zone, the motion of the packet calculated for the two-band model is very close to the motion of the packet calculated directly for the Kronig-Penney model. This result justifies the claim by the authors that the ZB oscillations in crystalline solids are related to the oscillations of instantaneous electron (packet) velocity due to variation of the periodic potential. In both cases, the instantaneous velocity of the electron oscillates around the average velocity. It is also shown that the average electron velocity calculated for the two-band model is very close to the average velocity obtained for the Kronig-Penney model. The analysis of small differences in the values of velocities for the Kronig-Penney model and the two-band model is carried out.

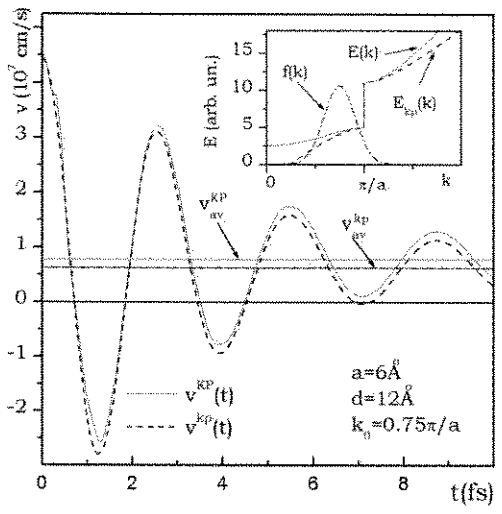

Fig. 8: Electron velocity in a periodic potential. Solid lines: Kronig-Penney model, dashed lines: two-band k·p model. Inset: energy band structure in two models for *k* = *π/a*. Wave packet is centralized at *k* =0.75 *π/a*.

Key results and suggestions:

- Despite of a formal similarity between the ZB in solids and the ZB in a vacuum, the nature of this phenomenon is different in the two cases.
- The Zitterbewegung of electrons in crystalline solids occurs because of the periodic potential. The electron slows down and speeds up depending on the value of periodic potential at various points in space.

- Numerical calculations for the Kronig-Penney model and an approximate two-band **k·p** model confirm the above thesis.

6) W. Zawadzki i T. M. Rusin

„Zitterbewegung (trembling motion) of electrons in semiconductors: a review"

J. Phys.: Condens. Matter **23**, 143201 (2011) [19 pages].

DOI: 10.1088/0953-8984/23/14/143201

16 citations (Web Of Science), Impact Factor = 2.546

Participation of the author: **50%.**

Author's input: review of literature, participation in selection of discussed papers, preparing figures, participation in text preparation.

The paper reviews the research on the Zitterbewegung phenomenon in semiconductors and other periodic systems at the beginning of 2011. Results of approximately 70 works on the Zitterbewegung in solids, atomic systems, models of transport and wave phenomena are collected and systematized. The paper, based mostly on works of the authors, describes and explains the main aspects of problem of Zitterbewegung: operator-like approach introduced by Schrodinger, wave packet oscillations, importance of the interference between states of positive and negative energies, main results for graphene and other simple systems, role of the periodic potential in the ZB in solids and possibilities of observation of ZB. In addition, the results for the ZB in theory of transport in solids, in wave phenomena and in relativistic systems are systematized. A separate section deals with the simulation of ZB in atomic and ionic systems as well as in optical superlattices. The paper stirred a great interest and has been downloaded more than 500 times from the IOP website. In early September 2012 it was selected by the editors as one of the IOP major works in years 2010-2011, to which the free internet access is provided by the IOP.

7) T. M. Rusin i W. Zawadzki

„Zitterbewegung of relativistic electrons in a magnetic field and its simulation by trapped ions"

Phys. Rev. D **82**, 125031 (2010) [20 pages].

DOI:10.1103/PhysRevD.82.125031

11 citations (Web Of Science), Impact Factor = 4.964

Participation of the author: **75%.**

Author's input: participation in determining the subject, performing calculations, preparing figures, participation in discussion of results, participation in text preparation.

The paper analyzes the trembling motion of a Dirac electron in a homogeneous magnetic field. This problem was considered in 1985 by Barut and Thacker[4] who obtained equations of motion for the Dirac electron in a magnetic field using the proper time formalism. The results have complicated algebraic structure and predict an existence of two frequencies of the cyclotron oscillations and two frequencies of the ZB oscillations. When the splitting of the cyclotron levels in a magnetic field exceeds the energy gap $2m_0c^2$ (i.e. $B > 4.4\times10^9$ T), two of the four frequencies become imaginary and the electron motion includes two components diverging exponentially in time. This result is qualitatively different from the results of the author and Zawadzki for an electron in graphene in a magnetic field (paper "3" of the current presentation). The motion of electron in graphene consists of an infinite number of components with cyclotron frequencies and interband frequencies (the ZB oscillations), without any singularity in the whole range of magnetic fields. In the limit $2m_0c^2 \rightarrow 0$ the Hamiltonian of electron in graphene is similar to the Hamiltonian of the two-dimensional Dirac electron. Thus, for $2m_0c^2 \rightarrow 0$ the time dependence of position of the relativistic electron should be similar to that of electron in graphene. However, in this limit the results of the two models are significantly different. This discrepancy has prompted the author and Zawadzki to re-examine the problem of ZB of the relativistic electron in a vacuum.

The first part of paper analyzes time evolution of generalized creation and annihilation operators A^+ and A for the Dirac electron in a magnetic field. The position operators X and Y are linear combinations of A^+ and A. Calculating twice the commutator of $A^+(t)$ and $A(t)$ with the Hamiltonian one obtains differential equations for time derivatives of $A^+(t)$ and $A(t)$. The solutions of these equations give time dependence of $A^+(t)$, $A(t)$, $X(t)$ and $Y(t)$. They are analytical functions of operators H, $A^+(0)$ and $A(0)$ and have no singularity for any value of a magnetic field. The results are formal exact solutions of the equations of motion for the electron in a magnetic field at the operator form. The oscillations of position result from the interference of the *four* sub-packets, not two as in the zero-field case. The motion of the relativistic electron consists of an infinite number of components with intraband frequencies $\omega_{n,kz}^c = (E_{n+1,kz} - E_{n,kz})/\hbar$ describing the cyclotron motion, and an infinite number

of components with interband frequencies $\omega_{n,kz}^{Z} = (E_{n+1,kz} + E_{n,kz})/\hbar$ describing the ZB oscillations. For a given Landau level n the amplitudes of cyclotron motion are larger than the amplitudes of ZB oscillations. For high magnetic fields the amplitudes of both components are comparable. For a low fields the amplitudes of cyclotron components exceed those of ZB components by many orders of magnitude. Therefore it seems impossible to observe the ZB of electron in a vacuum at magnetic field intensities available at present.

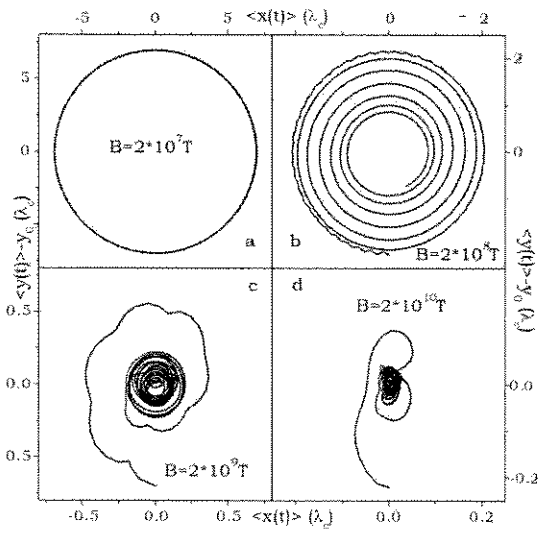

Fig. 9: Trajectories of the Dirac electron prepared in form of a Gaussian packet calculated for four values of a magnetic field in 3D. Packet's parameters are given in paper "7".

Further the paper shows calculations of time evolution of the Gaussian packet for the Dirac electron in a magnetic field. The average values of the position operators $\langle x(t)\rangle$ and $\langle y(t)\rangle$ are calculated in the Heisenberg picture. The unity operator $1=\Sigma_{\underline{n}}|\underline{n}\rangle\langle\underline{n}|$ is inserted twice, where $|\underline{n}\rangle=|n,k_x,k_z,\varepsilon,s\rangle$ is the eigenstate of the relativistic electron in a magnetic field, described by five quantum numbers. The summations over the quantum numbers ε, s and the integration over k_x are performed analytically, the summation over the Landau levels and the integration over k_z require numerical calculations. Numerical procedures with very high precision of calculations are used (hundreds of significant digits). To avoid accidental symmetries, an elliptical Gaussian packet of non-commensurable main ellipse axes is considered. It is assumed that at the beginning the spinor describing the wave packet has one or two non-zero components. In three dimensions the motion of the packet has evanescent character, in two dimensions the motion is persistent and, similarly to the motion of an electron in graphene in a magnetic field, it does not disappear in time. Time evolution of individual position components are calculated for different values of a magnetic field and packet parameters. Trajectories of the packet are calculated and optimal combinations of model parameters leading to the ZB oscillations are found: high magnetic fields (the cyclotron energy on the order of $2m_0c^2$), comparable scales of d_x, d_y, d_z (packet widths in three directions), the magnetic length $L=(eB/\hbar)^{1/2}$ and inverse of k_{0x}. An interesting result is the dependence of the packet trajectory on a magnetic field. For low fields (in this problem a field $B=2\times10^7$ T is considered as low) the packet trajectory is a

classical cyclotron orbit. For fields exceeding the Schwinger field $B=4.4\times10^9$ T the trajectory of the packet becomes an irregular and vanishing spiral. For low fields the ZB still exists but its amplitude is many orders of magnitude smaller than the amplitude of cyclotron motion and the ZB oscillations are not visible.

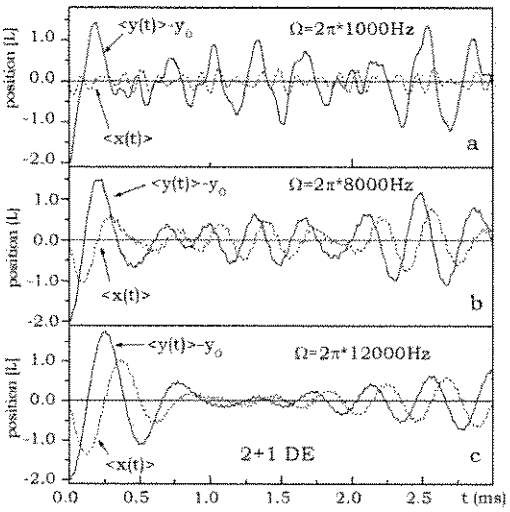

Fig. 10: Calculated motion of a wave packet for two-dimensional simulation of Dirac equation in ion traps $^{40}Ca^{+}$ for three values of simulated of energy gap $\hbar\Omega$.

The rest of paper deals with the possibility of simulation of ZB for the Dirac Hamiltonian with the use of atomic ions in the so-called atomic traps. The inspiration of this part of paper is an experiment of Gerritsma *et al.*[10], who simulated the one-dimensional Dirac Hamiltonian and observed the Zitterbewegung oscillations for the wave packet. This paper proposes a simulation of the Dirac equation for an electron in a magnetic field in two dimensions. Assuming parameters of the magnetic trap from the work of Gerritsma *et al.* [10], the ZB oscillations of the wave packet are calculated for different values of simulated magnetic fields. The obtained results confirm the possibility of simulation of ZB in the proposed system.

Key results and suggestions:

- The motion of the Dirac electron in a magnetic field consists of an infinite number of oscillations with cyclotron frequencies and an infinite number of oscillations with interband frequencies interpreted as the ZB. In three dimensions the oscillations disappear in time, in two dimensions the oscillations are permanent presenting sequences of the "collapse and revival" events.
- The obtained results describe motion of the electron (packet) for any magnetic field strength. The motion has no singularity in the whole range of magnetic fields.
- In the low-field limit the amplitude of ZB oscillations is many orders of magnitude smaller than the amplitude of cyclotron oscillations, so for low fields the ZB oscillations are invisible.

- The equations of motion for the position operators of the Dirac electron in a magnetic fields are solved exactly and analytically.
- It is possible to simulate the ZB for the two-dimensional Dirac Hamiltonian in a magnetic field using the technique of atomic ions in magnetic traps.

8) T. M. Rusin i W. Zawadzki

„Zitterbewegung of Klein-Gordon particles and its simulation by classical systems"

Phys. Rev. A **86**, 032103 (2012) [15 pages]

DOI: 10.1103/PhysRevA.86.032103

0 citations (Web Of Science), Impact Factor =2.878

Participation of the author: **75%.**

Author's input: participation in determining the subject, performing calculations, preparing figures, participation in discussion of results, participation in text preparation.

Existence of the Fermi sea and related many-body effects (e.g. electron-hole creation) present practical difficulties in an observation of ZB for relativistic electrons in a vacuum. These problems do not occur when one deals with the ZB for bosons, because in this case there is no Fermi sea and other effects associated with the fermion's statistics. The paper analyzes the ZB of the charged spin-0 bosons described with the use of Klein-Gordon (KG) equation. This problem was considered in the literature for the ZB in the operator form in the absence of a magnetic field[13,14]. The present paper deals with new aspects of ZB phenomenon for the KG equation: motion of the Gaussian wave packet, disappearance of oscillations, comparison of ZB in Hamiltonian and wave formalisms, analysis of ZB in a magnetic field and a possibility to simulate ZB in mechanical systems.

In the literature, the KG equation appears in two forms: as a second order differential equation with respect to time, similar to the wave equation but with an additional term including the mass of particle, and as a first-order equation with respect to time, resembling the Schrodinger equation, in which the Hamiltonian operator is a 2×2 matrix. The Klein-Gordon model allows one to calculate ZB oscillations by using either the Heisenberg equations of motion or in terms of the wave formalism. The specific feature of KG equation is a difficulty in the definition of the position operator for spin-0 particles. For this reason the

[13] R. F. Guertin and E. Guth, Phys. Rev. D **7**, 1057 (1973).
[14] M. G. Fuda and E. Furlani, Am. J. Phys. **50**, 545 (1982).

present paper considers properties of the velocity and of the current operators, which are well defined in the KG formalism.

At the beginning the paper analyzes the motion of a free spin-0 particle by calculating the velocity operator in the Heisenberg picture. Then the average value of the velocity operator over a Gaussian wave packet is obtained. For the KG model the ZB oscillations arise from the interference of two sub-packets involving states of positive and negative energies. The sub-packets move in the direction parallel to the direction of the initial wave vector of the packet, but with different velocities, resulting in a disappearance of ZB oscillations in time. Let us recall that for graphene the sub-packets move in a direction perpendicular to the direction of the initial wave vector of the packet. An unexplained feature of the motion of spin-0 particle indicated in the paper is a possibility of exceeding the speed of light by the particle at high momenta. There is no such feature in the Dirac equation; it is shown that the velocity of the Dirac electron can not exceed the speed of light for any value of momentum.

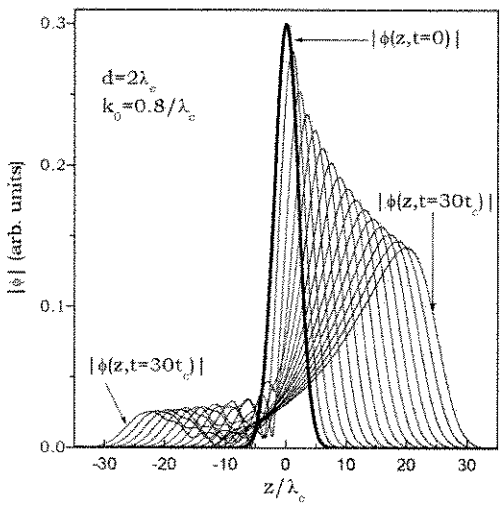

Fig. 11: Time evolution of absolute value of one-dimensional Gaussian wave packet for wave KG equation. Packet separates into two sub-packets moving with different velocities.

The next issue analyzed in the paper is a comparison of ZB oscillations of the wave packet in the Hamiltonian representation with the packet motion described in the wave form of the KG equation. The transformation from the wave form to the Hamiltonian form is equivalent to the conversion of a second-order differential equation into a set of two first-order equations with appropriate initial conditions. One should also transform the packet in the wave representation to the packet in the Hamiltonian form. Performing the two above mentioned transformations and calculating the average value of the current operator in the two representations one obtains the same result. The equivalence of the two approaches takes place for *average* values of the current and velocity operators.

The following part of paper considers the packet motion for the KG equation in a homogeneous magnetic field. The motion consists of a sum of components with cyclotron frequencies and a sum of components with interband frequencies. In the low-field limit the

motion reduces to the classical cyclotron motion, while at high fields the motion resembles irregular oscillations. For high magnetic fields the ZB components give significant contribution to the motion, while for low fields (in respect to the Schwinger field for the particle of mass m) their contributions are negligible. Calculations are carried out for an elliptical Gaussian packet and trajectories of the packet experience continuous transition from the cyclotron motion for low fields to the irregular vanishing spirals at high fields. Finally, the equation of motion for the current density operator in a magnetic field is solved exactly and analytically in the Heisenberg picture.

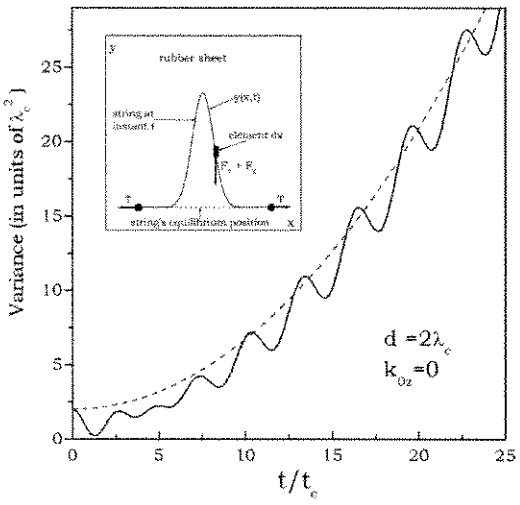

Fig. 12: Variance of packet shape in mechanical model simulating ZB for KG equation. Solid line: total variance, dashed line: non-oscillating part of variance. For model in paper "8" there is λ_c=4.47 mm and t_c=2.37 × 10^{-5}s.

The last part of paper considers simulation of ZB phenomenon by other physical systems. The method mentioned above (see paper "7") for the simulation of the Dirac equation with ion traps is not useful for the Hamiltonians containing higher powers of the momentum operator. Therefore, the paper proposes to simulate the phenomenon of ZB by other systems. A simple model described by the KG equation in the wave form was proposed by Morse and Feschbach[15]. In this model, the metal string (for example, the piano string) is attached in the x-y plane to a thin sheet of rubber. If the string is stretched in x direction and prepared in a shape of a Gaussian function in the y direction then, after releasing the string, it begins to oscillate in the y direction in a way described by the wave KG equation. It its shown that the variance of string's shape performs oscillations similar to the ZB oscillations of the relativistic spin-0 particle. For typical parameters of a piano string and a rubber, the frequency of oscillations is on the order of 13 kHz, which is in the range accessible to the human ear.

Key results and suggestions:

[15] P. M. Morse and H. Feshbach *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953).

- The Zitterbewegung for the charged spin-0 particles is a result of the interference of two sub-packets consisting of states having positive and negative energies, moving with different velocities in the direction parallel to the initial direction of the packet.
- The average values of the velocity or current operators calculated for the wave packet in the Hamiltonian formalism are equal to the average values of velocity or current obtained for the wave formalism. The agreement of the two formalisms takes place for mean values.
- For low magnetic fields the motion of charged spin-0 particles is described by the cyclotron motion and the ZB components are negligible. For high magnetic fields the ZB components are comparable to the cyclotron components.
- The equation of motion for the free KG particle allows, theoretically, a possibility of exceeding the speed of light by a particle. The cause of this feature in the KG theory is unclear. There is no such possibility for the Dirac equation.
- The ZB oscillations can be simulated by a simple mechanical model. Under suitable conditions the variance of shape of an oscillating string simulates the ZB oscillations of spin-0 particles in a vacuum.

Summary

At present the understanding of the Zitterbewegung phenomenon is as follows:

1. The Zitterbewegung occurs when eigenvalues of the Hamiltonian form at least two energy bands. Then the Hamiltonian H does not commute with the velocity operator $v_i = \partial H/\partial p_i$, (where **p** is particle's momentum). In this case, *even in absence of external forces*, the velocity of particle varies in time. Then, in addition to the rectilinear uniform motion, there appear also oscillations of particle's velocity and position.
2. The ZB appears frequently in systems having two energy bands separated by a gap, e.g. for the Dirac Hamiltonian, the Klein-Gordon equation, the two-band **k·p** model for semiconductors. The ZB phenomenon also exists for systems with more energy bands (e.g. the models of Kane, Kronig-Penney etc.) or for systems with two energy bands and a zero energy gap (e.g. for graphene).
3. The ZB is revealed by an existence of motion components oscillating with the interband frequency. Usually the amplitude of ZB is small, so one can imagine ZB as small oscillations around the classical trajectory of the particle.

4. For the wave packet, the ZB occurs as a result of the interference between the sub-packets containing states having positive or negative energies. When the sub-packets move away from each other and cease to overlap, the ZB oscillations disappear.
5. For a majority of cases the ZB oscillations of a wave packet disappear in time. Mathematically, it is a consequence of the Riemann-Lebesgue theorem. Physical processes responsible for the decay of ZB are: sub-packets moving away from each other, packet's spreading in time, broadening of energy levels and effects of radiation associated with an oscillating dipole moment of the packet.
6. In some cases the ZB oscillations do not disappear in time (e.g. packet's motion in graphene in a magnetic field or motion of the Dirac electron in a magnetic field in two dimensions). Then there appear sequences of the type "collapse and revival".
7. A physical reason for the occurrence of ZB in crystalline solids is the presence of a periodic potential of the lattice, which alters the instantaneous velocity of the moving particle. In formalism of the **k·p** matrix, the periodic potential does not appear explicitly but is "hidden" in the matrix elements of momentum and in the energy gap. Physical "trace" of the periodic potential in the **k·p** matrix formalism is the lack of commutation between the Hamiltonian and the velocity operator.
8. The reason for ZB in a vacuum is the presence of two energy bands in the Dirac and the Klein-Gordon Hamiltonians. Therefore the ZB in a vacuum has *a different nature* than the ZB in crystalline solids. However, mathematically, the phenomenon of ZB in a vacuum and in solid state can be described using similar formalisms.
9. The Zitterbewegung occurs also in wave phenomena, when the system is periodic and the dispersion $\omega(k)$ consists of at least two bands. The oscillations of the average velocity or average position of the wave packet having frequencies on the order of the energy gap correspond to the electron ZB in a vacuum or in crystalline solids.
10. In the absence of a magnetic field the amplitude of ZB oscillations in a vacuum is on the order of the Compton wavelength λ_c. In crystalline solids λ_Z plays the same role as λ_c in a vacuum and it can be on the order of tens of Angstroms. For a wave packet the amplitude of oscillations depends on the packet width and it is usually smaller than λ_Z.
11. In a magnetic field, the packet motion consists of many components of intraband frequencies corresponding to the cyclotron motion and many components of interband frequencies corresponding to the ZB. For a given Landau level n the amplitude of ZB oscillations is always smaller than the amplitude of cyclotron motion. For the Dirac

electrons and spin-0 Klein-Gordon particles in high magnetic fields, amplitudes of both motion components are comparable. For low fields the amplitude of ZB oscillations is many orders of magnitude smaller than the amplitude of cyclotron motion. For the electron in graphene in a magnetic field the amplitudes of ZB oscillations are comparable to the amplitudes of cyclotron oscillations in the whole range of fields.

12. The ZB oscillations can be simulated in other systems: ion or atomic traps, optical superlattices, mechanical models. In all of these systems one can simulate the ZB oscillations of the packet in more suitable experimental conditions than in a vacuum. However, so far the electron ZB in solids has not been observed. The difficulty is that one either to observe the motion of a single electron or would need to prepare many electrons oscillating in phase.

5. Discussion of other scientific achievements

T. M. Rusin i W. Zawadzki

„Quantum theory of symmetric screening in the Hartree approximation"

Phys. Stat. Sol. (b) **243**, 1286–1295 (2006).

Participation of the author:**75**%.

An interaction of two electric charges in the electron gas in the Hartree approximation is considered. In the standard approach it is assumed that one charge (e.g. ion of a donor) polarizes the electron gas, and the second charge moves in the screened potential of first charge without disturbing the potential (i.e. the second charge is assumed as an arbitrary small test charge). The described approach is conceptually inconsistent when both charges are of the same absolute magnitude, as in the theory of screened donor. In this case, the concept of the test charge is incorrect because both charges polarize the electron gas in the same way. In the paper, the energy of interaction between the charges is calculated symmetrically, i.e. taking into account the polarization of the electron gas by *both* charges. Including all contributions to the energy, in particular the change of energy of screened electron gas, and calculating in a self-consisted way the screened potential derived from the two charges, the obtained results are identical to these in the "classical" approach (which assumes the existence of the test charge). The paper shows a sequence of cancellations of individual contributions to the energy, which finally leads to the known results. One can summarize by saying that in the textbooks the correct result is obtained using an incorrect method.

T. M. Rusin i W. Zawadzki

"Non-locality of Foldy-Wouthuysen and related transformations for the Dirac equation"

Phys. Rev. A **84**, 062124 (2011).

Participation of the author: **75%**.

Foldy and Wouthuysen[16] (FW) have shown that for the Dirac equation in the absence of external fields it is possible to separate exactly the states with positive and negative energies. In the real space, the kernel of FW transformation is a non-local operator and the measure of non-locality is λ_c. This work deals with further analysis of the non-locality of FW transformation and a similar transformation proposed by Moss and Okniński[17] (MO). The following results are obtained. In the absence of external fields the kernel of MO transformation is given by the Dirac delta function and the Bessel function $K_1(r)$. The kernel of FW transformation is expressed by the Dirac delta function and a function vanishing exponentially with the distance. Second moments of transformation kernels for the MO and FW transformation are calculated and it is shown, that the variances of both kernels are on the order of λ_c^2. A widening of the Gaussian packet for the MO and FW transformations is computed numerically. The variances of transformed Gaussian packet for both transformations are calculated analytically in terms of special functions. In the limiting case of wide packets the variance of the packet after the MO or FW transformations remains unchanged (i.e., $V=3/2\ d^2$), but for a narrow packets the variances of transformed packet tend to $V= 7/2\ d^2$.

T. M. Rusin i W. Zawadzki

"Non-locality of energy separating transformations for Dirac electrons in a magnetic field"

J. Phys. A: Math. Theor. **45** 315301, (2012).

Participation of the author: **80%**.

Properties of the Moss and Okniński[17] (MO) transformation for the Dirac Hamiltonian are analyzed. This transformation separates exactly states with positive and negative energies also

[16] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).
[17] R. E. Moss and A. Okninski, Phys. Rev. D **14**, 3358 (1976).

for non-zero magnetic fields. The following results are obtained. The non-locality of transformation kernel in a direction parallel to the magnetic field is on the order of λ_c and it weakly depends on the field intensity. The non-locality of the transformation kernel in the direction perpendicular to the field strongly depends on field intensity. For low fields the non-locality is close to λ_c, it reaches a maximum for the Schwinger field $B=4.4\times10^9$ T, and for still larger fields it is on the order of the magnetic length $L=(\hbar/eB)^{1/2}$. Thus at high fields it decreases with a magnetic field. The MO transformation of a Gaussian packet is calculated for high fields, and a variance of the transformed packet is analyzed as a function of the initial packet width and field intensity. The largest relative widening of transformation kernel is obtained for narrow packets and a magnetic field close to the Schwinger field.

T. M. Rusin i W. Zawadzki

"Green function of electron in monolayer graphene in a magnetic field"

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Participation of the author: **80%**.

The Green function operators for monolayer and bilayer graphene in the presence of a magnetic field at the K point of the Brillouin zone are calculated analytically. In both cases the Green functions are 2×2 matrices. For example, for a monolayer graphene the (2,2) element of Green function matrix is (notation from the paper is used)

$$G_{00}^{M}(\rho,\rho',E)=\frac{\tilde{E}e^{i\chi}}{2\pi\hbar\omega L^2|r|}\Gamma\left(-\tilde{E}^2\right)W_{\tilde{E}^2+0.5,0}\left(r^2\right),$$

where: ω is the cyclotron frequency, L is the magnetic length, ρ is a two-dimensional position vector, $r^2=(\rho-\rho')^2/(2L^2)$, χ is a gauge-dependent phase factor, $\tilde{E}=E/\hbar\omega$ is a dimensionless energy, $\Gamma(z)$ is the gamma function, $W_{\mu,\nu}(z)$ is the Whittaker function in the standard notation. The paper provides numerical examples of the obtained Green functions and discusses the Green function at the K' point of the Brillouin zone.

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Chapter: „Zitterbewegung (trembling motion) of electrons in graphene"

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E-book is available on: http://www.intechopen.com/articles/show/title/zitterbewegung-trembling-motion-of-electrons-in-graphene
Participation of the author: **70%**.

This is a chapter of the book published electronically and in the paper form. It reviews the results of the authors for the phenomenon of ZB in graphene. The paper also contains a new result: exact and analytical solutions of the equations of motion for the operators $a(t)$ and $a^{+}(t)$ for graphene electron in a magnetic field. The electron motion consists of many components with intraband as well as interband frequencies (the ZB oscillations). The solutions are obtained at the operator form without introducing the wave packet as in paper "3" of the present work. After averaging the equations of motion over the wave packet, the same results as those in paper "3" are obtained.

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