1. What are topological insulators?
2. What is topological about them?
3. Experimental consequences: quantum spin hall effect in 2D.
4. Experiments on QSHE.
Surface/edge states and topological insulators

Band insulator:

Its surface:

Surface states
(Tamm, Shockley…)

They can appear, but they don’t have to – their existence depends on the kind of surface (crystal plane, reconstruction, oxidization…)

Kane & Mele ’05, Bernevig, Hughes, Zhang, Science ’06:

• There are materials for which the surface states always exist, and their dispersion crosses the whole bulk bandgap.

• Zero magnetic field is necessary!

• These states have interesting spin properties.
Surface for 3D bulk vs edge for 2D bulk

2D bulk – a quantum well $\rightarrow$ 1D edge states

3D bulk $\rightarrow$ 2D surface states

Both edge and surface states have interesting spin and transport properties.
What is the basic physical phenomenon?

Edge states in 2D TI: they conduct when the inside of the QW is insulating. Theoretically, at low temperature their conductance should be quantized.

„Quantum spin Hall effect”

Koenig, ... , Molenkamp, Science ’07
HgTe/HgCdTe quantum well

Suzuki,..., Muraki, PRB ’13
Heterostructure of InAs/GaSb
Topological insulators

Topological narrow-gap semiconductors (TNGS)

Conduction and valence bands trade places due to relativistic corrections to band structure ("inverted bandgap")

Relativistic corrections: $\Delta E \sim 1 \text{ eV}$

$E_g - \Delta E < 0 \rightarrow$ nonrelativistic $E_g < 1 \text{ eV}$

Real $|E_g|$ is even smaller…

Zero-gap topological semiconductor
Why „topological”?

Topology: properties of geometric structures that do not change under continuous transformations

What kind of geometric structure are we talking about?

Function from a torus → Hilbert space of dimension $N$

Continuous transformations: modifications of $H(k)$ without closing of the bandgap (shuffling of valence band states)
Topological invariants

Global (topological) property hidden in local geometrical quantity

Genus $g$ – number of holes (topological defects) in a manifold

Gaussian curvature $K = 1/r_1 r_2$ ($r_1$ and $r_2$ are two curvature radii at a given point)

Gauss-Bonnet theorem:

$$\frac{1}{4\pi} \int K \, dS = 1 - g$$
Band inversion

Continuous transformations: modifying $H(k)$ without bandgap closing (shuffling of valence bands)

**Band inversion $\rightarrow$ topological defect**

Bernevig, Hughes, Zhang, Science ’06

\[
\begin{pmatrix}
\frac{E_g}{2} + \frac{\hbar^2 k^2}{m} \\
\nu k
\end{pmatrix}
\]

\[
\begin{pmatrix}
-\frac{E_g}{2} - \frac{\hbar^2 k^2}{2m}
\end{pmatrix}
\]

$E_g \cdot m > 0$ \quad $E_g = 0$ \quad $E_g \cdot m < 0$

When leaving the crystal we go back to the defect-free situation $\rightarrow$ gap has to close at the boundary $\rightarrow$ surface state
Thought experiment: breaking up of a crystal

If YES – then we have a „trivial” insulator
If NO – we have a „topological” insulator
There is a topological invariant for bandstructure of an insulator in absence of time-reversal symmetry breaking (no B field).

\( \nu = 0 \) – „trivial” insulator
\( \nu = 1 \) – „topological” insulator (with edge/surface states)

Don't ask for a formula… It’s easier to count the number of band inversions…
Quantum Hall Effect

2D system in a **strong magnetic field**: Landau levels

Skipping orbits: unidirectional currents along the edges

„chiral” edge states

Another topological invariant:
Chern number $n=0$, $\pm 1 \pm 2$

Perfect quantization!
(much better than in TIs)

TI $\approx$ two copies of QHE with opposite chiralities
Edge states and their properties

2D TI: a quantum well with inverted bandgap at zero B field

One-dimensional spin nondegenerate states at the edge of the well „Quantum spin Hall effect”

States with the same energy at the same edge having $k_x$ and $-k_x$ are Kramers partners: they are connected by time-reversal operation $T$, so they have opposite spins.

Scattering on T-symmetric potential

$$\langle \sigma, \mathbf{k} | \hat{V} | \hat{T} \sigma, \mathbf{k} \rangle = \langle \sigma, \mathbf{k} | \hat{V} | -\sigma, -\mathbf{k} \rangle = 0$$

Impossible between two such states!
„Ideal” 1D channels – „HELICAL” states

Scattering on T-symmetric potential
\[ \langle \sigma, k | \hat{V} | \hat{T} \sigma, k \rangle = \langle \sigma, k | \hat{V} | -\sigma, -k \rangle = 0 \]
Impossible between such states

Each edge has conductance \( G = e^2/h \) – ideal ballistic 1D channel

Bernevig, Hughes, Zhang, Science ’06
Topological protection?

Scattering on T-symmetric potential

$$\langle \sigma, k | \hat{V} | \hat{T} \sigma, k \rangle = \langle \sigma, k | \hat{V} | -\sigma, -k \rangle = 0$$

Impossible between such states

This „topological protection” works only against elastic scattering!

At surface of 3D TI:

Only elastic exact backscattering is forbidden!
Anderson’s localization:
At T=0, in presence of ANY amount of disorder, all electronic states in 1D and 2D are LOCALIZED in limit of infinite sample.

„Normal” 1D channel with some disorder: G will go to zero for L>L_{localization}.

Not true for the helical edge state!
First experiments

Band-inverted HgTe/CdHgTe quantum well

Bernevig, Hughes, Zhang, Science ’06

Koenig, ..., Molenkamp, Science ’07

Imperfect quantization for channel length L~1-2 μm
Other experiments on HgTe

Roth et al., Science ’09 – multiterminal devices

Nowack et al., Nat. Phys. ’13 – micro-SQUID

Ma et al., Nature Comm ’09

inverted device. At zero magnetic field, only the inverted device shows clear edge conduction in its local conductivity profile, consistent with theory. Surprisingly, the edge conduction persists up to 9 T with little change. This indicates physics beyond simple quantum spin Hall model, including material-specific properties and possibly many-body effects.
Experiments on InAs/GaSb

Type-III (broken gap) heterostructure

Liu et al. PRL ’08

Suzuki et al. PRB ’13

Du et al. PRL ’15
Inelastic scattering

Lots of theory on:
- Electron-electron interaction leading to backscattering
- Interaction with localized spins (magnetic impurities, nuclear spins)
- Scattering on lattice vibrations (phonons)
- ...

All predict strong dependence of $R$ on $T$ (at least $R \sim 1/T^2$)
HgTe/HgCdTe structures investigated at IPPAS

HgTe/Hg$_{0.3}$Cd$_{0.7}$Te QW, $d=8$ nm.

Edge channel length between contacts $L \sim 100 \mu m$.

„Normal“ Quantum Hall Effect at high $B$ for n and p type well

depletion – transport thorough edge states at $B=0$?
Nonlocal transport

Nonlocal voltage (or so-called nonlocal resistance)

For a 2D conductor:

\[ R_{nl} = R_{sq} e^{-\pi \frac{L}{W}} \]

In presence of edge states:
Much larger signal, quantized in the ideal case
Does the current go around the sample?
Does the current go around the sample?

Graph showing $R_{ij,kl}$ vs. gate voltage $V$ for $T = 0.46$ K, with a line indicating $R_{82,54}$. The diagram includes a circuit with nodes marked 1 to 8 and a voltage bias $U$.
Does the current go around the sample?

\[ T = 0.46 \text{ K} \]

\[ R_{82,64} \]

\[ R_{i,j,k,l} \text{ (k}\Omega\text{)} \]

GATE VOLTAGE (V)
Does the current go around the sample?
Does the current go around the sample? YES!

- Lack of quantization
- Large resistance
- Reproducible fluctuations
What is going on?

- No quantization
- Large resistance
- Reproducible fluctuations

Weak temperature dependence!
Charge puddles in the QW

Very narrow bandgap of the QW: $E_g \approx 10$ meV
Random potential from ionized donors (modulation doping)

Graphene: extreme case ($E_g=0$), but for small $E_g$ everything should look similar

Puddles of electrons and holes can appear

Graphene group, Nat Phys ’07
Backscattering caused by puddles

Tunneling from the edge states into the puddle.

Electron gets trapped for some time – during this time it can lose its phase memory (undergo an inelastic scattering process).

For very small bandgap, it is hard to make the 2D system **locally** insulating!
SUMMARY

• There are narrow-gap semiconductors in which relativistic effects cause an “inverted” ordering of conduction and valence bands.

• At their edges/surfaces propagating states are guaranteed to appear, as long as time reversal symmetry is not broken (B=0).

• For 2D TI there are edge states, with linear dispersion, and opposite spins propagating in opposite directions.

• At low temperatures (when there should be little inelastic scattering) these 1D edge states should show conductance quantization

• The reality is more complicated (and interesting)…