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Determination of Strength of Solid Porous Body

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Abstract

The problem of destruction of solid bodies is investigated by the science of strength of materials and constructions — mechanics of destruction, expounded on the base of modern conceptions about destruction of different nature bodies. Two kinds of material obtained in different ways have been investigated in this work. A method of alloying the micropowder composition by ultradispersed admixture was used. It increased a value of the Veibull parameter and of average strength of these materials. All these results were calculated and presented in diagrams.

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1. Up-to-date notions about fragile destruction of solid porous bodies

There are two stages in the process of the solid destruction to be considered — the nucleation of the crack and its propagation up to the body's breaking down into parts. Behaviour of the material at each stage of the destruction process obeys its own laws and it is put into the foundation of classification of the main types of destruction. It is impossible to estimate body's strength under the load only by the distribution of stresses, analytically determined from the solution of the proper problem. The criteria of the strength — the characteristics of the material that are determined experimentally and establish the limit of fluidity, the limit of strength, real destruction strength, etc. are also necessary. Dependent on the type of destruction some criteria of strength characterize conditions of nucleating the crack (the appearance of local rather dangerous state), the other ones proceed from the existence of cracks in the material and describe the conditions of their development.

The distinctive feature of the solid porous bodies is a ramified system of through and dead-end pores. This enables us to consider them as "bodies with cracks" that are in the state of partial destruction with no stresses. The permanent increase of cracks begins in the process of the porous body loading, at a certain level of stresses, on account of microdestruction of weak links of structure, statistically distributed in body's bulk. In this case mechanism of destruction does not become localized near one of the cracks but spreads up on the great area of the tense part of material. Monotonous loading results in pulverization of the material and finally ends in a full fragile destruction.

Therefore, the mechanism of the fragile destruction of porous bodies is stipulated by "imperfection" of structure as one of body's physical properties and confirmed by nonlinear dependence between stress and deformation typical of this class of bodies. The curved part of the diagram of deformation begins at the low level of stresses and corresponds to the beginning of steady increasing of numerous microcracks. It embraces a greater part of boundary stress than the initial linear part of the diagram. From this emerges the fact that solid porous bodies offer less resistance to the originating of cracks and more resistance to the developing of cracks.

This peculiarity distinguishes the mechanical behaviour of solid porous materials from the behaviour of majority of constructional materials.

Specific features of deformation and destruction of porous bodies are counted by the sensitive to material's structure characteristic — the measure of brittle. A definition of the measure of brittleness is based not on the strength but on the energetical notions about deformation and destruction of the material

$$x = \frac{U}{W}, \tag{1}$$

where W — full energy of deformation of one unit of bulk of studied sample, U — energy of elastic deformation of one unit of bulk.

To elucidate physical substance of the measure of brittleness, let us assume that a little-deformed fragile body is monotonously and steady loaded. The energy W brought up per unit of volume of tense part of body is accumulated as potential energy of elastic deformation. At a certain level of stress, corresponding to microcracks formation, releasing the elastic energy from the body begins, and it turns into the work of micro-rifts nucleating. From this moment, accumulation and partial spending of elastic energy on the work of steady destruction of material is going on constantly in the process of loading. On reaching the limit of strength, the work of body unsteady destruction to parts is being carried out, and the remaining part U of the elastic energy, accumulated in the body, is escaping into the environment, the total work of the destruction of material is equal to the irrevocable part of full energy: $A = W - U$. Every material is characterized by full energy of deformation and its certain distribution between the work of destruction and elastic part. The more resistance material offers to the breakdown the bigger part of full energy is spent on the work of destruction.

Therefore, the measure of brittleness is an energetic characteristics of the material that takes into account resistance to the destruction on the base of real (not model) law connecting stress and deformation.

The measure of brittleness tallies with Grifits's concept of destruction, according to which, the released elastic energy is irretrievably spent on creation of free from surface's loading main (arterial) crack. But as an integral characteristics of the porous body, the measure of brittleness takes into account the spendings of the elastic energy on the origination and propagation of a large number of microrifts.

The statistical description of fragile strength, made for the first time by Veibull [1], is based on the following prerequisites. In heterogeneous material there are always weak links of structure — "microdefects", statistically distributed in body's bulk. The defects are the centres of destruction and can be a danger under loading. A certain critical stress — "local strength" — corresponds to every defect. The destruction takes place when the average stress reaches the "local strength" of the weakest defect and does not depend on the dimensions of the body.

The probability P of the fragile destruction of the body under the tension load, that does not exceed the value of σ, is taken as the main factor in the statistical model by Veibull. The bigger is the size of the body, the bigger is probability of detecting in it the dangerous defect and the smaller is the strength of the body. Even under the different types of stresses, the most dangerous is that strained state, under which the bigger volume of investigated sample is in a zone of maximum stresses.

Considering this factors, the integral function of distribution of sporadic values of strength σ, obtained during the test of samples of the material, takes the following look:

$$P(\sigma) = 1 - \exp\left[-V\left(\frac{\sigma}{\gamma\sigma_0}\right)^m\right], \tag{2}$$

where V is the volume of the sample in the zone of stresses, m — the Veibull modulus or the coefficient of homogeneity of the material that characterises the degree of strength dispersing, σ_0 — the normalized value, constant for the given material (has the dimensional representation of stress), γ — a dimensionless constant that characterises the strained state of the material dependent on the type of the load.

The probability that the sample does not fail under the stress σ (the probability of "surviving"), according to formula (2), is equal to $1 - P(\sigma)$.

Under the distribution (2), the average strength of the group of samples is determined by the following expression:

$$\langle\sigma\rangle = \int_0^\infty \sigma \mathrm{d}p = \int_0^\infty m \left(\frac{V^{1/m}\sigma}{\gamma\sigma_0}\right)^{2m} \exp\left[-\left(\frac{V^{1/m}\sigma}{\gamma\sigma_0}\right)^m\right] \mathrm{d}\sigma,$$

after rearranging it becomes

$$\langle\sigma\rangle = \frac{\gamma\sigma_0}{V^{1/m}}\Gamma(1+m^{-1}), \tag{3}$$

where $\Gamma(1+m^{-1})$ is the Gamma function, the value of which versus the argument is given in Ref. [2]. The calculation of Veibull's parameters m and σ_0 is carried out using the test results and least squares method.

2. The objects of investigation

In our work, a research on the mechanical properties of solid porous bodies that are used for making abrasive tools for final processing of the part surfaces is carried out.

The blanks of the investigated abrasive material are manufactured by the slip casting method (water suspension), prepared from a micropowder mixture. The mixture contains green abrasive silicone carbide, with 7 micrometer grain, ceramic binder, temporary binder, etc. After drying, the blanks are heated to a high temperature and during this process the temporary binder is burned away, while the ceramic binder forms the matrix in which the abrasive grains of silicone carbide are fixed.

The burned out blanks are mechanically working up, cut into bars of a needed form that are used at the bearing and engineering plants.

The technical level of grinding processes, to a considerable extent, is determined by the quality and operation characteristics of the abrasive tools, among which the main are: strength, hardness, even distribution of hardness over the working layer (the class of precision), wear of the working layer, removal of metal from the processed surface, roughness of the surface.

The low strength of the abrasive bars is explained by the uncontrolled distribution of microcracks in the material. The embryonic cracks originate yet at the initial stage of micropowder mixing. They are bringing about by the existence in the micropowder mixture of agglomerates and flocculations that are preserved in the processes of preparing the slip, casting and burning out the blanks.

Uneven distribution of hardness over the working layer of abrasive tool causes the uneven scraping of the metal off the processed surface and worsens the geometry. The reduction of the precision class of the tool also caused by the formation of agglomerates of ceramic binder mainly of great sizes, uneven distributed in the abrasive material.

To abolish the causes of reduction of the strength and precision class of abrasive tools, the method of alloying the micropowder composition by ultradispersed admixture that enables the homogenization of abrasive material is elaborated.

In this connection we made the investigations of strength and elasticity characteristics of standard and doped by the ultradispersed admixture abrasive materials on the base of silicone carbide.

The samples for test, after the burning out blanks, were made in the form of bars with 90 mm length and 5×4 mm rectangular cross-section. For statistical processing of the results of measuring, two series of samples: standard and doped materials were tested (10 bars in each series).

3. Methods of mechanical tests

The investigations of mechanical properties of porous materials that have a low strength are carried out by the bend tests method.

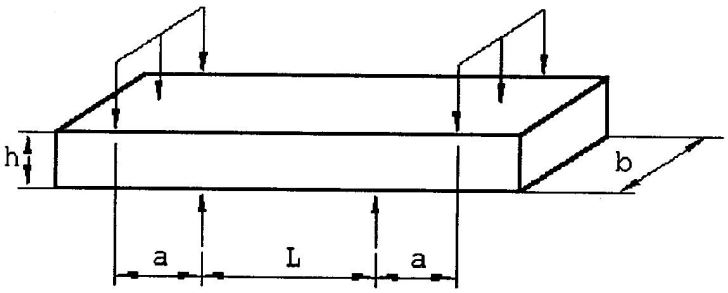

Fig. 1. 4-point system of beam loading.

The computation of stresses during the beam bending process can be made under pure bending that satisfies the hypothesis of flat sections. Pure bending is realized under the 4-points system of beam loading, shown in Fig. 1, where L is the zone of pure bending, equal to the distance between the inside points of loads application, a is the length of cantilever portion of the beam. For making this test, the investigated sample is placed in the so-called liberated loading support and, with the support, is put between the beams of the test machine.

During the loading process, the loads applied to the sample by the support and sag in the zone of pure bending are measured.

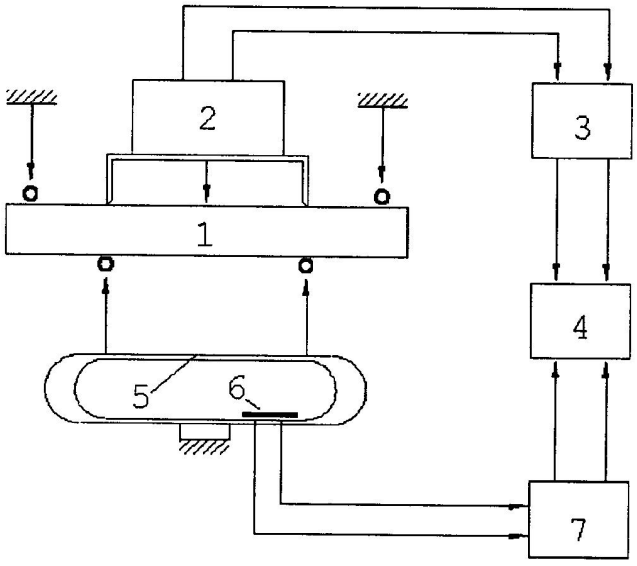

Fig. 2. Functional scheme.

A functional scheme of the installation for bending tests is shown in Fig. 2. Sample 1 is placed on the loading support, the distance between the inner rollers is 20 mm, and between the outside rollers is 40 mm. For measuring the sag σ, the sagmeter 2, completed with transducer of small shifts, is used. An electric signal from the transducer comes to the converter 3 and after amplifying is given to the input port "X" of fast-acting two-coordinates potentiometer 4. The load applied to the sample is measured by the dynamometer 5 with the aid of strain gauge 6. The alteration of the strain gauge resistance under the elastic deformation of dynamometer is converted by strain station 7 into voltage, proportional to the load P. This voltage is given to the input port "Y" of the potentiometer 4.

In the process of sample loading, the potentiometer records the diagram of deformation in coordinates loads – sag on the millimetre paper, that is to say the dependence $P(\sigma)$. In the end of the test the sample breaks that is marked on the diagram by the abrupt fall of the load.

The speed of shifting the beam of the test machine in working regime was constant in all tests and was equal to 0.2 mm/min.

4. Determination of the strength of silicone carbide material

Primary results of bending tests of silicone carbide material, as curves $P(\sigma)$, were used for plotting the diagrams of deformation $\sigma(\varepsilon)$ that provide full information about mechanical properties of the material.

In the case of pure bending, the relative deformation of the sample is proportional to the sag and is determined by the formula

$$\varepsilon = \frac{4h}{L^2}\delta, \tag{4}$$

where L is the distance between the inner rollers of loading support, h — the height of the sample (size in the loading direction), σ — the sag.

The dependence between the average stress and average deformation is as follows:

$$\sigma = \frac{2a}{bh^2}\left(P + \frac{\varepsilon}{2}\frac{\mathrm{d}P}{\mathrm{d}\varepsilon}\right), \tag{5}$$

where P — load, a — the length of cantilever portion, b — the width of the sample (size in the direction, perpendicular to the direction of the applied load).

The derivative $\mathrm{d}P/\mathrm{d}\varepsilon$ was computed by the formulae of numerical differentiation using experimental values P in nodal points $i = 1, \ldots, n$ in the interval $(0, \varepsilon)$ of relative deformation.

The strength of the material is equal to the boundary stress σ_{lim}, calculated by the formula (5) with the values P_{lim}, $\varepsilon_{\mathrm{lim}}$ and derivative $\mathrm{d}P/\mathrm{d}E$ at the point $i = n$.

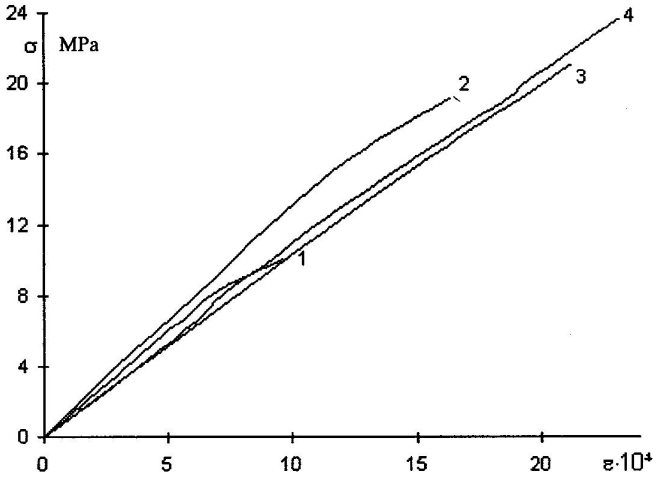

Fig. 3. Diagrams of silicone carbide abrasive material deformation, obtained by the formulae (4) and (5). Curves 1 and 2 represent standard material, the samples of which displayed correspondingly the minimum and maximum strength, curves 3 and 4 — the same for the material doped by the ultradispersed admixture.

A comparison of the curves from the diagrams 3 shows that scattering of strength in standard samples is several times bigger than in the samples of alloyed material.

The average strength of samples from each of these two series was determined by Veibull's statistic.

To do this, we should place the values of strength of number N samples of the given series, obtained from the experiment, in a form of variational series — in succession of strength growth. Each element of the series is put in correspondence to the probability P of sample break-down under the stress G. This probability is defined by the formula

$$P_i = (i - 0.5)/N, \qquad i = 1, \ldots, N.$$

The sampling of the dependence P on σ straightened in such a way, represents the integral distribution of sporadic strength values of investigated material.

For calculating the average strength of each of these materials by the formula (3), it is necessary to determine, on the base of experimental data, Veibull's parameters m and σ, and the constant V, which depend on the way of samples loading. To do this, we determine the "surviving" probability of the given series samples under the stress σ from Veibull's allocation

$$1 - P_i(\sigma_i) = \exp\left[-V\left(\frac{\sigma}{\gamma\sigma_0}\right)^m\right]. \tag{6}$$

Taking twice the logarithm of the expression (6), after some manipulations we get the following:

$$\ln\ln[1 - P_i(\sigma_i)]^{-1} = m\ln\sigma_i + m\ln\frac{V^{1/m}}{\gamma\sigma_0}. \tag{7}$$

The left part of the expression (7) is a linear function of $\ln(\sigma)$, so the expression can be approximated by the linear equation

$$y = a + bx, \tag{8}$$

where $y = \ln\ln[1 - P_i(\sigma_i)]^{-1}$, $x = \ln\sigma_i$,

$$a = m\ln\frac{V^{1/m}}{\gamma\sigma_0}, \quad b = m. \tag{9}$$

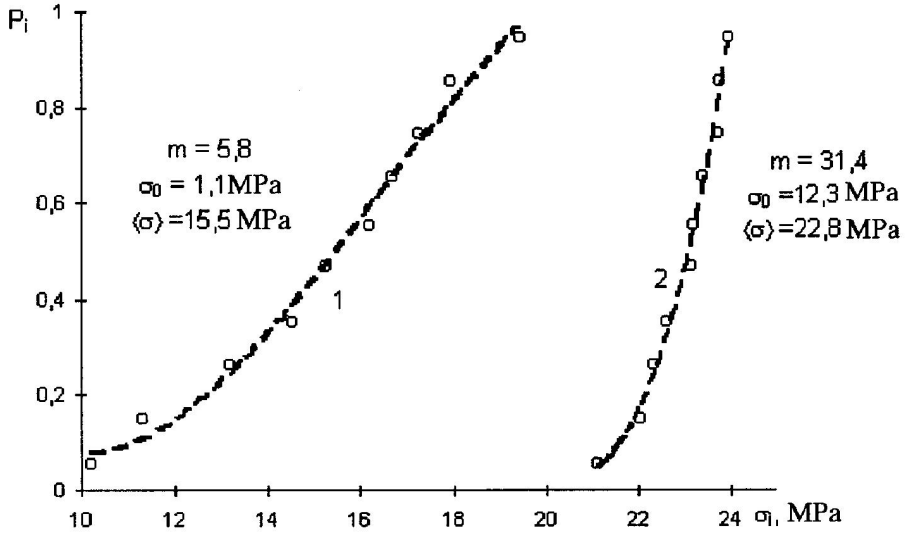

Fig. 4. Strength distributions for the abrasive materials: curve 1 is for the standard silicone carbide material, curve 2 is for the material doped by ultradispersed admixture.

Using the data files σ and $P(\sigma)$ for the given series of samples ($i = 1, \ldots, N$, $N = 10$), the coefficients a and b of the linear approximation (8) were determined by the least squares method. Veibull's parameters m and σ were calculated from Eq. (9) on the base of these values.

Beforehand the values γ and V were determined. For the scheme of 4-currents loading of the rectangular sections samples, which were used in our tests, γ was determined by the formula

$$\gamma = \left[\frac{2(m+1)^2}{(km+1)}\right]^{1/m},$$

where k is the relation of distances between the inner and outer rollers of loading support, V — the volume of the sample between the outer rollers of support.

For the studied materials we got the following values for Veibull's parameters and average strength (Fig. 4), calculated by the formula (3) for the standard silicone carbide material

$$m = 5.8, \quad \sigma_0 = 1.1 \text{ MPa}, \quad \langle\sigma\rangle = 15.2 \text{ MPa},$$

and for the doped silicone carbide material

$$m = 31.4, \quad \sigma_0 = 12.3 \text{ MPa}, \quad \langle\sigma\rangle = 22.8 \text{ MPa}.$$

5. Conclusions

Veibull's modulus m in the statistical sampling (2) of sporadic strength values can be regarded as an auxiliary mechanical characteristics of homogeneity of solid porous material structure. From the correlation (3) emerges that under the big values of m, the dependence between the fragile strength of the material and the volume V becomes weaker. This means that with an increase in m the number of weak links of structure decreases.

Veibull's modulus for the doped silicone carbide material is 5.4 times bigger than for the standard material, which testifies to homogenization of the material by the ultradispersed admixture. The ultradispersed particles of the admixture contribute to the destruction of agglomerates of ceramical binder and even affect the distribution of hard silicone carbide grains in micropowder mixture. On the stage of slip preparing, they prevent the flocculations, structurize water suspension and during the drying of casting decrease the speed of sedimentation of abrasive silicone carbide grains. The homogenization of the doped material results in the strength increase of 1.5 times.

Referances

1. V. Veibull, *Ustalostnye Ispytanya i Analiz ikh Rezultatov*, Mashynostroyenye, Moskva 1964, p. 275 (in Russian)
2. M. Abramovitz, I.A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, in *Applied Mathematics Series*, Vol. 55, National Bureau of Standards, Washington 1965