This paper was awarded in the III International Competition (1994/95) "First Step to Nobel Prize in Physics" and published in the competition proceedings (*Acta Phys. Pol. A* **89** Supplement, S-43 (1996)). The paper is reproduced here due to kind agreement of the Editorial Board of "Acta Physica Polonica A".

HIGH MASS RESOLUTION WITH MAGNETIC FIELD APPLIED IN PAUL'S TRAP

A. Toropova

Advanced Education and Science Centre of Moscow State University Kremenchugskaya st. 11, Moscow, Russia

In this paper the Paul's trap placed in a homogeneous static magnetic field is presented. The motion of ions in such system is investigated. It is shown that such system can work in two regimes: as a mass-spectrometer and as a device which separates a particle beam depending on the mass and specific charge. An application of the external static magnetic field gives a possibility to work for a new set of parameters of the system which was not available before. Apart from that the sensitivity of the device is improved.
PACS numbers: 07.75.+h, 07.77.+p

1. Introduction

Electrons and ions placed in the electromagnetic trap are a unique object which gives a possibility to perform a wide range of experiments in the area of fundamental and applied physics [1]. The two-dimensional field of the quadrupole capacitor allows us to realize the radio-frequency sensitive mass-spectrometer [2, 3].

In this study we investigate the motion of particles in a quadrupole capacitor and magnetic field. Application of the homogeneous magnetic field leads to a new regime of work of such a device. It is shown that the two-dimensional trap with magnetic field can be used for separation of particles beam according to the specific charge and mass of the ions. Such device can be particularly attractive for fabrication of a high-resolution mass-spectrometer.

The application of the magnetic field leads to the splitting of ion states with different values of specific charge in the space of system parameters.

2. Electromagnetic field of the system

Let us consider the electromagnetic system — the quadrupole capacitor placed in the static magnetic field $\vec{B} = (0, B, 0)$ (Fig. 1). The electrostatic field of the quadrupole in vicinity of the z-axis $\vec{E} = (-x, y, 0)V_0/R^2$ (with the potential $\varphi = V_0(x^2 - y^2)/2R^2$) determines equipotential hyperbolic surfaces expressed by the following formulae: $y^2 = x^2 + 2R^2$ and $y^2 = x^2 - 2R^2$; V_0 — electrostatic potential applied to the electrodes, R — space separation of the electrodes.

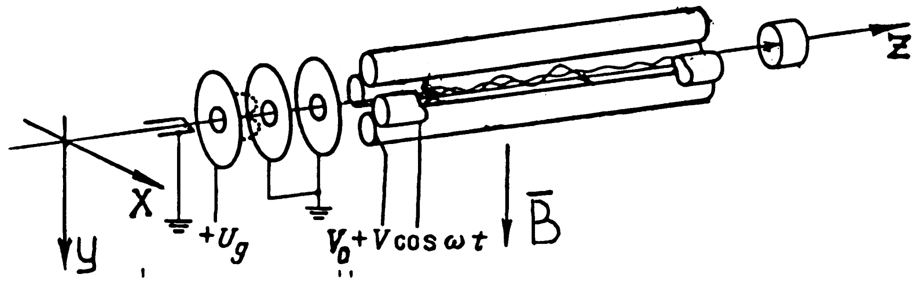

Fig. 1. The experimental setup. The quadrupole capacitor with the length L is situated along the z-axis. Its beginning and end correspond to $z = 0$ and $z = L$, respectively. The source of accelerated ions and the detector are localized on the z-axis at certain $z < 0$ (source) and $z > L$ (detector). The applied field $\vec{B}$ is directed along the y-axis.

The volume restricted by electrodes between the planes $z = 0$ and $z = L$, is a waveguide or an open resonator. In such system the basic transversal electromagnetic wave can propagate with a critical frequency equal to zero. This wave excited by the source can propagate for any frequency ω. A unique feature of such an open system is that the structure of a basic wave field is analogous to the structure of electric and magnetic fields in the static case.

The solution of Maxwell's equations, satisfying the boundary conditions on the surface of the electrodes are as follows:

$$\vec{e} = \frac{V}{R^2}(-x, y, 0)\cos kz \cos \omega t,$$

$$\vec{b} = \frac{V}{R^2}(y, x, 0)\sin kz \sin \omega t.$$

Let us limit the radio-frequency range to $\omega L \ll 1$. In such case $\cos kz \approx 1$ and $\sin kz \approx 0$ and the electromagnetic field of such system has the following form:

$$\vec{E} = (-x, y, 0)V/R^2, \qquad \vec{B} = (0, B, 0), \tag{1}$$

where $V(t) = V_0 + V\cos\omega t$ — the variable voltage between electrode couples.

3. Solution of equations of ion's motion

Let us consider the motion of nonrelativistic ions with a mass m and charge e in a quadrupole trap with fields expressed by the formula (1). Taking into account the electric and magnetic fields (1) we obtain the following equations of motion:

$$m\ddot{x} = -eB\dot{z} - ex(V_0 + V\cos\omega t), \tag{2}$$

$$m\ddot{y} = ey(V_0 + V\cos\omega t), \tag{3}$$

$$m\ddot{z} = eB\dot{x}. \tag{4}$$

The initial conditions are $\vec{x}(0) = (x_0, y_0, 0)$, $\vec{\nu}(0) = (\nu_1, \nu_2, \nu_3)$, $\nu_3 > 0$, $|\nu_2| \ll \nu_3$. By introducing the dimensionless variables expressed by

$$s = \omega t/2, \qquad a = \frac{4eV_0}{mR^2\omega^2}, \qquad b = \frac{2eV}{mR^2\omega^2}, \tag{5}$$

Eq. (3) may be written in the following form:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}s^2} - [a + 2b\cos(2s)]\, y = 0. \tag{6}$$

The first integral is defined by Eq. (4). Substituting

$$\dot{z} = \Omega(x - x_0) + \nu_3, \qquad \Omega = eB/m,$$

to Eq. (2), we obtain the equation in dimesionless variables (5):

$$\frac{\mathrm{d}^2 x}{\mathrm{d}s^2} + [g + a + 2b\cos(2s)]\, x = F, \tag{7}$$

where

$$g = 4\Omega^2/\omega^2, \qquad F = 4\Omega(\Omega x_0 - \nu_3)/\omega^2.$$

Equations (6) and (7) are of the Mathieu's type. The homogeneous Mathieu's equations has the following standard form:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}s^2} + [\mu + 2q\cos(2s)] = 0. \tag{8}$$

Let us assume that $u_1(\mu, q, s)$ and $u_2(\mu, q, s)$ are two linear independent solutions of Eq. (8). Then $y = y_0 u_1(\mu, q, s) + (2\nu_2/\omega)u_2(\mu, q, s)$ and coordinates $x(t)$ and $z(t)$ can represent the general solutions expressed by the functions of $u_1(a+g, q, s)$ and $u_2(a+g, q, s)$.

4. Conditions for confined motion of ions in xy-plane

Let us consider the system shown in Fig. 1. We would like to formulate conditions for the passing of the ions through the quadrupole without collision with the electrodes and conditions for splitting the beam in dependence on mass and charge of the ions.

It is well known that the Mathieu's equation has two classes of solutions — stable and unstable ones. Regions of stability of solutions of Eq. (8) in (μ, q)-plane (available zones) are restricted by $\mu = \mu_n(q)$ curves (Fig. 2). We are interested in values of the parameters for which the solutions of both Eqs. (6) and (7) are stable.

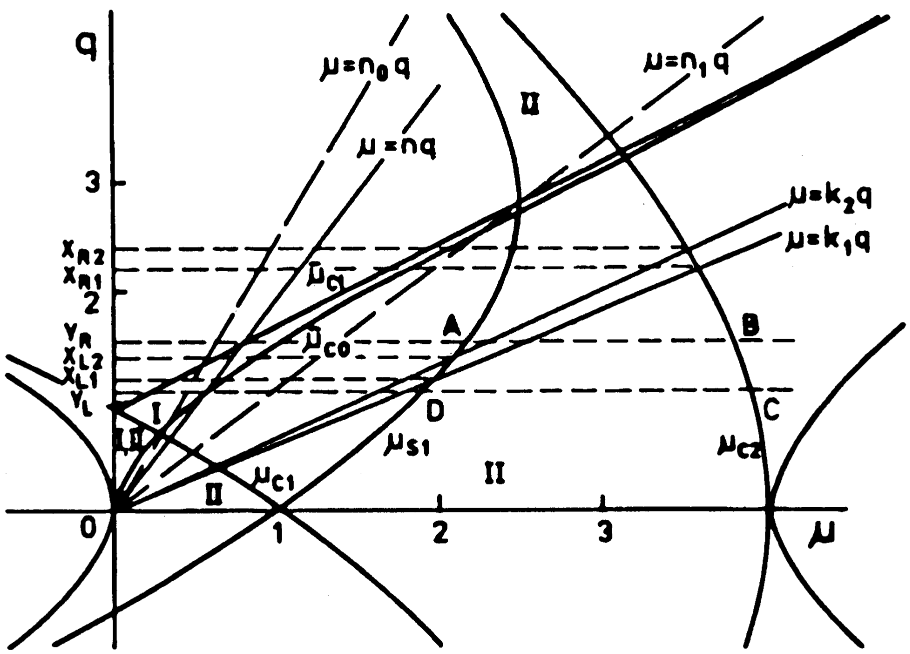

Fig. 2. The Mathieu's equation has two classes of solutions — stable and unstable ones. The regions corresponding to the stable solutions in the plane (μ, q) are limited by the lines $\mu = \mu_n(q)$.

Let us consider a beam, containing ions with a charge e_i and mass m_i ($i = 1, 2$) entering into the system. From the formula (5) it is seen that $n = a/b = 2V_0/V$ and this ratio does not depend on the ion mass and the frequency ω. When the frequency ω changes, as long as V_0 and V change in the same way, n remains constant and then the parameters $\mu = -a$ and $q = -b$ in Eq. (6) fall into the line $\mu = nq$ (where $q > 0$ if $e_i < 0$ and $q < 0$ if $e_i > 0$). Let us assume $M_y[-a, -b]$ is the stable solution of Eq. (6) for $q = -b$ in the range of $[-y_R, -y_L]$.

It is seen that the values of y_R and y_L do not depend on the ratio e_i/m_i and that they are determined by the value of n only and configuration of available zones. The parameters $\mu = a + g$ and $q = b$ in Eq. (7) lies on the lines

$$\mu_i = k_i q, \qquad k_i = e_i h/m_i + n, \qquad h = 2B^2R^2/V \tag{9}$$

with an angular coefficient, depending on the specific charge e_i/m_i.

The magnetic field removes space degeneracy with respect to the specific charge. In the domain of stability the solution of Eq. (7) expressed by $M_x^{(i)}(a+g, b)$ for b taken from $[x_{Ri}, x_{Li}]$ interval, corresponds to different values of frequency at the fixed n. The motion in the (x, y)-plane is confined to the values b from the intersection of $[q_{Li}, q_{Ri}]$, $[y_L, y_R]$ and $[x_{Li}, x_{Ri}]$.

An analysis of the ion movement can be simplified by introducing a central-symmetrical reflection in (μ, a)-plane. In this way we change a stable area of Eq. (8) from the third quarter to the first with a modified Mathieu diagram form.

Let us consider the dependence of ion separation on the specific charge. The ions with different e_i/m_i values should fulfill the stability conditions described by Eqs. (6) and (7), respectively. Every point on (μ, q)-plane represents a single state of a particle. By increasing the frequency ω the points can be moved along the straight lines $\mu = nq$ and $\mu_i = k_i q$ towards the origin. The slope of $\mu_i = k_i q$ can be controlled by changing induction of the magnetic field.

In this study we will consider the separation of single charged ions with masses m_1 and m_2: $m_2 > m_1$. We assume that $V > 0$, $V_0 > 0$. For proper work of the device within the stables areas the following parameters should be chosen $|\mu| < 5$, $|q| < 4$.

- The region of stability of the solutions of Eq. (7) is denoted in Fig. 2 with the symbol "I". In the next step we will consider in detail the case for one n ($n_0 < n < n_1$) only. The limits of the interval $[y_L, y_R]$ are determined by the intersection of $\mu = nq$ line with the $\bar{\mu}_{C0}$ and $\bar{\mu}_{S2}$ curves. The limits of $[x_{Li}, x_{Ri}]$ intervals are determined by the intersection of lines (9) and μ_{C2} and μ_{S1} curves. The stable area of Eqs. (6) and (7) is defined by $[q_{Li}, q_{Ri}] \in [x_{Li}, x_{Ri}] \cap [y_L, y_R]$.

For $\omega = \omega_{\min}(m_2)$, which is the root of equation $b(m_2) = y_R$, ions with a mass m_2 fall within the stable area. The parameter $b(m_2)$ takes the value from the $[y_R, y_{L2}]$ interval if the frequency changes as follows:

$$\omega_{\min}(m_2) \le \omega \le \omega_{\max}(m_2). \tag{10}$$

For $\omega = \omega_{\max}(m_2)$ we have $b(m_1) > y_R$. For such case only ions with a mass m_2 fall within the collector. The parameter $b(m_1)$ takes the values $[y_R, x_{Li}]$ when the frequency is changing within the interval

$$\omega_{\max}(m_2) < \omega_{\min}(m_1) \le \omega \le \omega_{\max}(m_1). \tag{11}$$

The boundary conditions of Eqs. (10) and (11) are the following:

$$\omega_{\min}{}^2(m_i) = 2eV/(m_i R^2 y_R), \qquad \omega_{\max}{}^2(m_i) = 2eV/(m_i R^2 x_{Li}).$$

To separate the ions depending on their masses the parameters n and B should satisfy the following condition: $\omega_{\max}(m_2) < \omega_{\min}(m_1)$. This inequality is equivalent to inequality $m_1 y_R < m_2 x_{L2}$, which determines allowable n, B values.

• Assuming that $m_1 = m$, $m_2 = m + \Delta m$ the ratio $m/\Delta m = Q(m, B, n)$ is obtained, where Q is the mass resolution of the system. For $\Delta m \ll m$ we obtain

$$Q = [x_L(m) + m\partial x_L(m)/\partial m][y_R - x_L(m)]^{-1}.$$

Based on the analysis of the data presented in Fig. 1 it is possible to show that the changes of B give the possibilities to extract optimal areas when $Q \ll 1$. Such an optimal range is shown in the figure by the rectangle ABCD.

One should add that for large values of n the regime of separation of ions is realized for $|\mu| > 4$, $|q| > 4$ (it is not shown in Fig. 2).

Paul has considered the regions $0 < |\mu| < 0.4$, $|q| < 1$ for the ion separation depending on the mass. The application of the external static magnetic field improves the mass-spectrometer sensitivity.

References

[1] R.S.Jr. Van Dyck, H. Ekstrom, H. Dehmelt, Phys. Rev. **59**, 26 (1987)

[2] W. Paul, Usp. Fiz. Nauk **160**, 109 (1990)

[3] N.W. McLachlan, *Theory and Application of Mathieu Equations*, Clarendon Press, Oxford 1947