Fermi and non-Fermi liquids: 2

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Fermi liquid theory

- The Fermi liquid as a quantum protectorate, adiabatic continuity.
- The energy functional, self-consistency, zero-sound.
- The electron and the quasiparticle.

Beyond the Fermi liquid

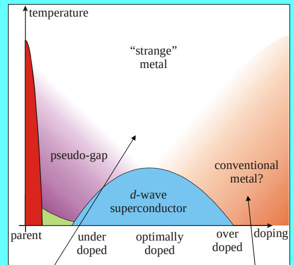
- Luttinger liquids:
  - Special features of 1D, spin-charge separation.
- Quantum criticality:
  - The Reizer singularity, metals on the border of magnetism.

References:

A. J. Schofield, "Non-Fermi liquids", Cont. Phys. **40**, 95 (1999);
C. M. Varma *et al.* "Singular Fermi liquids", Phys. Rep. **361**, 267 (2002).

The search for "non-Fermi liquid" metals

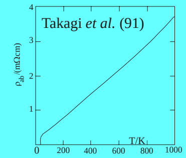
Motivated by the puzzle of the "high temperature" cuprate superconductors



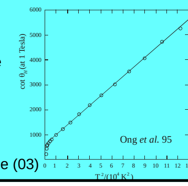
No quasiparticles ever seen here

Quasiparticles seen here: Hussey et al. Nature (03)

Electric currents scatter like T



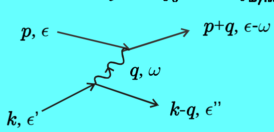
Hall currents scatter like T^2



Destabilizing the Fermi liquid:

Change variables in the Fermi golden rule scattering rate:

$$\frac{1}{\tau_\epsilon} = \frac{2\pi}{\hbar}\int_0^\epsilon g_F \omega d\omega \int_{\omega/\hbar v_F}^{2k_F} \frac{q^{d-1}dq}{(2\pi/L)^d}\frac{|D(q,\omega)|^2}{(\hbar v_F q)^2}.$$



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Consider first energy/momentum independent scattering: $D(q,\omega)=V$

- 1D $\quad \frac{1}{\tau_\epsilon^{1D}} \sim \int_0^\epsilon \omega d\omega \int_{\omega/\hbar v_F}^{2k_F} \frac{dq}{q^2}|V|^2 \sim \int_0^\epsilon d\omega \sim \epsilon\,.$

So the Landau criterion (that the decay rate vanishes faster than the energy of the quasiparticle) is not satisfied at lowest order. This is an indication of a sickness in one-dimensional Fermi liquids: Luttinger liquids.

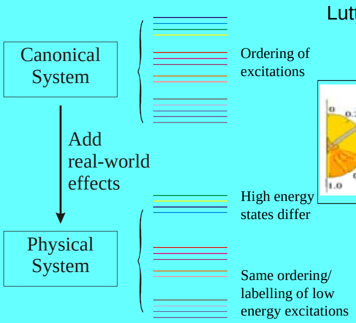
- 2D $\quad \frac{1}{\tau_\epsilon^{2D}} \sim \int_0^\epsilon \omega d\omega \int_{\omega/\hbar v_F}^{2k_F} \frac{q dq}{q^2}|V|^2 \sim \int_0^\epsilon \omega \ln\omega d\omega \sim \epsilon^2 \ln\epsilon\,.$

Forward scattering (small q) in two-dimensions leads to non-analytic corrections to Fermi liquid theory. Surprisingly this seems to have been noticed only very recently: A. V. Chukukov et al. cond-mat/041228.

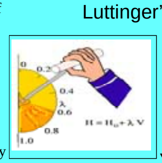
- 3D Need to make D(q,ω) singular: quantum criticality.

Luttinger liquids – a new protectorate in 1D

Luttinger liquid theory



Luttinger's model



Add real world effects

Luttinger liquid

Excitations: spinons and holons.

Characteristics: spinon and holon velocity and two "anomalous exponents": κ_ρ, κ_σ

F. D. M. Haldane, J. Phys. C SSP **14** 2585 (1981)

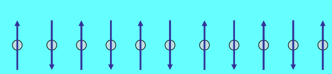
Simple picture of spin-charge separation

(Proper treatment: bosonization of the 1D interacting metal)

$$\hat{\mathcal{H}}_{tJ} = \hat{\mathcal{P}}_{n_i=2}\left[-t\sum_{\sigma,\langle i,j\rangle}\hat{c}^\dagger_{i,\sigma}\hat{c}_{j,\sigma} + J\sum_{\langle i,j\rangle}\hat{\mathbf{S}}_i\cdot\hat{\mathbf{S}}_j\right]\hat{\mathcal{P}}_{n_i=2}$$

tJ model in 1D

- no double occupancy
- spins on neighbouring sites want to be anti-parallel
- electrons want to move from site to site



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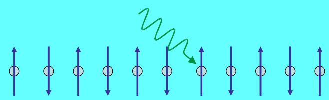

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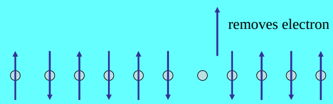

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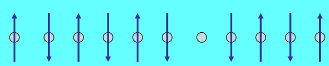

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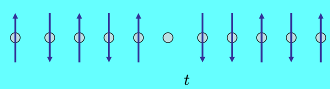

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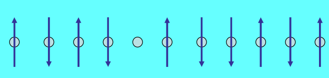

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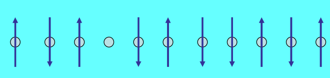

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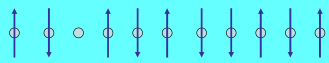

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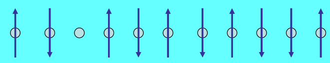

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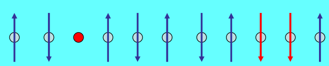
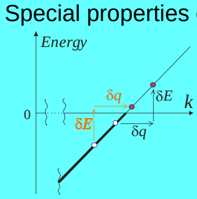


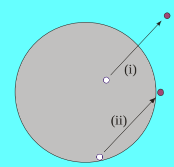
holon spinon

electron has "dissolved"

Special properties of 1D



Particle hole excitations at fixed q "all" have the same kinetic energy.



Not the case in high dimensions.

So what? Interactions can typically be written in terms of density waves

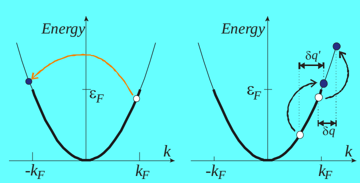
$$\sum_{\vec r,\vec r'} V(\vec r-\vec r')\hat\rho(\vec r)\hat\rho(\vec r') \to \sum_{\vec k,\vec p,\vec q}\tilde V(\vec q)\hat c^\dagger_{\vec k+\vec q}\hat c^\dagger_{\vec p-\vec q}\hat c_{\vec p}\hat c_{\vec k} = \sum_{\vec q}\tilde V(\vec q)\hat\rho_{\vec q}\hat\rho_{-\vec q}$$

A density wave is a linear combination of particle-hole excitations all with the same wavevector but different starting points $\hat\rho_{\vec q} = \sum_{\vec k}\hat c^\dagger_{\vec k+\vec q}\hat c_{\vec k}$

→ In 1D a density wave can be an eigenstate of both potential and kinetic energy

Not strictly correct…

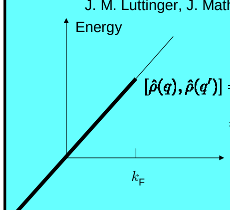
Inter-branch processes and band curvature spoil this property.



The Luttinger model explicitly removes these processes. Putting them back is part of the "real-world" effects added adiabatically.

1 branch Luttinger model:

J. M. Luttinger, J. Math Phys. **4**, 1154 (1963).



$$\hat\rho(q) = \sum_k \hat c^\dagger_{k+q}\hat c_k$$

$$[\hat\rho(q),\hat\rho(q')] = \delta_{q,-q'}\sum_k(\hat n_{k-q}-\hat n_k)$$

$$= \delta_{q,-q}\sum_k(\hat n_{k-q}-\hat n_k) \to \delta_{q,-q}\sum_{k>k_\Lambda}(\hat n_{k-q}-\hat n_k)$$

$$= \delta_{q,-q}\left\{\sum_{k_\Lambda>\tilde k>k_\Lambda-q}\hat n_{\tilde k} - \sum_{k>k_\Lambda}(\hat n_k-\hat n_k)\right\} = \frac{\delta_{q,-q}q}{(2\pi/L)}$$

$$[\hat\rho(q),\hat\rho(q')] = \delta_{q,-q'}\frac{qL}{2\pi}$$

So we can define a set of bosons $\hat b^\dagger_q = \sqrt{\frac{2\pi}{Lq}}\hat\rho_q \qquad \hat b_q = \sqrt{\frac{2\pi}{Lq}}\hat\rho_{-q}$

and use them to write (restoring spin) the interacting Hamiltonian

$$\hat H = \sum_{q,\sigma} v_F q\,\hat b^\dagger_{q,\sigma}\hat b_{q,\sigma} + \sum_{q,\sigma,\sigma'}\frac{V(q)q}{2\pi}\hat b^\dagger_{q,\sigma}\hat b_{q,\sigma'}$$

Solution of the 1 branch Luttinger model

$$\hat{H} = \sum_q v_F q \left(b^+_{q\uparrow} b_{q\uparrow} + b^+_{q\downarrow} b_{q\downarrow}\right) + \frac{Sq}{2\pi}\left(b^+_{q\uparrow} b_{q\uparrow} + b^+_{q\downarrow} b_{q\downarrow}\right) + \frac{Aq}{2\pi}\left(b^+_{q\uparrow} b_{q\downarrow} + b^+_{q\downarrow} b_{q\uparrow}\right)$$

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Now define new operators: $\alpha^+_q = \frac{1}{\sqrt{2}}\left(b^+_{q\uparrow} + b^+_{q\downarrow}\right)$ charge density

$\beta^+_q = \frac{1}{\sqrt{2}}\left(b^+_{q\uparrow} - b^+_{q\downarrow}\right)$ spin density

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Now

$$\alpha^+_q \alpha_q + \beta^+_q \beta_q = b^+_{q\uparrow} b_{q\uparrow} + b^+_{q\downarrow} b_{q\downarrow} \; ; \; \alpha^+_q \alpha_q - \beta^+_q \beta_q = b^+_{q\uparrow} b_{q\downarrow} + b^+_{q\downarrow} b_{q\uparrow}$$

So

$$\hat{H} = \sum_q \left(v_F q + \frac{Sq}{2\pi}\right)\left(\alpha^+_q \alpha_q + \beta^+_q \beta_q\right) + \frac{Aq}{2\pi}\left(\alpha^+_q \alpha_q - \beta^+_q \beta_q\right)$$

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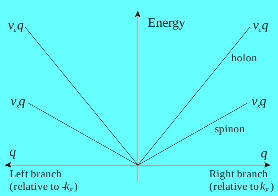
$$= \sum_q v_\alpha q \, \alpha^+_q \alpha_q + v_\beta q \, \beta^+_q \beta_q$$

where $v_\alpha = v_F + \frac{S}{2\pi} + \frac{A}{2\pi}$ $\qquad v_\beta = v_F + \frac{S}{2\pi} - \frac{A}{2\pi}$

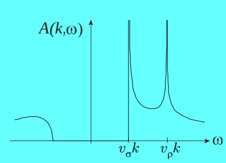
holon velocity $\qquad$ spinon velocity

Consequences:

Excitation spectrum



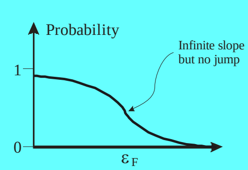
Spectral function



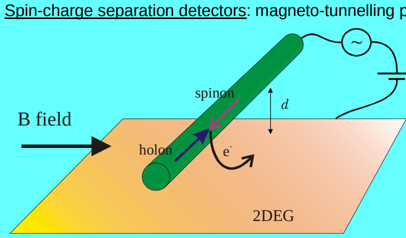
J. Voit, Phys Rev B **47**, 6740 (1993)

Thermodynamic quantities very similar to those of a Fermi liquid.

To observe the differences you need to probe electron like quantities: ARPES or tunnelling.



Spin-charge separation detectors: magneto-tunnelling proposal



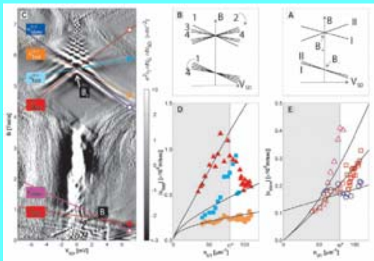
A Altland, CW Barnes, F Hekking & AJ Schofield, Phys Rev Lett, **83**, 1203 (1999).

$$I(V,B) = \frac{4I_0}{m}\int dq \int \frac{d\epsilon}{2\pi}\left[n_F(\epsilon - eV) - n_F(\epsilon)\right] A_{1D}(q,\epsilon) A_{2D}(q - eBd - \Delta q_F, \epsilon - eV)$$

Best experiments:

- Some ARPES:
  - e.g. H. Ishii et al., Nature **426**, 540 (2003); P. Segovia, Nature **402**, 504 (1999); J. D. Denlinger et al., PRL **82**, 2540 (1999); C. Kim et al, PRL **77**, 4054 (1996).
- Some magneto -tunnelling (1D to 1D)

O. M. Auslaender *et al.*, Science **308**, 88 (2005).



Destruction of the Fermi liquid in D>1

$$\frac{1}{\tau_\epsilon} = \frac{2\pi}{\hbar}\int_0^\epsilon g_F \omega d\omega \int_{\omega/\hbar v_F}^{2k_F} \frac{q^{d-1}dq}{(2\pi/L)^d}\frac{|D(q,\omega)|^2}{(\hbar v_F q)^2}.$$

Need to make D(q,ω) more singular:

One possibility – the Coulomb interaction:

$$\frac{\rho_e(\vec{r})\rho_e(\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|} \longrightarrow D_c(q) = \frac{\rho_e(q)\rho_e(-q)}{4\pi\epsilon_0 q^2}$$

Long range implies singular at small q.

However, metals screen electric fields, so ultimately there is no singularity.

$$\frac{\rho_e(\vec{r})\rho_e(\vec{r}')}{4\pi\epsilon_0|\vec{r}-\vec{r}'|}e^{-|\vec{r}-\vec{r}'|/\xi} \longrightarrow D_{sc}(q) = \frac{\rho_e(q)\rho_e(-q)}{4\pi\epsilon_0(q^2+\xi^{-2})}$$

However, metals do not screen magnetic fields so the current-current interaction in a metal leads to a long range interaction.

$$D_{\rm Amp}(\vec{q},\omega) = \vec{A}(\vec{j})\cdot\vec{J} = \frac{\mu_0 \vec{j}_{\vec{q},\omega}\cdot\vec{j}_{-\vec{q},-\omega}}{q^2+i\omega\sigma\mu_0-\omega^2/c^2}$$

This does lead to the breakdown of the Fermi liquid – the Reizer singularity M. Reizer (1989)…buut is very weak effect so would only be observed at microkelvin temperatures. $\frac{D_{\rm Amp}}{D_c} \sim \frac{v_F^2}{c^2} \sim 10^{-6}$

Quantum critical points

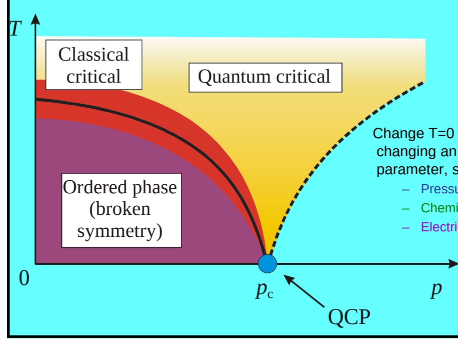
Reizer singularity physics at accessible temperatures. Create an effective long range force…

Enhance the scattering matrix elements for quasi-particles.

Use a medium where "vortex trails" are long lived.

Quantum critical points in itinerant Fermi systems

[J Hertz (1976), AJ Millis (1993), …]

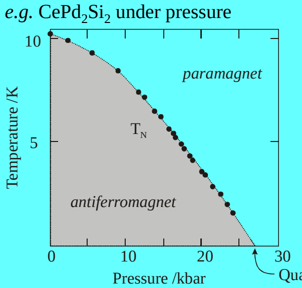


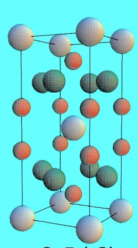
Change T=0 groundstate by changing an external parameter, such as:
- Pressure
- Chemical composition
- Electric field doping

Quantum critical points in itinerant Fermi systems

[J Hertz (1976), AJ Millis (1993), …]

e.g. $CePd_2Si_2$ under pressure

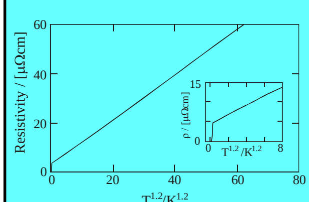


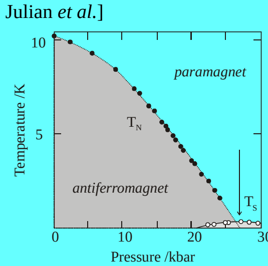


S. R. Julian *et al.* J. Phys: Condens. Matt., **8**, 9675 (1996)

The effect of these quantum critical fluctuations:

e.g. $CePd_2Si_2$ [S. R. Julian *et al.*]





non-Fermi liquid metal:
• $T^{1.2}$ resistivity over 2-3 decades

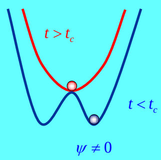
New correlated states induced near the quantum critical point
• Superconductivity.

…and the theory should be tractable!

Universality at phase transitions

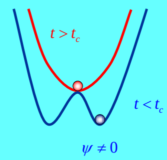
$$F = \qquad (t - t_c)|\psi|^2 + b|\psi|^4$$

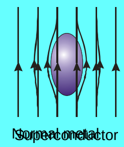
An order parameter: ψ.



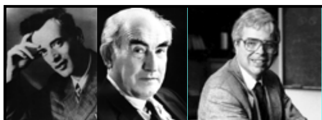
Universality at phase transitions

$$F = \int d^3x \left[(t - t_c)|\psi|^2 + b|\psi|^4 + |(-i\vec{\nabla} - e^* \vec{A})\psi|^2 \right]$$





Diverging correlation length: $\xi \sim \frac{\xi_0}{(t - t_c)^{1/2}}$



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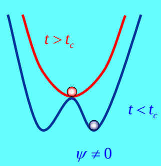
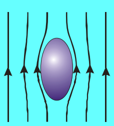




Role of (interacting) fluctuations.

$d<d_u$ (upper critical dimension)

- Dominate…scaling, modified exponents…

$d>d_u$

- Negligible…Gaussian results okay.

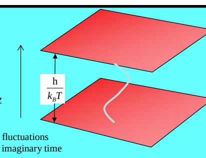
Diverging correlation length: $\xi \sim \frac{\xi_0}{(t - t_c)^{\nu}}$

Quantum criticality

$$Z = e^{-\beta F} = \sum_i e^{-\beta \epsilon_i}$$

$$Z = \sum_i \langle \psi_i | e^{-\beta \hat{H}} | \psi_i \rangle$$

$[\tau]=[L]^z$



$$F = \int d\tau d^D x \left[(x - x_c)|\psi|^2 + b|\psi|^4 + |\vec{\nabla}\psi|^2 + \text{dissip.} \right]$$

Quantum criticality

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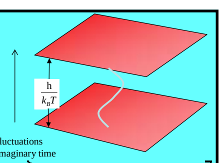

$$S = \int \sum_\omega d^D q \left[\left((x - x_c) + q^2 + \frac{i\omega}{q^{z-2}} \right) |\psi|^2 + O(b|\psi|^4) \right]$$

$D+z < d_u$

$$\tau^{-1} \propto \frac{k_B T}{h}$$

$$\chi''(E) = \frac{1}{E^{1-\alpha}} G\left(\frac{E}{T}\right)$$

α>0 E/T Scaling.

One energy scale- the temperature.

Sachdev and Ye, PRL 69, 2411 (92), Sachdev, QPT, pp234 (Cambridge, 99)

$D+z > d_u$

$$\tau^{-1} \propto T^{d/2} U, \quad (z = 2)$$

$$\alpha = 0$$

"Gaussian fixed point" α=0.

T is not the only energy scale.

Typically, z=2 (antiferromagnet) or 3 (ferromagnet) and d_u=4 so experimental examples should be Gaussian – no scaling..

Quantum criticality

$$Z = e^{-\beta F} = \sum_i e^{-\beta \epsilon_i}$$

$$Z = \sum_i \langle \psi_i | e^{-\beta \hat{H}} | \psi_i \rangle$$

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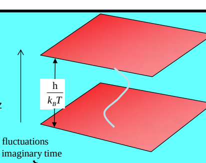


$$S = \int \sum_\omega d^D q \left[\left((x - x_c) + q^2 + \frac{i\omega}{q^{z-2}} \right) |\psi|^2 + O(b|\psi|^4) \right]$$

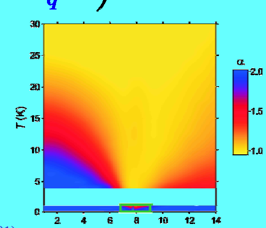
For the Gaussian fixed point this is the inverse scattering matrix element.

$$D_{GFP}(q,\omega) \sim \frac{1}{(x - x)_c + q^2 + i\frac{\omega}{q^{z-2}}}$$

Example: $Sr_3Ru_2O_7$

$$\rho = \rho_0 + AT^\alpha$$

S.A.Grigera, R.S.Perry, A.J.Schofield, M.Chiao, S.R.Julian, G.G.Lonzarich, S.I.Ikeda, Y.Maeno, A.J.Millis, A.P.Mackenzie, Science, **294**, 329 (2001).



Quantum criticality – a theory in crisis?

See P. Coleman and A. J. Schofield, Nature **433**, 226 (2005)

Some of the puzzles…

- Quantum critical antiferromagnets show anomalous powerlaws in the resistivity too. (eg $CePd_2Si_2$). Strictly the singular scattering should only occir between parts of the Fermi surface linked by the AFM wavevector (Hlubina and Rice).
- E/T scaling is seen: $CeCu_{6-x}Au_x$ [A. Schroeder *et al.* (2000)] but the theory should be above its upper critical dimension.
- Quantum critical power-laws seen over a range of pressures in MnSi not just emanating from a point. [Doiron-Leyraud *et al.* Nature 425, 595 (2003).]

Non-Fermi liquids: Summary

- Growing experimental evidence of metals that are not Fermi liquids:
  - High Tc cuprate metals,
  - UBe_{13} and other heavy fermion systems.
  - Various low dimensional organics.
- The Luttinger liquid state in one dimension:
  - New excitations – spinons and holons, characterized by 4 parameters.
  - Challenge to see this unambigiously in experiment…
- Quantum critical metals:
  - Many experimental examples, showing unusual metallic, superconducting (and other) transitions.
  - A theory in crisis?