Fermi and non-Fermi liquids: 1

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Fermi liquid theory

- The Fermi liquid as a quantum protectorate, adiabatic continuity.
- The energy functional, self-consistency, zero-sound.
- The electron and the quasiparticle.

Beyond the Fermi liquid

- Luttinger liquids:
  - Special features of 1D, spin-charge separation.
- Quantum criticality:
  - The Reizer singularity, metals on the border of magnetism.

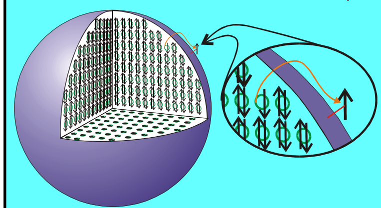
References:

A. J. Schofield, "Non-Fermi liquids", Cont. Phys. **40**, 95 (1999);
C. M. Varma *et al.* "Singular Fermi liquids", Phys. Rep. **361**, 267 (2002).

The surprising success of the free electron picture

Simple picture of a metal:
(ignore the interactions)

	Specific heat	Magnetisation
classical	$\sim N_e \frac{3}{2} k_B$	$\sim N_e \left(\frac{\mu_B B}{k_B T}\right)\mu_B$
free electrons	$\sim N_e \left(\frac{k_B T}{\epsilon_F}\right) k_B$	$\sim N_e \left(\frac{\mu_B B}{\epsilon_F}\right)\mu_B$



Why does this work so well for real metals?

This perplexed many of the founders of condensed matter physics like Sommerfeld.

Are the interactions small compared to the kinetic energy?

Define the mean electron separation, r_s, from the density, ρ_e: $\rho_e \frac{4}{3}\pi r_s^3 = 1$

Then the average Coulomb energy per electron is: $\langle \mathrm{P.E.}\rangle \sim \frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r_s}$

and the average kinetic energy per electron is: $\langle \mathrm{K.E.}\rangle \sim \frac{\hbar^2}{2m}\frac{1}{r_s^2}$

$$\frac{\langle \mathrm{P.E.}\rangle}{\langle \mathrm{K.E.}\rangle} \sim \frac{e^2}{8\pi\epsilon_0 r_s}\frac{2mr_s^2}{\hbar^2} = \frac{me^2}{4\pi\epsilon_0\hbar^2} r_s = \left(\frac{r_s}{a_0}\right)$$

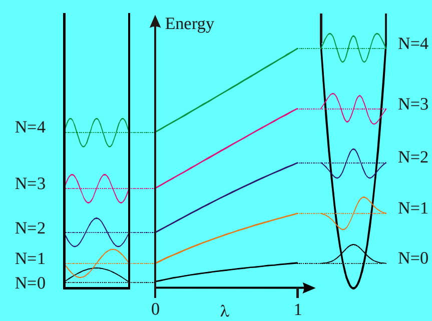
The ratio of potential to kinetic energy is of order the electron-electron separation measured in units of the Bohr radius, a_0.

Typical elemental values range from Be (1.9) to Cs (5.6) so *never* small!

Aside: the Wigner crystal

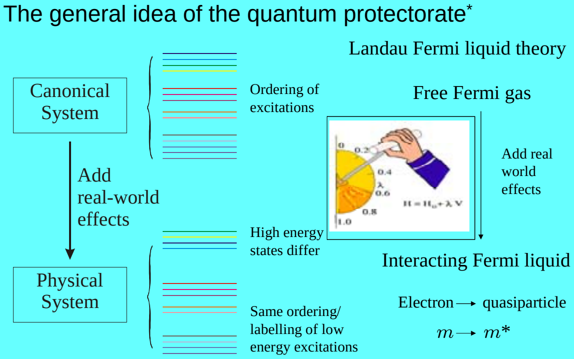
For sufficiently low densities it will become better to ignore the kinetic energy and minimize the potential energy by forming an ordered array of electrons. This Wigner crystal state is thought to occur for $r_s/a_0 > 20$(?).

Adiabatic continuity: "labels more robust than wavefunctions"



$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x) \quad V(x) = \begin{cases} \frac{1}{2}\lambda x^2, & |x| \le a; \\ \infty, & |x| > a. \end{cases}$$

The general idea of the quantum protectorate*



*R.B. Laughlin & D. Pines, PNAS, **97**, 28 (2001)

Fermi liquid theory (Landau 1956,1957):

- Entropy unchanged – since depends only on labels.

$$S = -k_B \sum_i p_i \ln p_i = -k_B \sum_{\vec k,\sigma} n_{\vec k\sigma}\ln n_{\vec k\sigma} + (1-n_{\vec k\sigma})\ln(1-n_{\vec k\sigma})$$

$$\Longrightarrow n_F(\epsilon^{qp}_{\vec k\sigma}) = \frac{1}{e^{(\epsilon^{qp}_{\vec k\sigma}-\epsilon_F)/k_BT}+1}$$

Comes from maximizing the entropy subject to a fixed energy and particle number

  - So there is a quasiparticle Fermi surface.

- Energy new form – but labelled by same quantum numbers

$$\epsilon_{\vec k\sigma} = \frac{\hbar^2 k_F}{m}(k-k_F) \longrightarrow \epsilon^{qp}_{\vec k\sigma} = \frac{\hbar^2 k_F}{m^*}(k-k_F) + \frac{1}{V}\sum_{\vec k'\sigma'} f(\vec k\sigma;\vec k'\sigma')\delta n_{\vec k'\sigma'}$$

  - energy of a single quasiparticle modified: $m \to m^*$.
  - also depends on the other quasiparticles: $\delta n_{k\sigma} = n_{k\sigma} - n_F(\epsilon_k)$ e.g. in a rotationally invariant system (^{3}He)

$$\frac{1}{V}\sum_{\vec k'\sigma'}(f_l^s + \vec\sigma\cdot\vec\sigma' f_l^a)\,\mathrm{P}_l\left(\vec k\cdot\vec k'/k_F^2\right)\delta n_{\vec k'\sigma'}$$

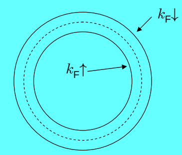
Example 1: specific heat of a Fermi liquid

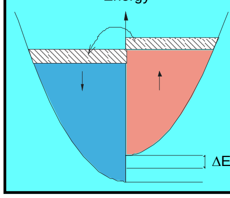
- Specific heat: quasiparticles always in thermal equilibrium so no δn=0.

Density of states: $N(0) = \dfrac{m^* k_F}{\pi^2 \hbar^2}$ so $c_v = \dfrac{m^* k_F}{3\hbar^2} k_B^2 T$

i.e. same as non-interacting result except for an effective mass.

Example 2: Pauli susceptibility





$$M = -\mu_B \sum_{\vec{k}} (\delta n_{\vec{k}\uparrow} - \delta n_{\vec{k}\downarrow})$$

$$0 = \sum_{\vec{k}} (\delta n_{\vec{k}\uparrow} + \delta n_{\vec{k}\downarrow})$$

$$\Rightarrow \sum_{\vec{k}} \delta n_{\vec{k}\uparrow} = -\sum_{\vec{k}} \delta n_{\vec{k}\downarrow} = -\frac{M}{2\mu_B}$$

$$M = \mu_B N(0) \Delta E$$

$$\Delta E = \mu_B B + \sum_{\vec{k}} (f_0^s + f_0^a) \delta n_{\vec{k}\uparrow} + \sum_{\vec{k}} (f_0^s - f_0^a) \delta n_{\vec{k}\downarrow}$$

$$= \mu_B B - \frac{M f_0^a}{\mu_B}$$

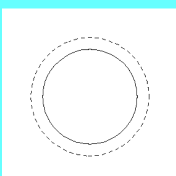
$$\Rightarrow M = \frac{\mu_B^2 N(0)}{1 + N(0) f_0^a} B = \frac{\mu_B^2 N(0)}{1 + F_0^a} B$$

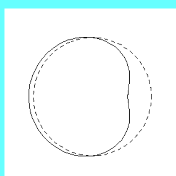
$$\Rightarrow \chi = \frac{M}{B} = \frac{1}{1 + F_0^a} \frac{m^*}{m} \chi_0$$

So Pauli susceptibility enhanced by a Landau parameter, F_0^a. Stoner ferromagnetism if F_0^a=-1

New collective modes: zero sound

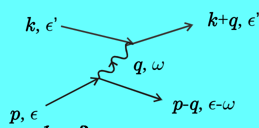
- Ordinary (first) sound
- Zero sound

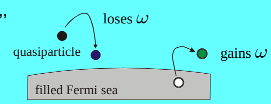




Why is the Fermi liquid "protected" from real world effects?

Landau: Fermi surface self-consistently protects by reducing scattering phase space





$$\frac{1}{\tau_\epsilon} = \frac{2\pi}{\hbar} \sum_f |V_{ij}|^2 \delta(\epsilon - \epsilon_f)$$ Fermi's golden rule

$$\frac{1}{\tau_\epsilon} \sim \frac{2\pi}{\hbar} |V|^2 \int_0^\epsilon N(0) d\omega \int_0^\omega N(0) d\epsilon' \int_{-\infty}^{+\infty} N(0) \delta(\epsilon - \omega - \epsilon' + \epsilon'') d\epsilon''$$

$$\sim \frac{\pi}{\hbar} |V|^2 N(0)^3 \epsilon^2$$ Scattering rate vanishes near Fermi surface

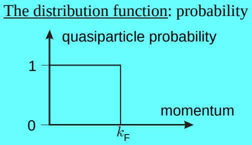
At finite temperatures

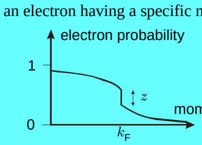
$$\frac{1}{\tau_\epsilon} \sim \frac{\pi}{\hbar} |V|^2 N(0)^3 \left[\epsilon^2 + (\pi k_B T)^2\right] \longrightarrow \rho \sim T^2$$

Connection between the quasiparticle and the electron?

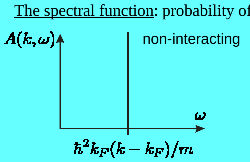
$$|\psi^{qp}_{k,\sigma}\rangle = \sqrt{z} |\psi^{el}_{k,\sigma}\rangle +$$ Particle-hole excitations and disturbance of the medium

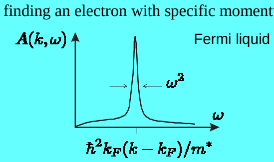
The distribution function: probability of an electron having a specific momentum



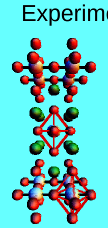


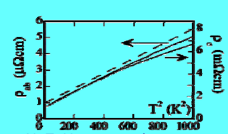
The spectral function: probability of finding an electron with specific momentum





Experimental examples: Sr_2RuO_4





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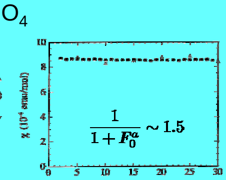
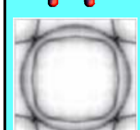
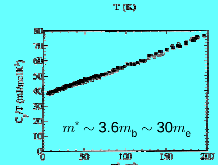




A. Damascelli *et al.*, Phys. Rev. Lett., **85**, 5194 (2000).

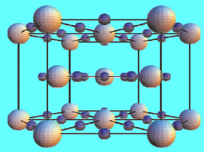
C. Bergemann *et al.*, Adv. Phys. 52, 639 (2004).



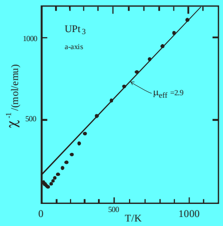
A.P. Mackenzie et al, J. Phys. Soc. Jpn. **67**, 385 (1998).

Experimental example: UPt_3

A heavy Fermi liquid: U ~ $5f^3$



High temp: f electrons bound to form a local moment via Hund's rule (~ 1eV):



Frings *et al.*,
JMMM 31-34, 240 (1983).

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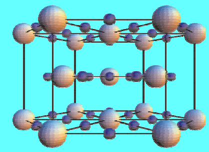
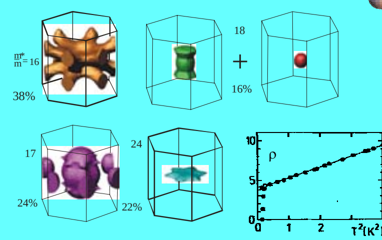
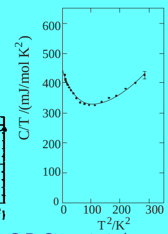


S.R. Julian and G. McMullan, unpublished (1998)





Low temp: f electrons delocalize to make up a heavy Fermi liquid

G.R. Stewart *et al.*,
PRL, 52, 679 (1984)

Fermi liquid theory - summary

- Adiabatic continuity with the free electron gas:
  - electron → quasiparticle,
  - modified properties: effective mass and Landau parameters.
  - Fermi surface (of quasiparticles) provides self-consistency.
- Experimental probes
  - Thermodynamics probe quasiparticles: specific heat, susceptibility, transport, quantum oscillations.
  - ARPES and tunnelling probe electron overlap with the quasiparticle excitations.