

Semiconductor few-electron quantum dots as spin qubits for quantum computing

“Delft Spin Qubit Team”

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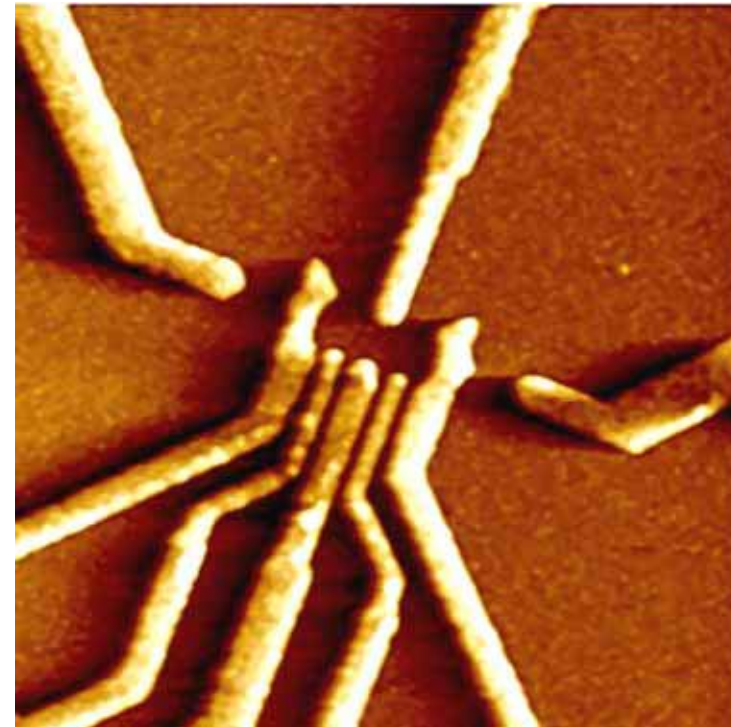
Collaborators

Tarucha group (Tokyo)

Loss group (Basel)

Fujisawa, Hirayama (NTT)

 TU Delft



NTT 

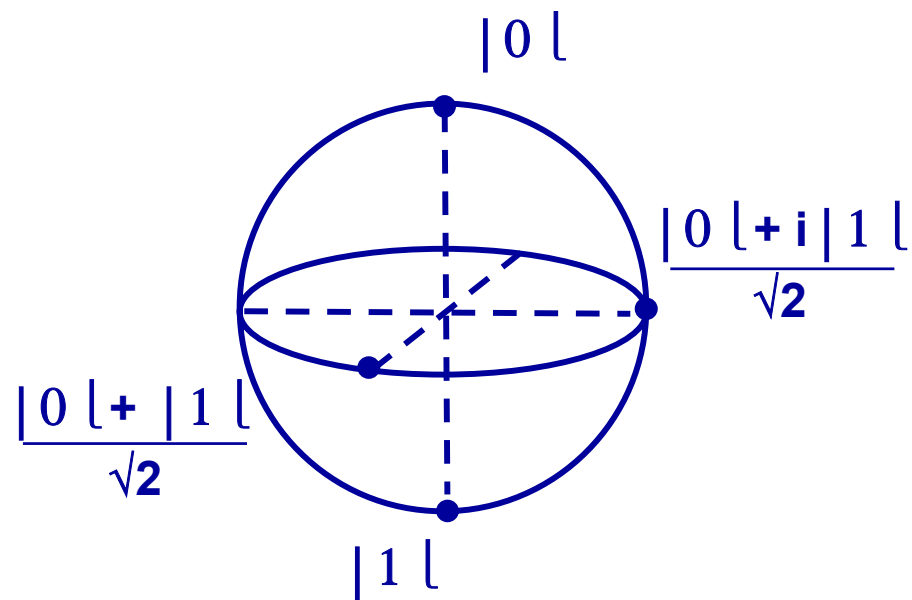


Outline

- I. Introduction to quantum computing**
- II. Quantum dots and spin**
- III. One-electron quantum dots as spin qubits**
- IV. Outlook**

Part I.

Introduction to quantum computing



“Hard” problems

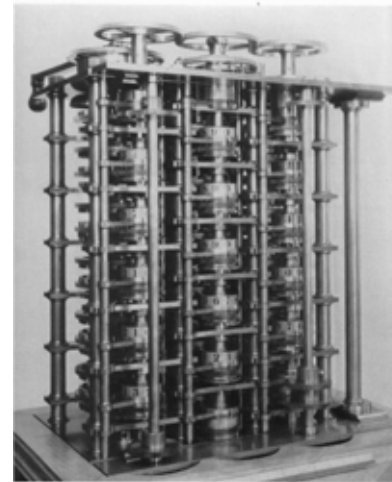
$$\begin{aligned} 15 &= 3 \times 5 \\ 91 &= \dots \times \dots ? \\ 437 &= \dots \times \dots ? \\ &???$$

No classical algorithm is known to *efficiently* factor integers



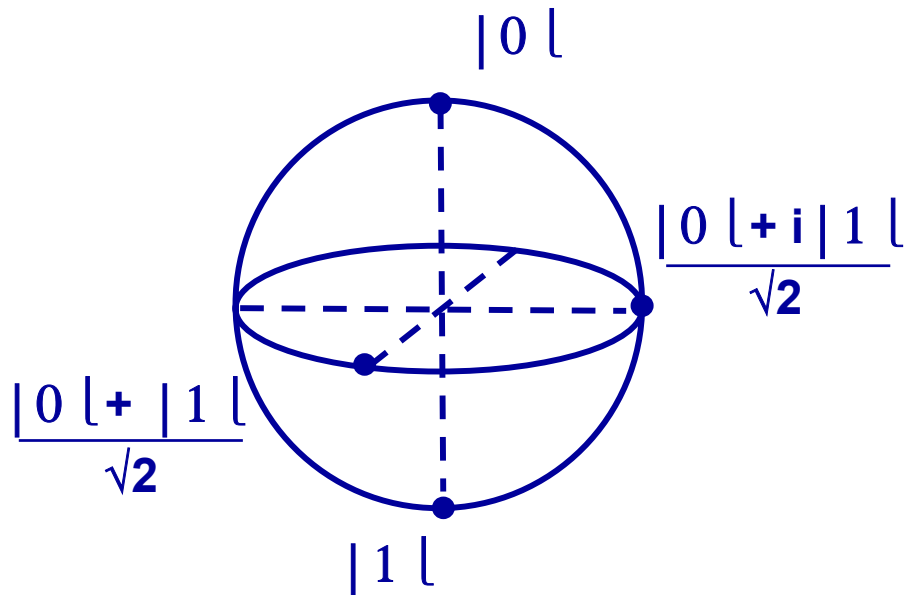
factoring takes exponential effort

“Hard” problems are beyond the reach of any machine relying on the classical laws of physics



Courtesy IBM Corporation

Complexity of Quantum Systems



n coupled quantum bits

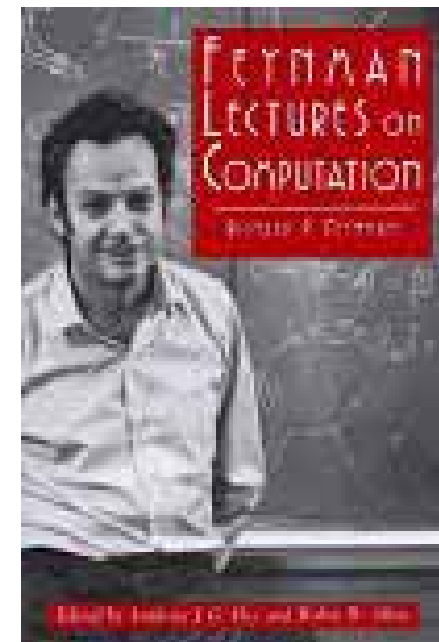


~~n spheres~~

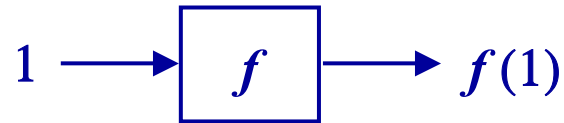
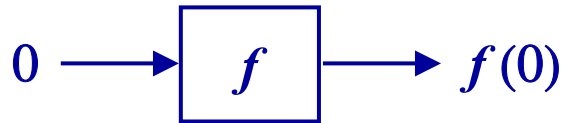
2^n degrees of freedom!

Could a quantum computer *efficiently* simulate quantum systems ?

Could it be used to solve hard problems ?

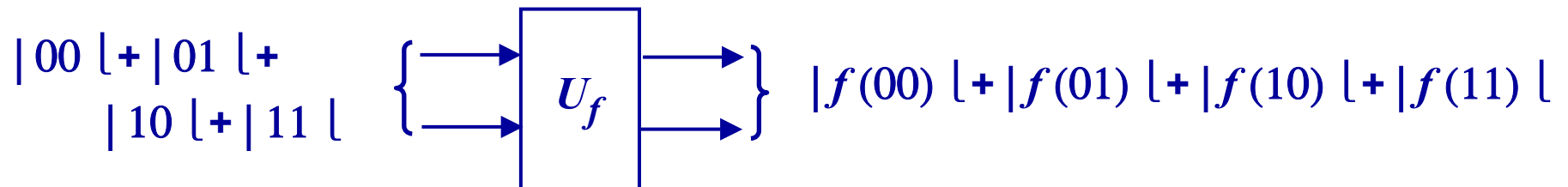
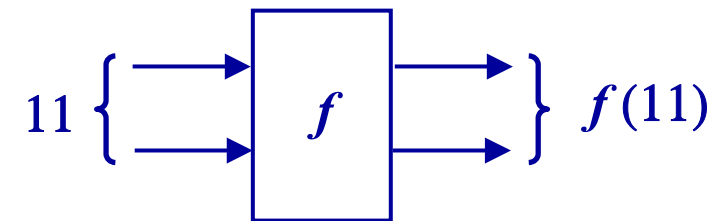
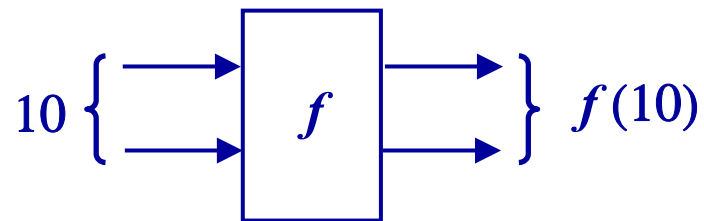
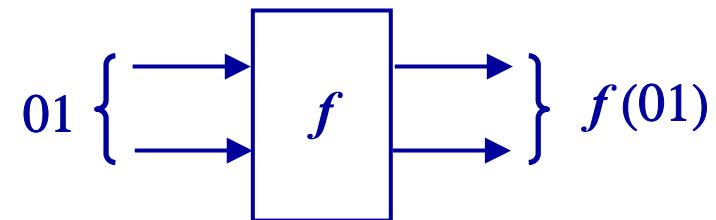
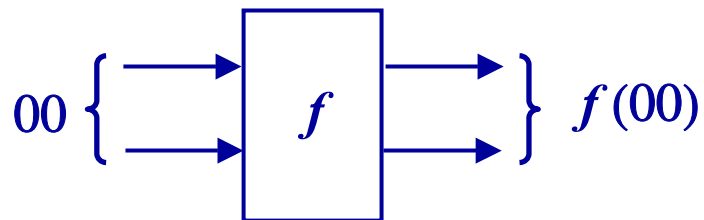


Quantum Parallelism



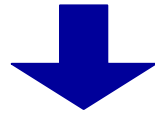
Computational power:
 classically $\propto n$
 quantum $\propto 2^n$

D. Deutsch, 1985



Quantum algorithms

Measurement of $|f(0)\rangle + |f(1)\rangle$ gives either $f(0)$ or $f(1)$



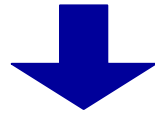
The exponential power appears inaccessible ...

Nevertheless: quantum algorithms make computational speed-ups possible !

- Exponential for factoring integers (P. Shor 1994)
- Quadratic for unstructured searches (L. Grover 1996)
- Exponential for quantum simulations (S. Lloyd 1996)

Quantum error correction

Decoherence destroys quantum parallelism



The exponential power appears limited in time ...

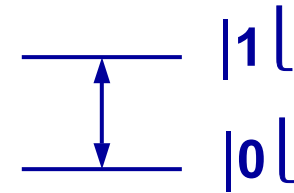
Nevertheless: quantum error correction makes arbitrarily long quantum computations possible !

- Quantum error correction (P. Shor 1996, A. Steane 1996)
- Accuracy threshold (D. Aharonov 1997, A. Kitaev 1997, ...): $\sim 10^{-4}$

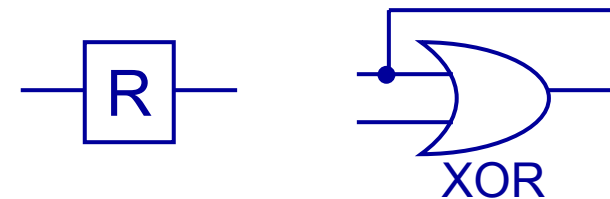
Implementation of quantum computers

D. DiVincenzo

1. Well-defined qubits



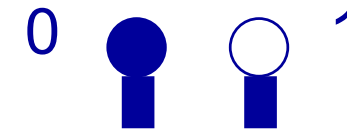
2. Universal set of quantum gates



3. Initialization to a pure state

$|00 \dots 0\rangle$

4. Qubit-specific measurement



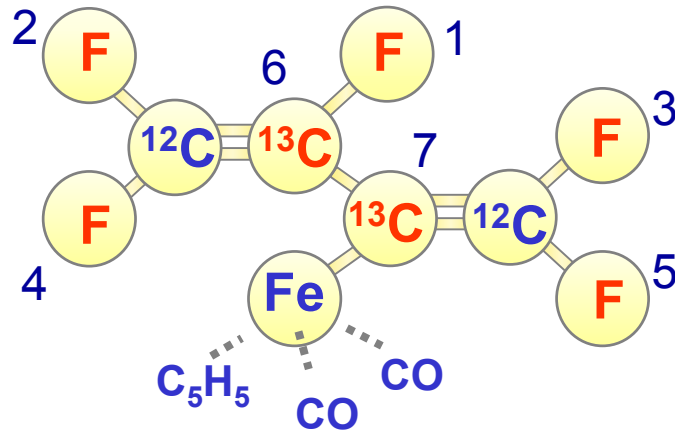
5. Long coherence times

$|0 \dots 0\rangle + |1 \dots 1\rangle$

Key challenge: combine access to qubits (initialization, control, readout) with high degree of isolation (coherence)

Factoring 15 with nuclear spins

Vandersypen et al., *Nature* 414, 883 (2001)

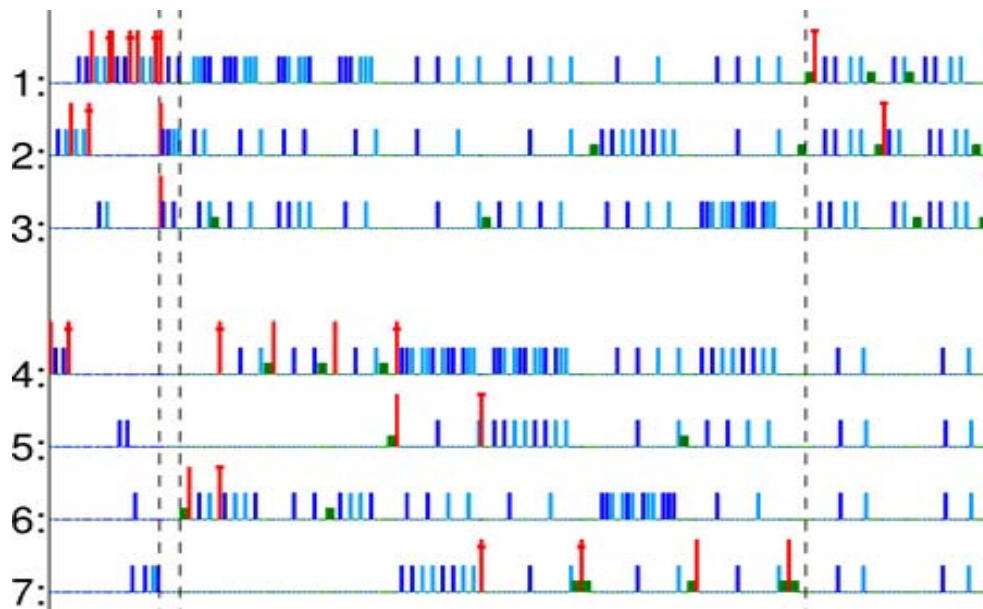


$$15 = 3 \times 5$$

Proof-of-principle of
quantum computing

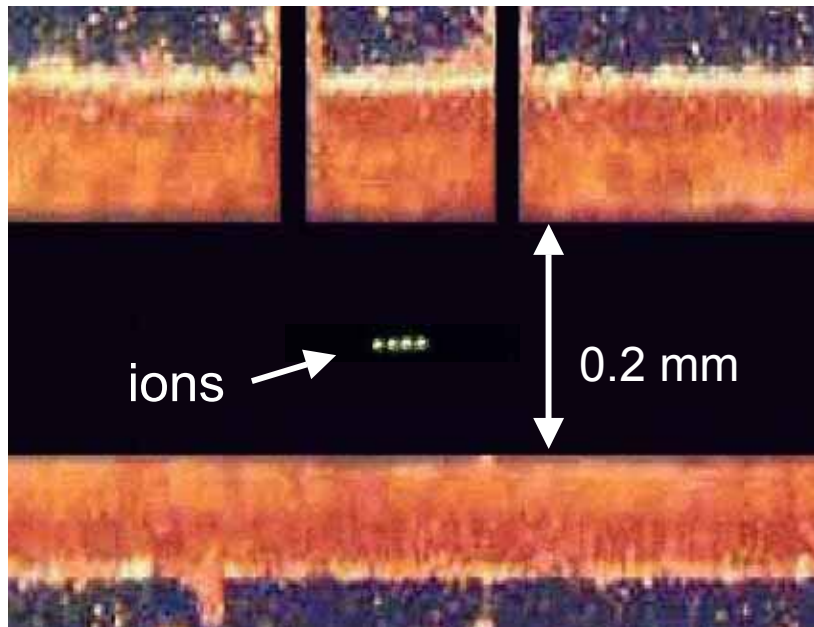
But:

No practical path for
scaling liquid NMR to
many more qubits



Other techniques

Trapped ions



Courtesy D. Wineland, NIST

Scalability is a big problem!

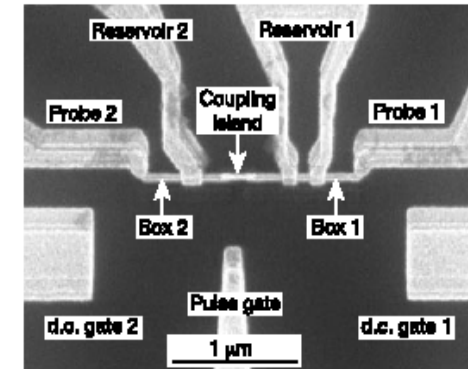
Neutral atoms in optical traps



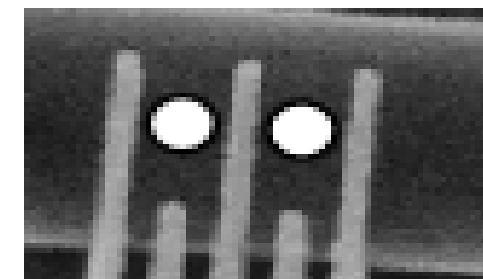
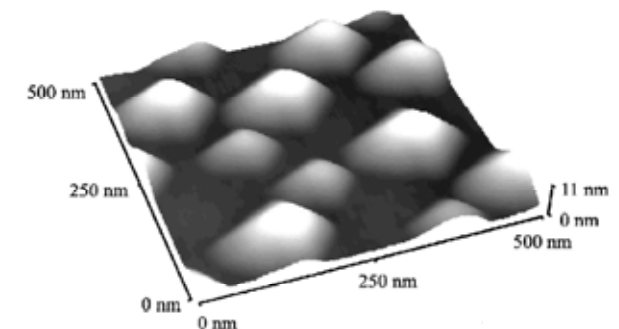
Courtesy J. Kimble, Caltech

Charge quantum bits in the solid state

- **Superconducting charge qubits**
 - One- and two-qubit operations
 - Single-shot read-out
- **Semiconductor quantum dots**
 - Excitons in self-assembled quantum dots
 - Double dot charge qubit



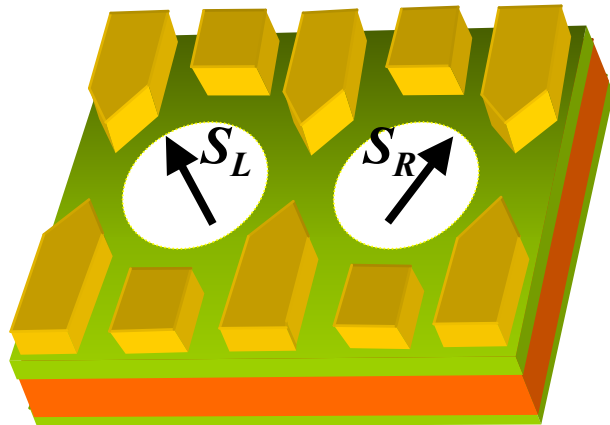
Courtesy Y. Nakamura, NEC



- Charge easy to manipulate and read out
- But coherence time only ~ ns

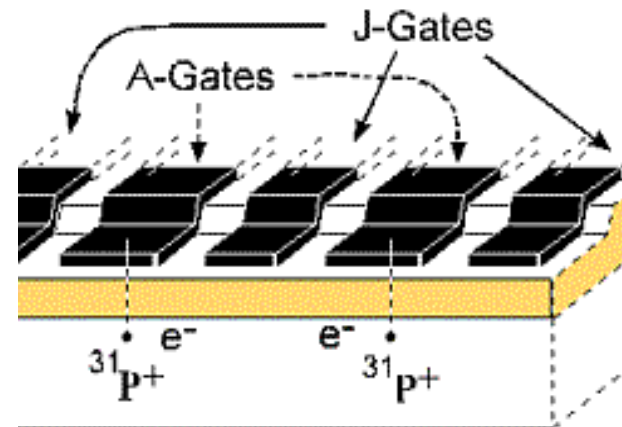
“Scalable” single-spin qubit proposals

Electron spin in gated quantum dots (electrical)



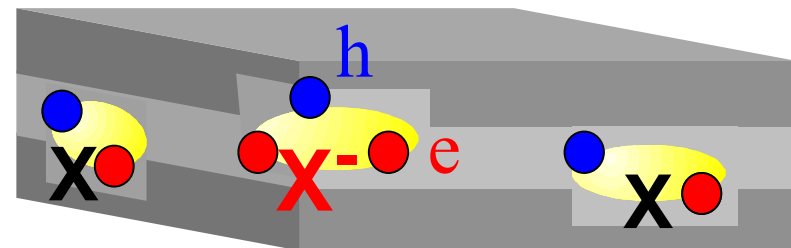
Loss & DiVincenzo, PRA 1998

Nuclear spin of P donors in Si



Kane, Nature 1998

Electron spin in self-assembled quantum dots (optical)



Gammon *et al.*
Imamoglu *et al.*, PRL 1999

Electron spin in a lateral quantum dot

Why electron spin in a lateral quantum dot?


- Electron spin is a natural 2-level system
- Well isolated from environment ♥
long coherence times expected
- Lateral quantum dots are very flexible and controllable systems, should be scalable

Of course, not only application-driven:

- Spin physics at the fundamental quantum limit: 1 spin !

Part II: Quantum dots and spin

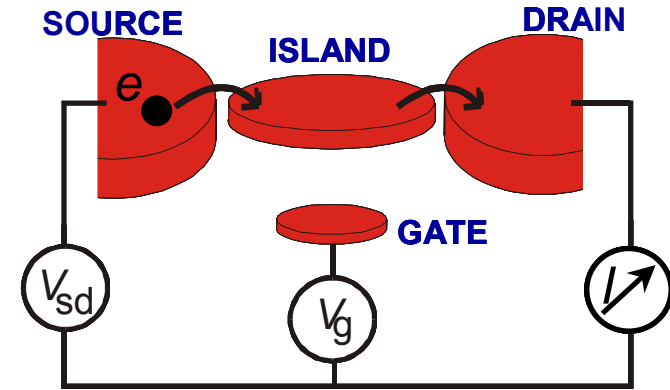
2D Artificial Atoms



1 Ta							2 Ha
3 Et	4 Au					5 Ko	6 Oo
7 Sa	8 To	9 Ho			10 Mi	11 Cr	12 Ja
13	14	15	16 Wi	17 Fr	18 El	19	20 Da

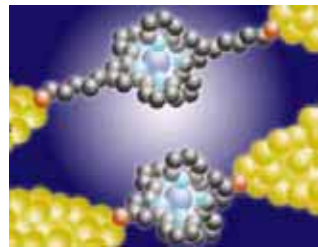
Quantum dots

- Small box occupied by electrons (holes)
- Coupled via tunnel barriers to source and drain reservoirs
- Coupled capacitively to gate electrode(s)
- Box (island) has discrete energy spectrum



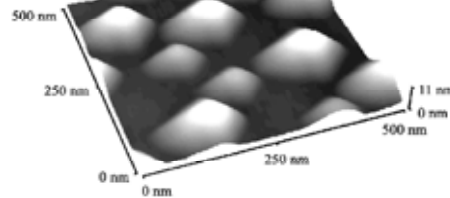
♥ **artificial atom!**

single molecule



1 nm

self-assembled QD



10 nm

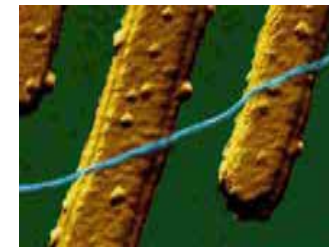
control & flexibility

lateral QD



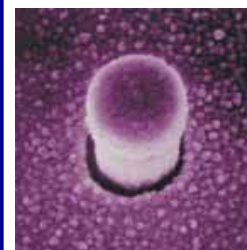
100 nm

nanotube

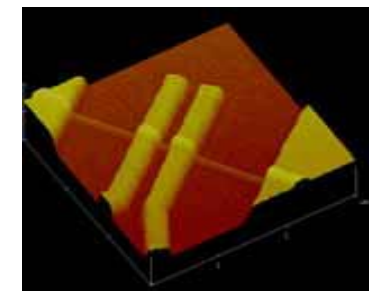


1 μm

metallic
(superconducting)
nanoparticle

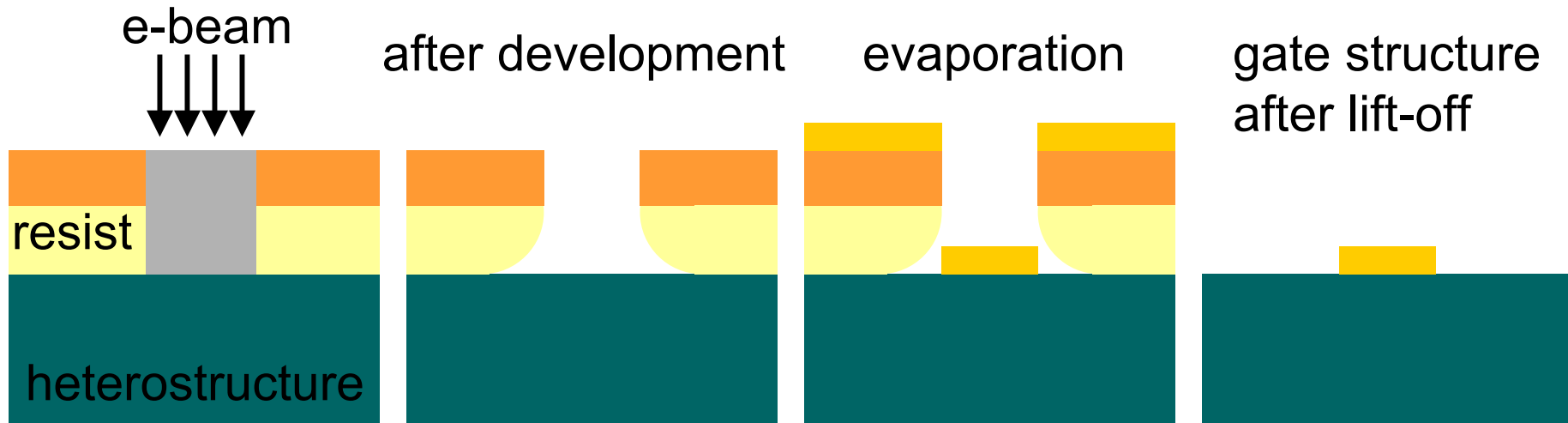
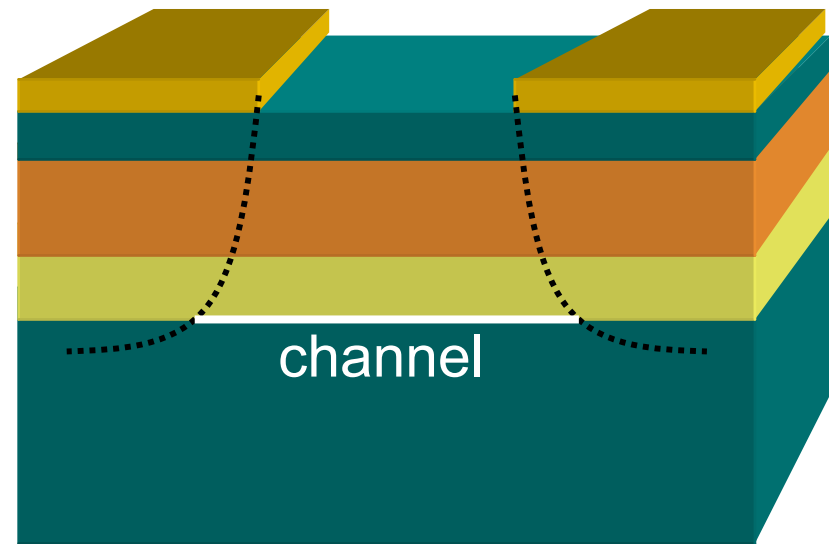


vertical QD

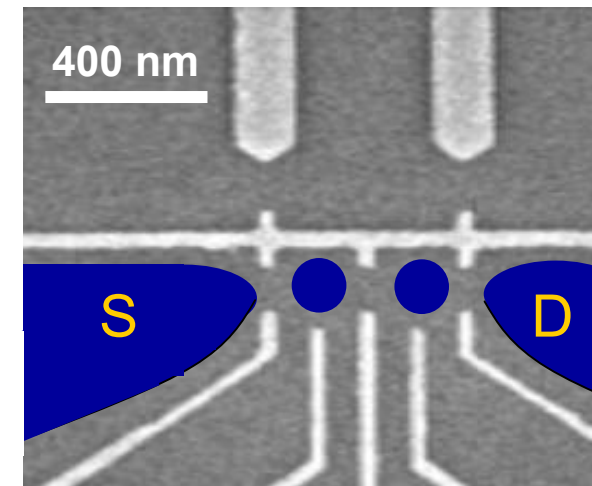
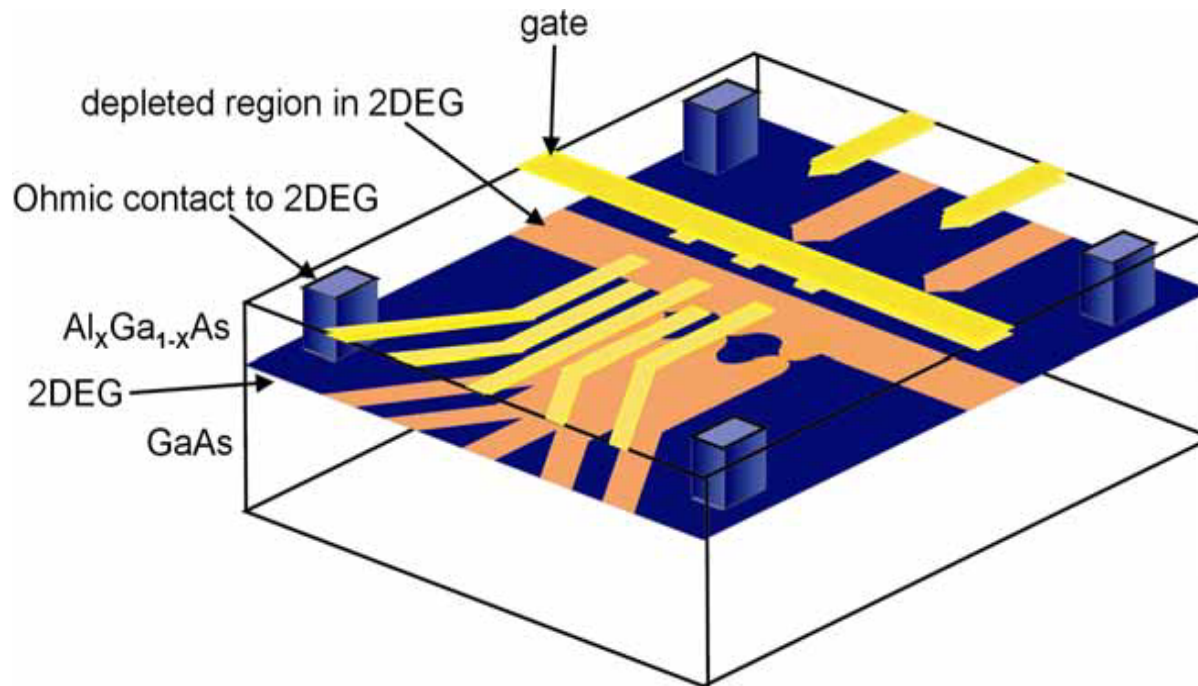


nanowire

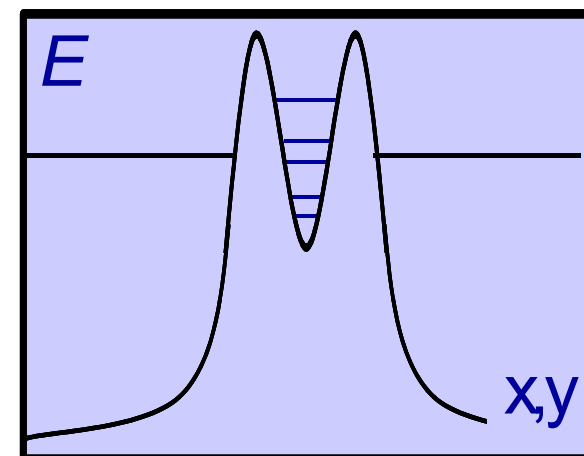
Heterostructure processing



Lateral QD fabrication

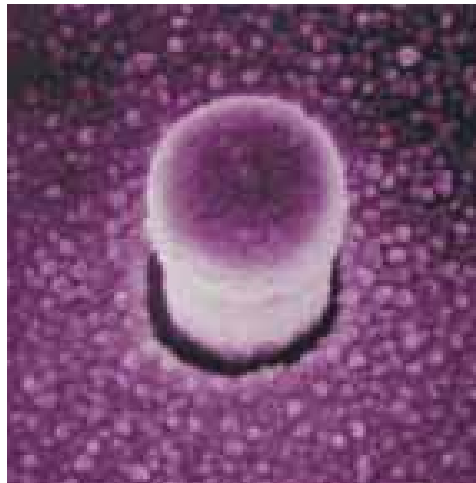


- High-mobility 2DEG ($\sim 10^6 \text{ cm}^2/\text{Vs}$)
- Density $\sim 10^{15} \text{ m}^{-2}$ ♥ $\lambda_F \sim 30 \text{ nm}$
- Resolution gate structure $\sim 20 \text{ nm}$
- Dot size $\sim 100 \text{ nm}$
- Comparable to electron wavelength ♥ **discrete energy spectrum**

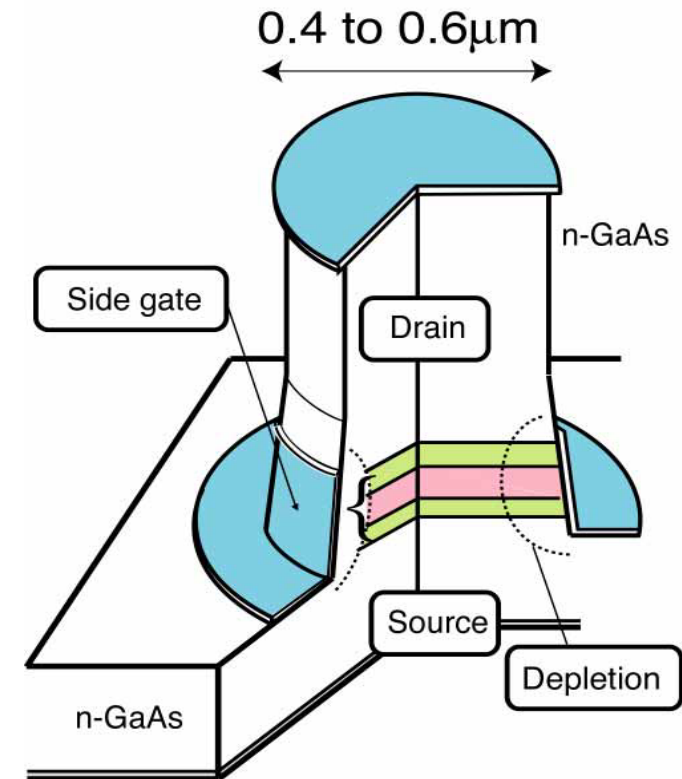
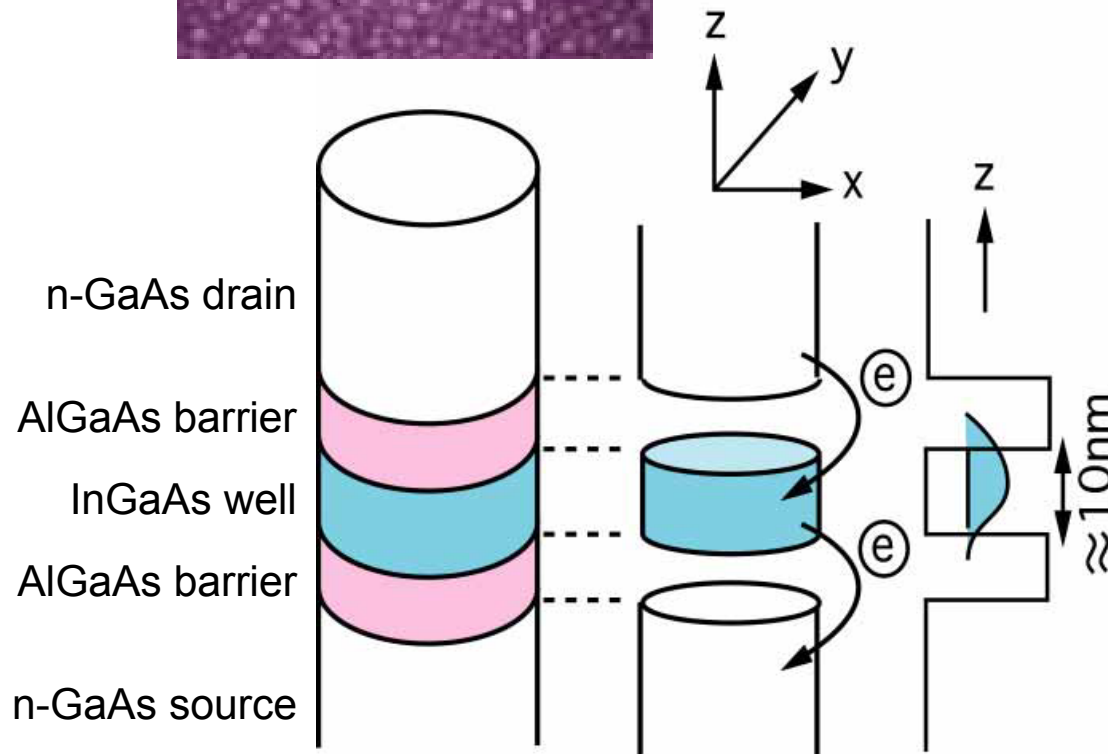


Vertical QD fabrication

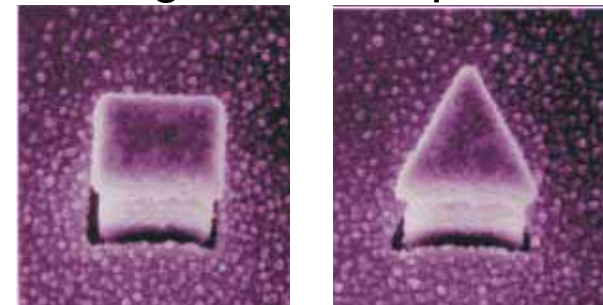
Tarucha PRL 77, 3613 ('96)



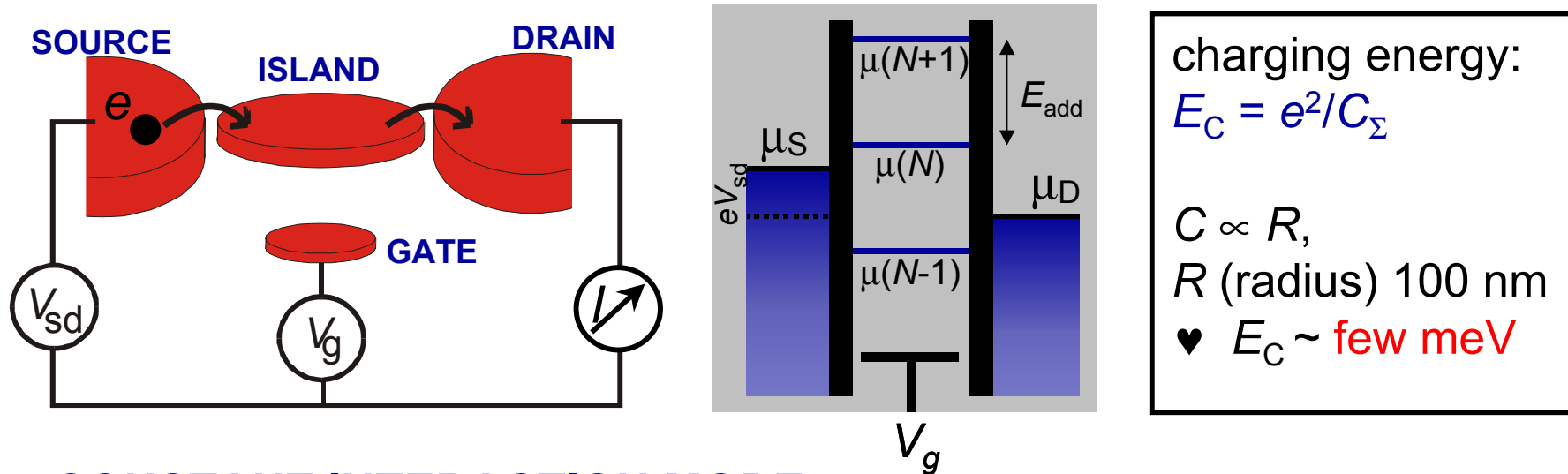
Circular pillar made from double barrier structure by dry and wet etching



Other geometries possible



Coulomb Blockade in transport



charging energy:

$$E_C = e^2/C_\Sigma$$

$$C \propto R,$$

R (radius) 100 nm

♥ $E_C \sim$ few meV

CONSTANT INTERACTION MODEL

$$U(N) = \frac{(-Ne + C_g V_g + q_0)^2}{2C_\Sigma} + E_{int}(N)$$

$$C_\Sigma = C_L + C_R + C_g$$

$$\mu(N) \equiv U(N) - U(N-1) = E_C [(N - 1/2 - q_0/e) - C_g V_g/e] + E_N \quad E_C = e^2/C_\Sigma$$

$$E_{add} \equiv \mu(N+1) - \mu(N) = E_C + \Delta E$$

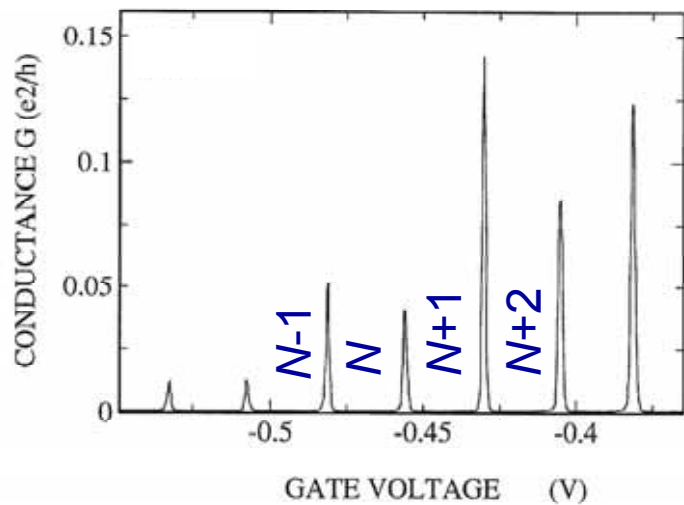
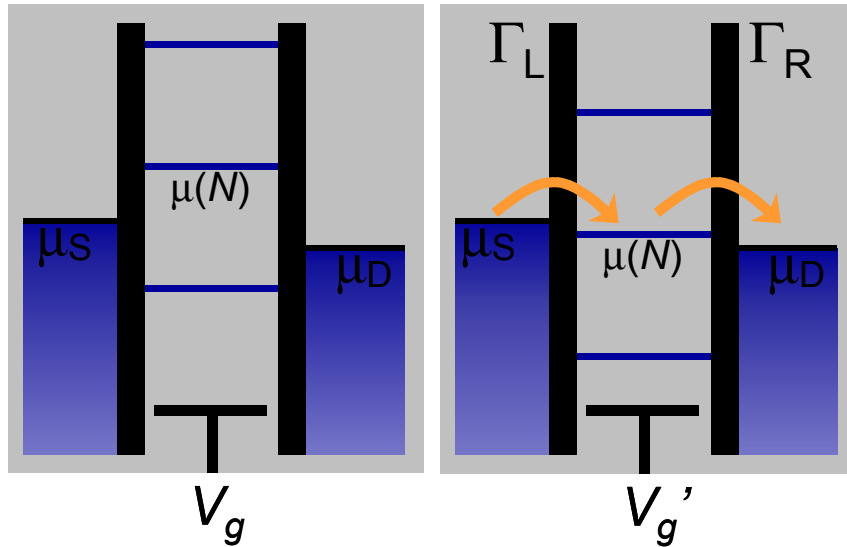
- Transport when $\mu(N) = \mu_S, \mu_D$
- Small C_Σ ♥ large addition energy
- At low T , small V_{sd} , energy not available



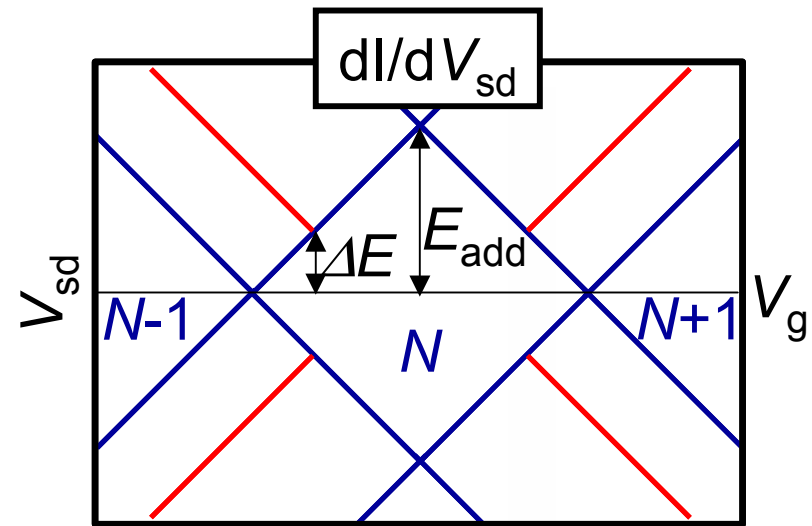
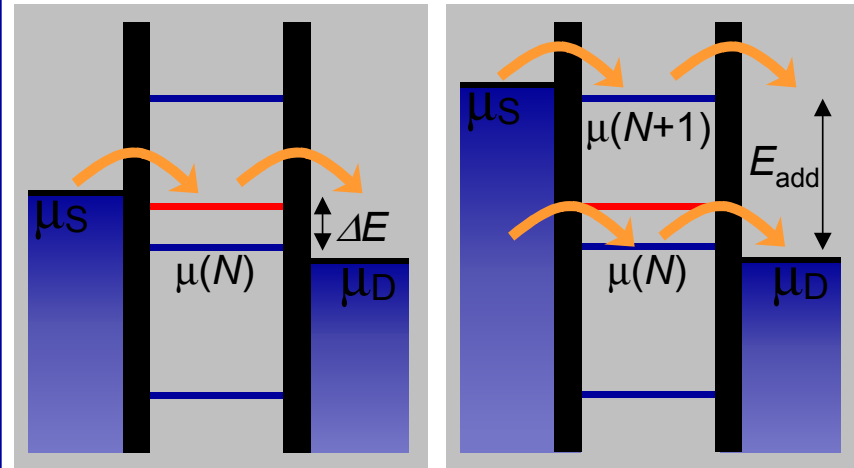
**CURRENT = 0
(COULOMB
BLOCKADE)**

Single electron tunneling

Linear response

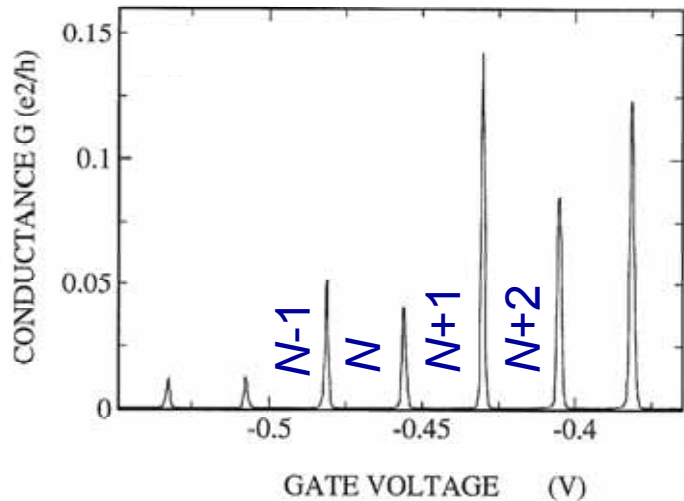


Non-linear transport

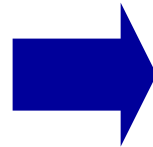
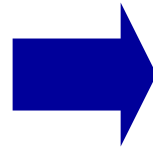


Large-bias spectroscopy!

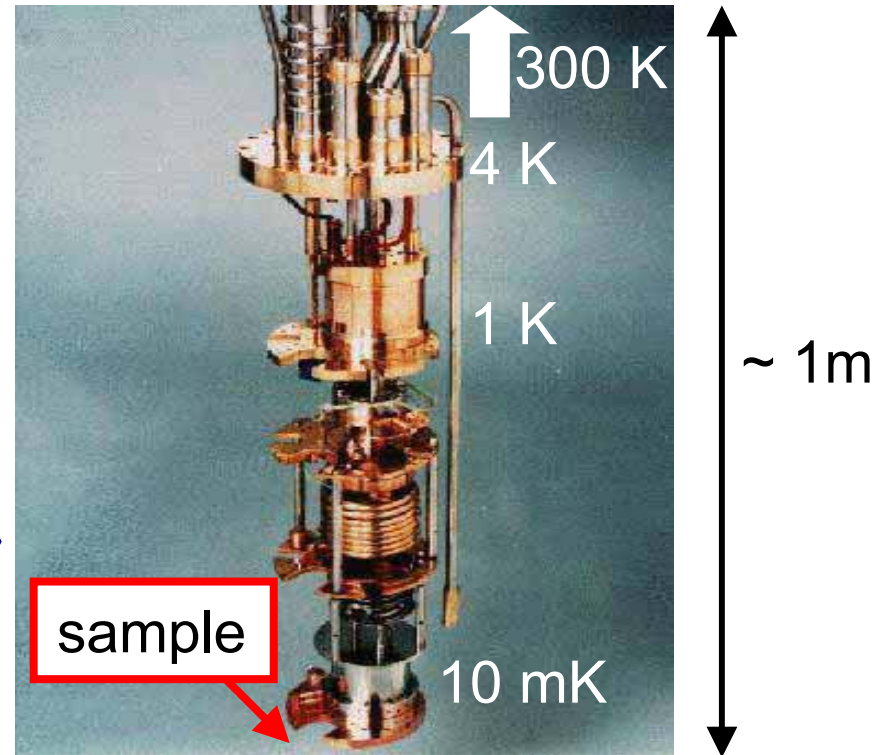
Experimental set-up



- Resonance width: lifetime ($\hbar\Gamma$), V_{sd} , temperature, noise
- Peak spacing: charging + single-particle spacing
- $E_C \sim$ few meV
- $\Delta E \sim$ few 0.1 meV
- $3 \text{ fA} / \text{Hz}^{1/2} \heartsuit 1 \text{ e}/50 \mu\text{s}$

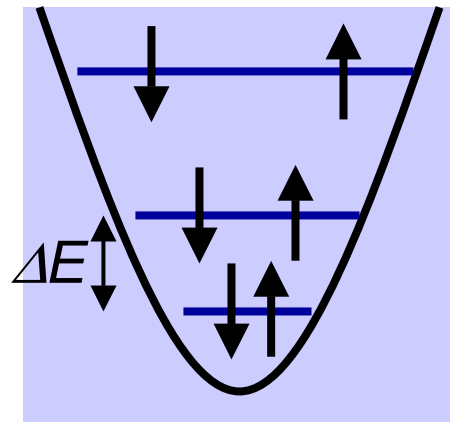


Dilution refrigerator:
10 mK (\sim few μeV)



low-noise
amplifiers
+
filtering

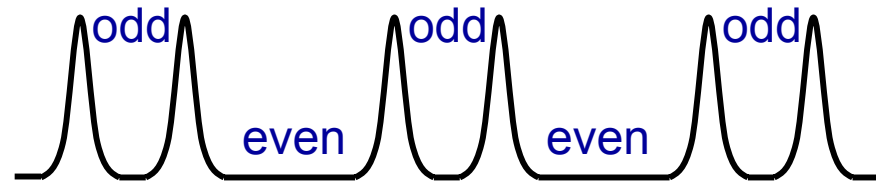
Single-particle levels & shell structure



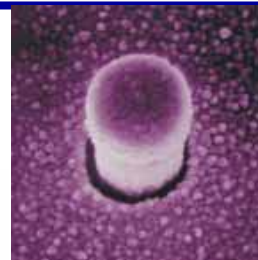
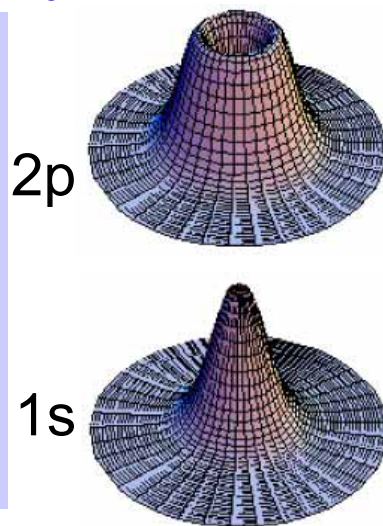
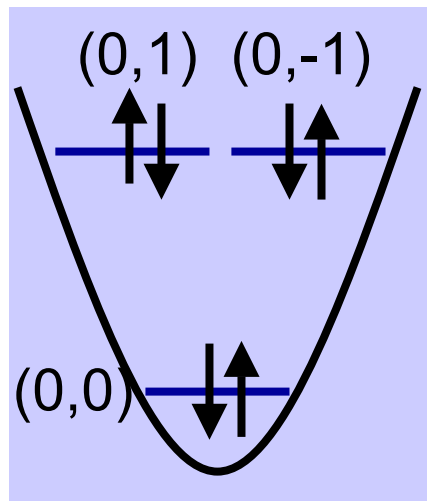
orbitals 2-fold degenerate ♥
 expect even/odd effects in peakspacing

$$U(N) = \frac{(-Ne + C_g V_g + q_0)^2}{2C_\Sigma} + E_{int}(N)$$

$$E_{add} = E_C + \Delta E$$

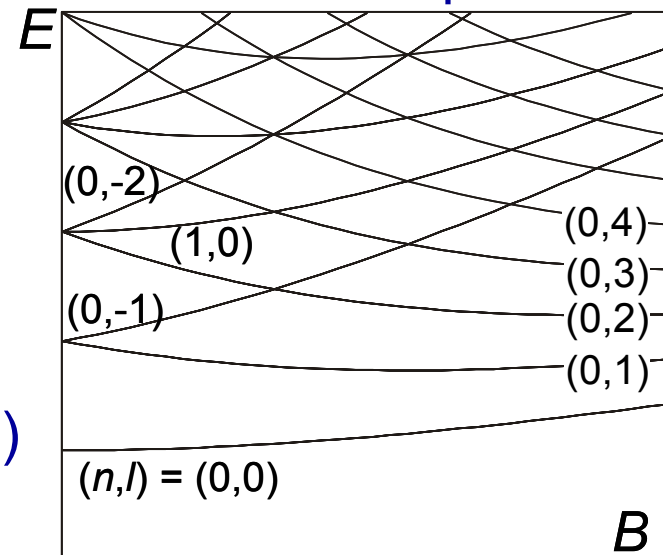


$B = 0$ T:
 $E_{nl} = (2n + |l| + 1) \hbar \omega_0$



$N=2,6,12$
 full shell
 (noble elements)

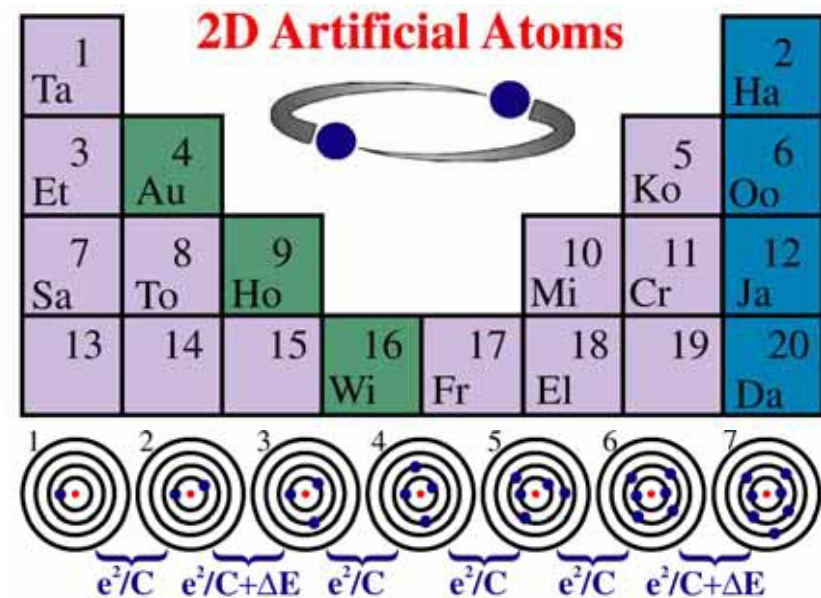
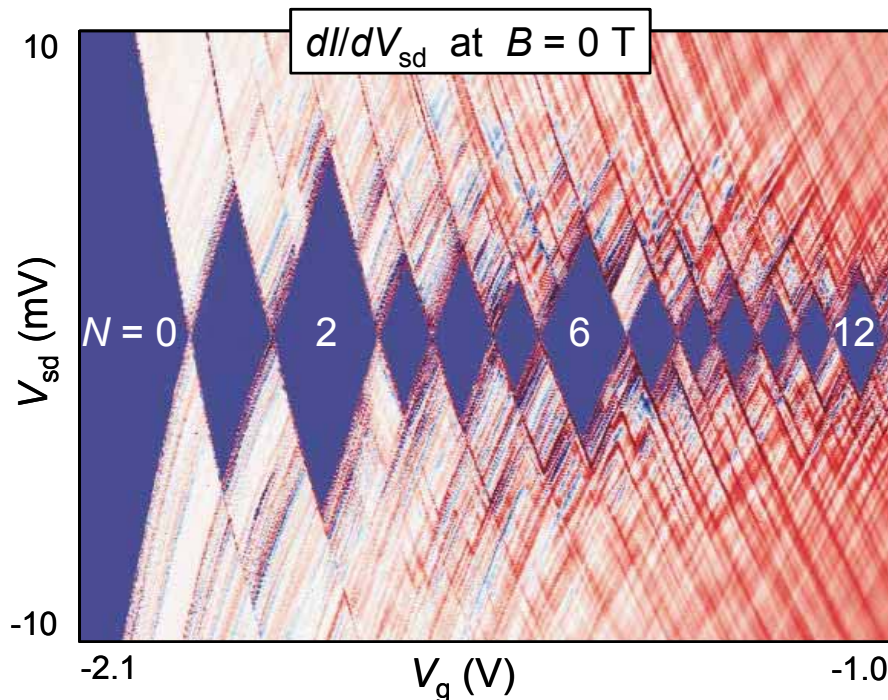
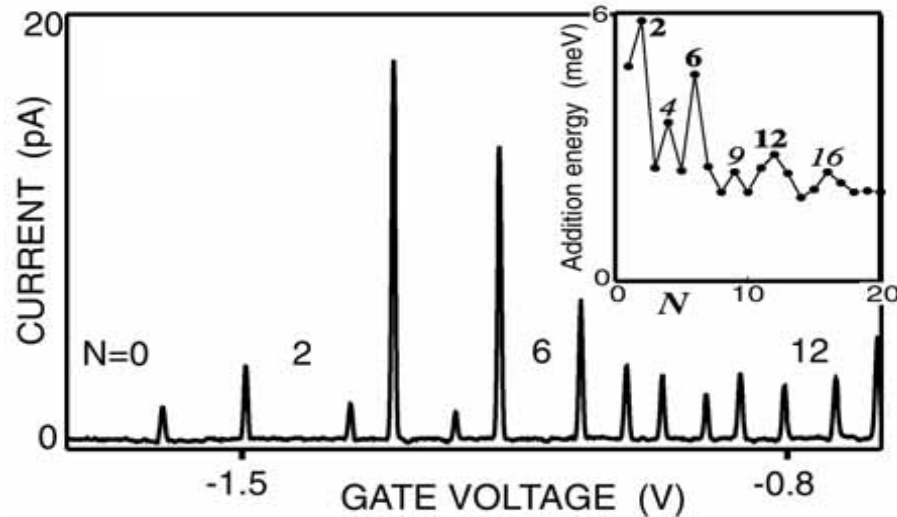
2D harmonic potential:
 Darwin-Fock spectrum



B

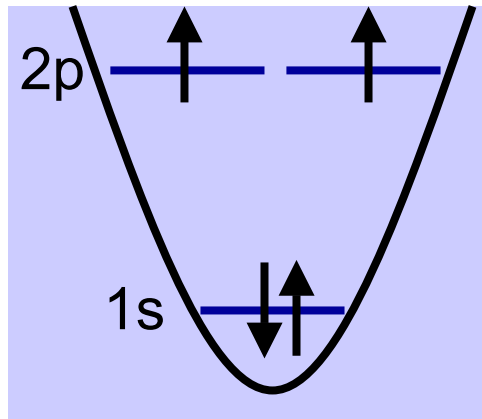
Artificial atoms

Kouwenhoven *et al.*, Science 278, 1788 ('97)

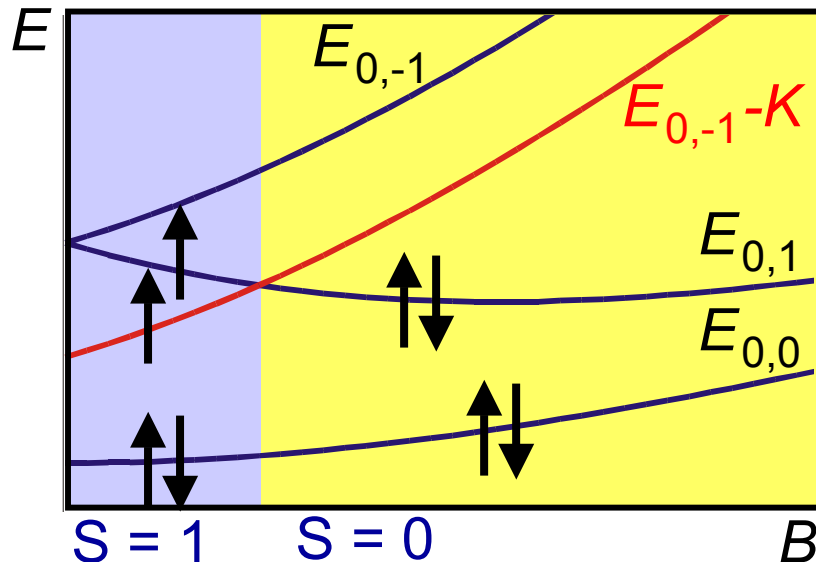
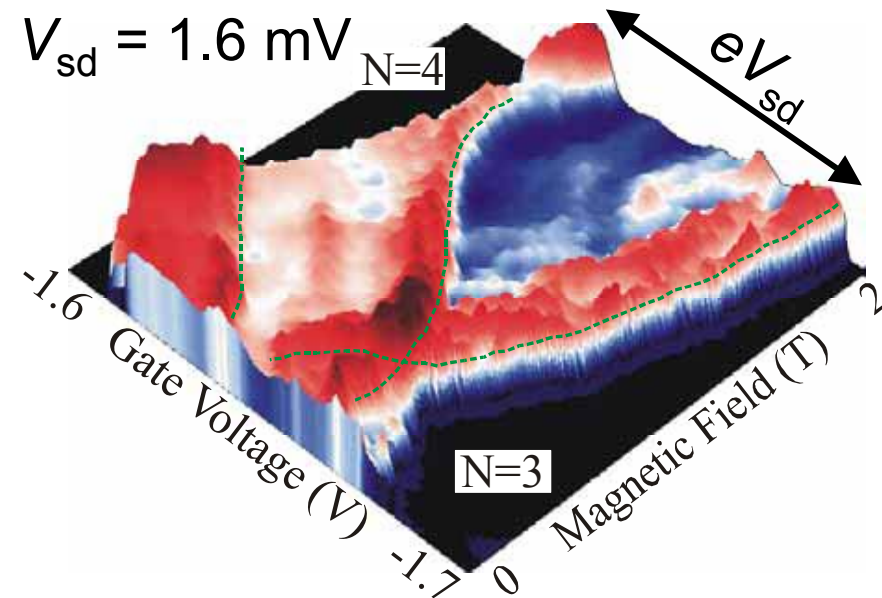


- Few-electron QDs are *artificial atoms*
- Change element by tuning V_g
- Full shell $N = 2, 6, 12, 20 \dots$ (noble elements)
- Half-full shell $N = 4, 9, 16 \dots$

Hund's rule for $N = 4$

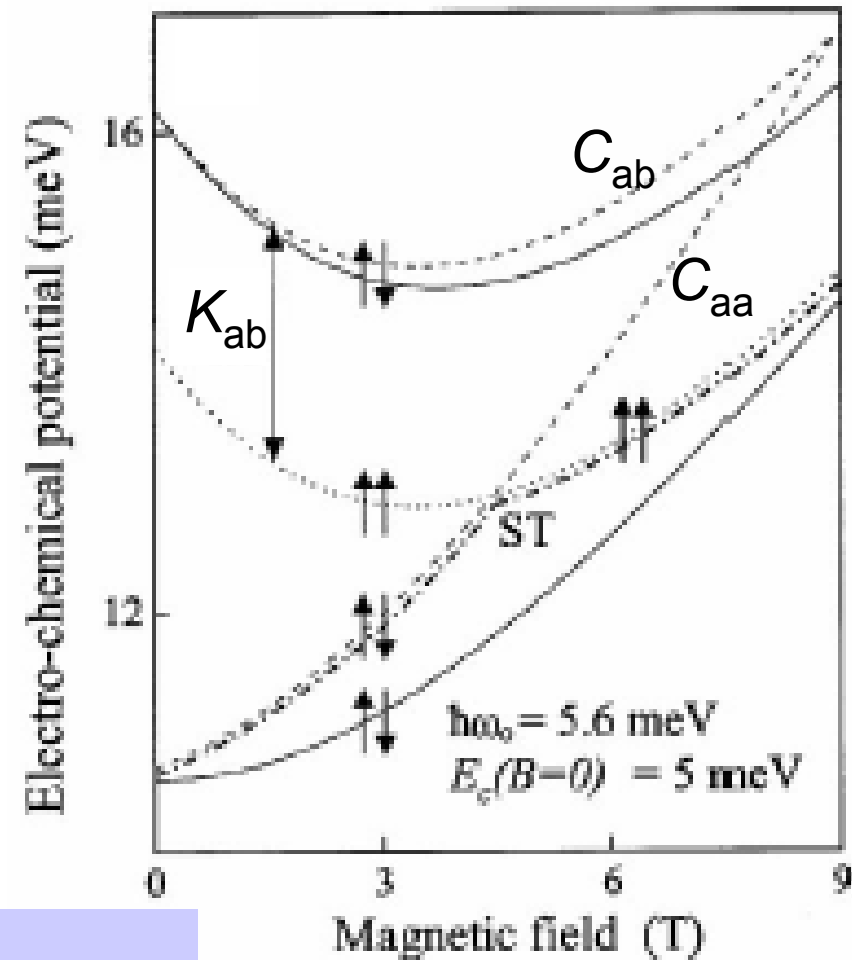
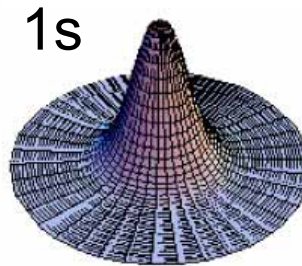
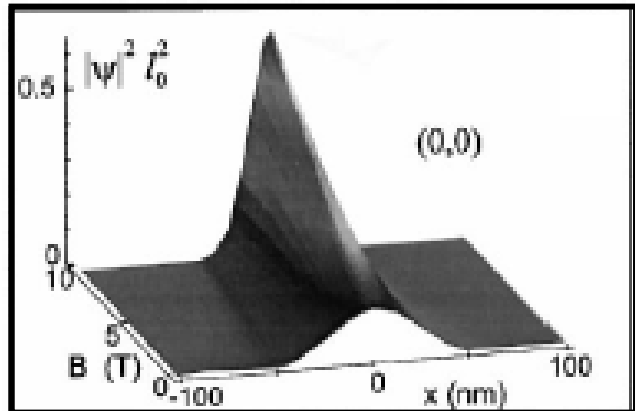
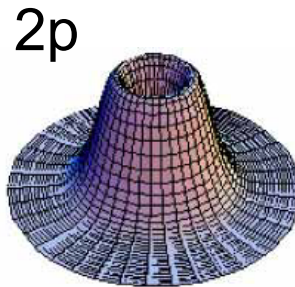
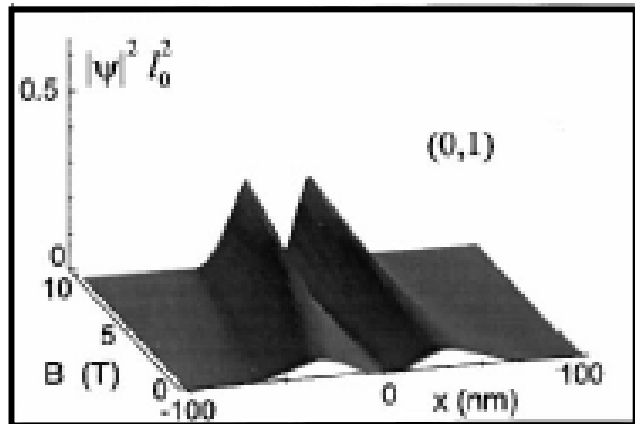


Half-filled shell for $N=4,9,16,..$
 ♥ total spin maximized (exchange)



- Transition in angular momentum (l) accompanied by spin transition
- Exchange energy ~ 0.8 meV
- Spin states well described by 2-electron states (S and T)

Singlet-triplet transition for $N = 2$

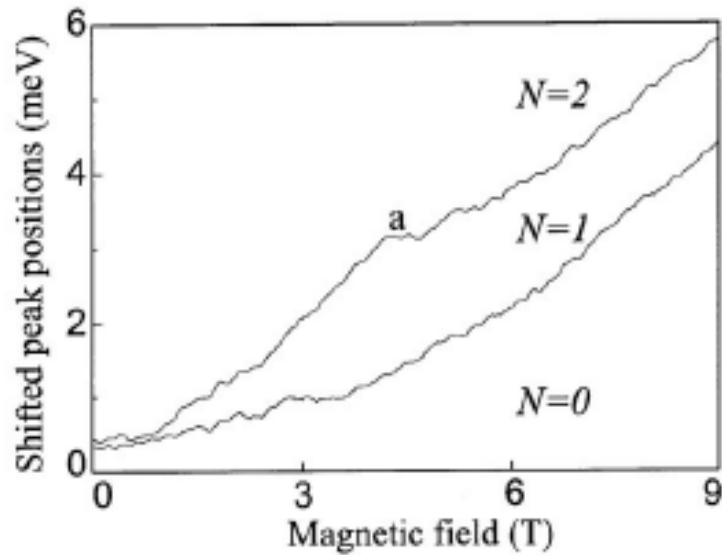


- B_{\perp} shrinks wavefunction ♥ increases Coulomb interaction
- C_{aa} grows faster than C_{ab}
- Exchange interaction favors parallel spins

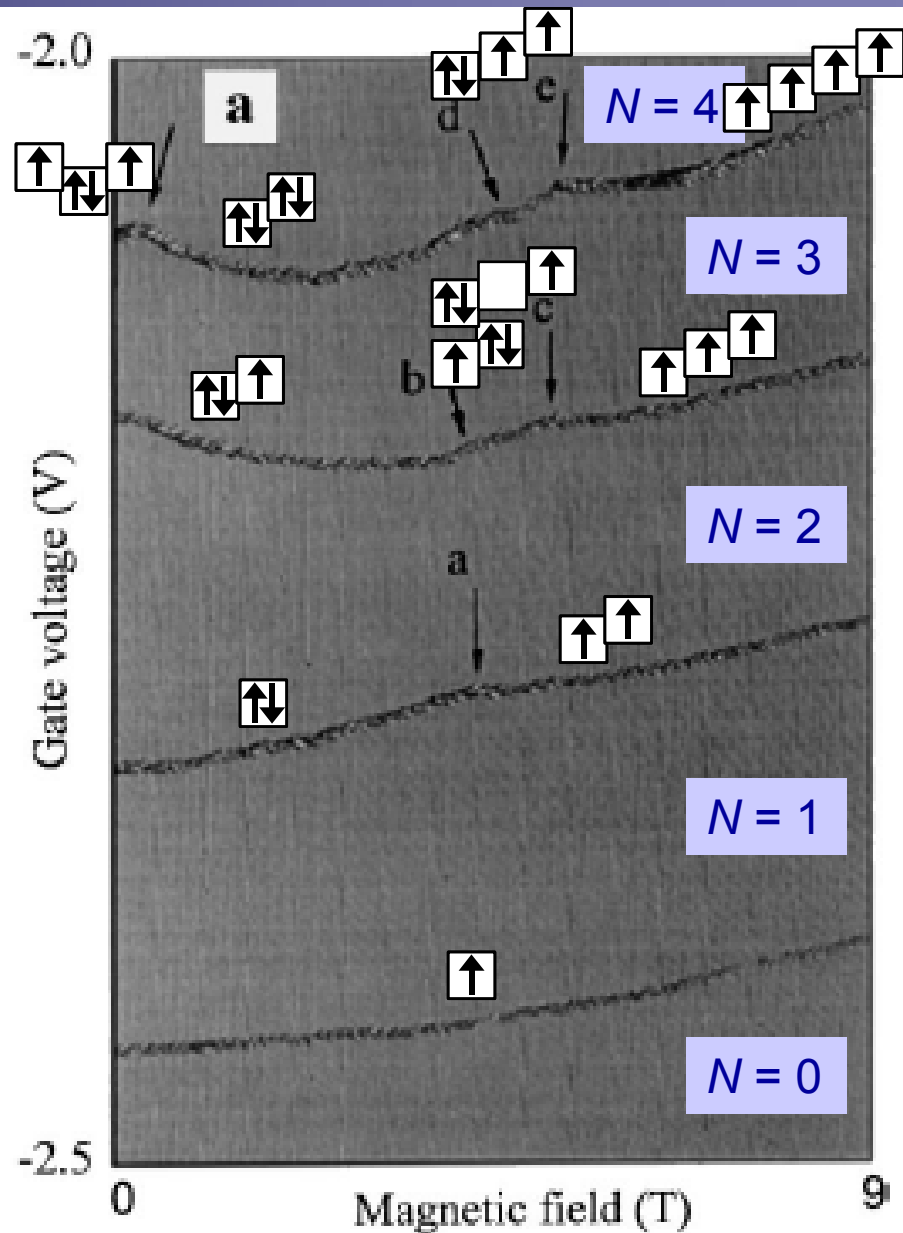


2nd electron in 2p orbital to form triplet

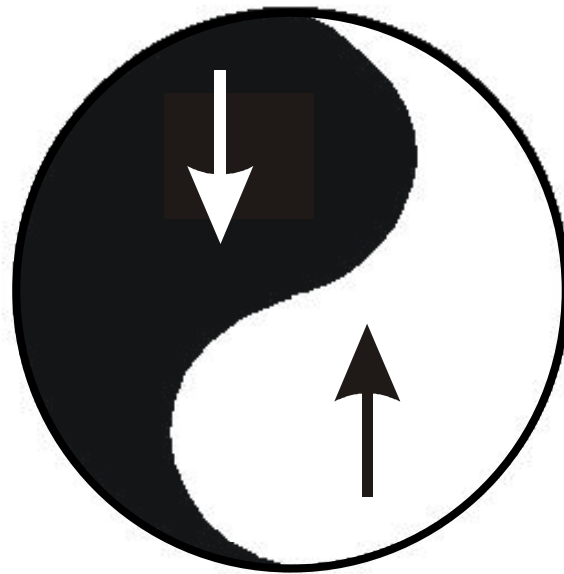
Spin transitions vs. B



- Spins gradually polarized as B increases
- Spin states more complicated as N increases
- **Single-spin state ($N = 1$) always simple!**

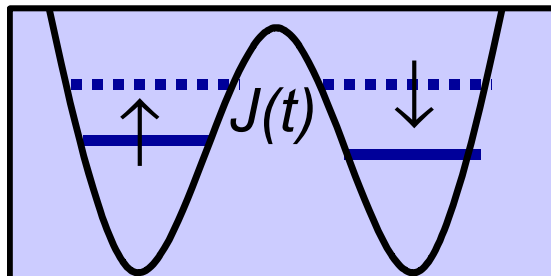
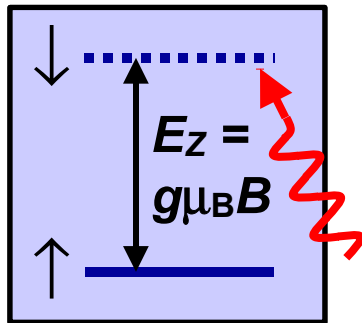
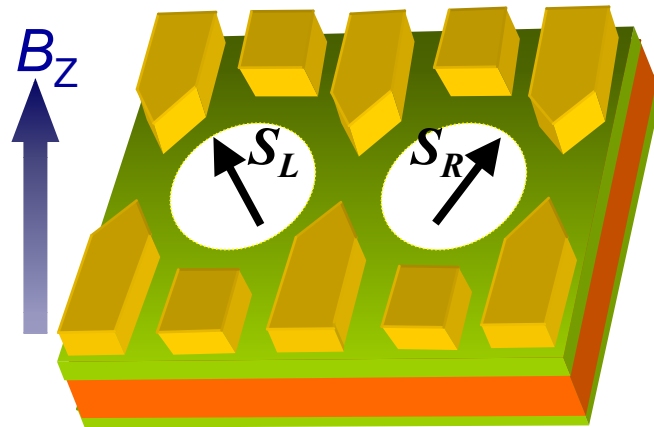


Part III: One-electron quantum dots as spin qubits



Spin qubits

Loss & DiVincenzo, PRA 57, 120 (1998)



$$H_s(t) = J(t) \mathbf{S}_L \cdot \mathbf{S}_R$$

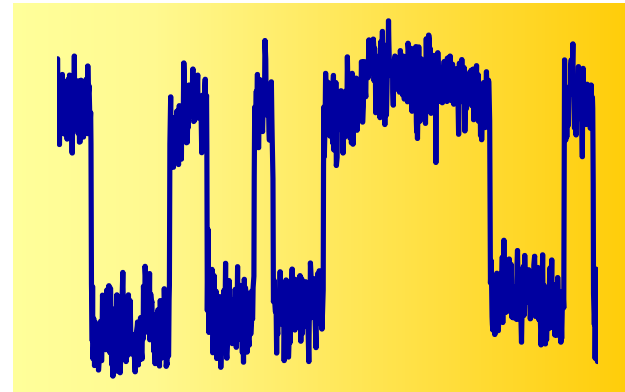
- Qubit defined by Zeeman-split levels of *single electron* in quantum dot
- 1-qubit control:
 - magnetic (ESR-field)
 - electric (modulate effective g-factor)
- 2-qubit coupling: electric (exchange interaction between dots)
- **Read-out of the qubit state!**

We need...

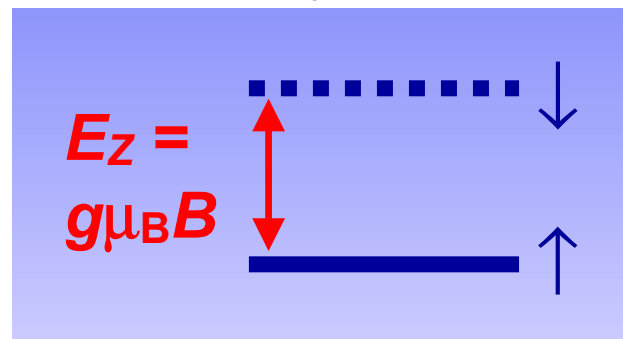
one-electron
double dots...



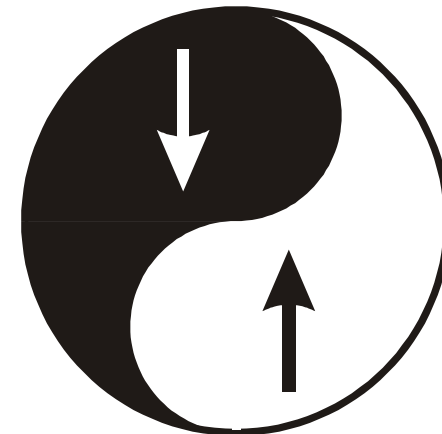
...fast charge
detection...



...two-level
system...



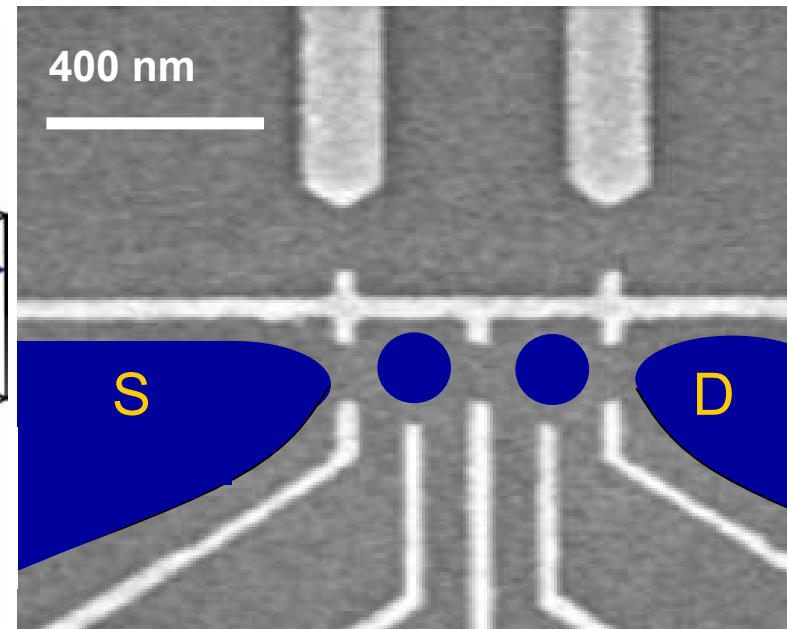
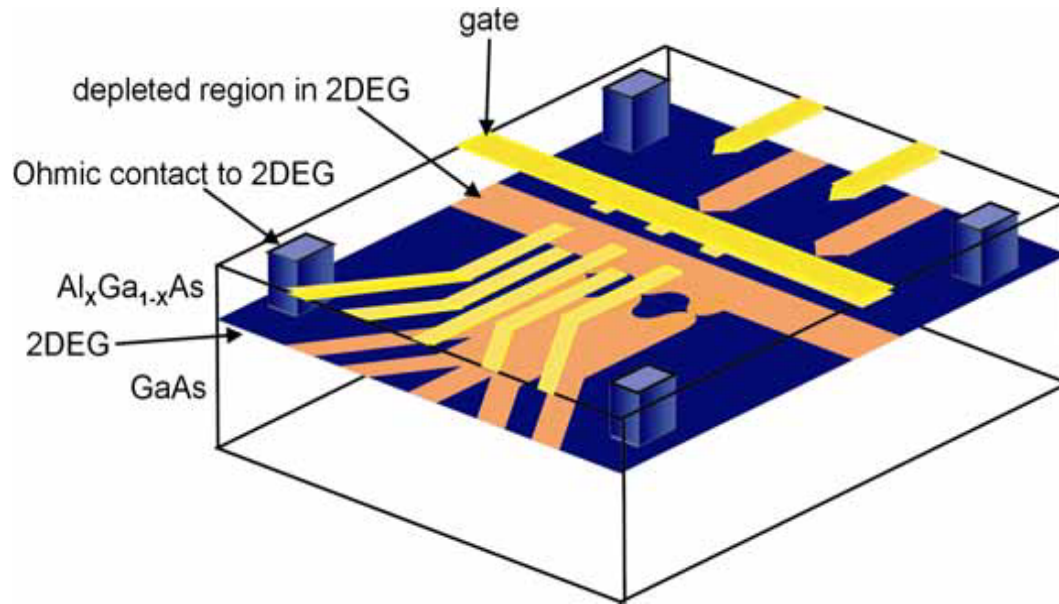
...single spin
measurement!



Few-electron quantum dot circuit with integrated charge detector



$N = 1$ in lateral quantum dots?



- High-mobility 2DEG ($\sim 10^6$ cm²/Vs)
- Density $\sim 10^{11}$ cm⁻² ♥ $\lambda_F \sim 30$ nm
- Smallest gate structure ~ 40 nm
- Dot size ~ 200 nm
- ♥ discrete energy spectrum

BUT: Barriers close as dot is depleted ♥ current too small to measure!

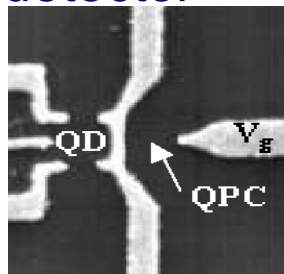
Lateral few-electron double dot

few-electron
single dot



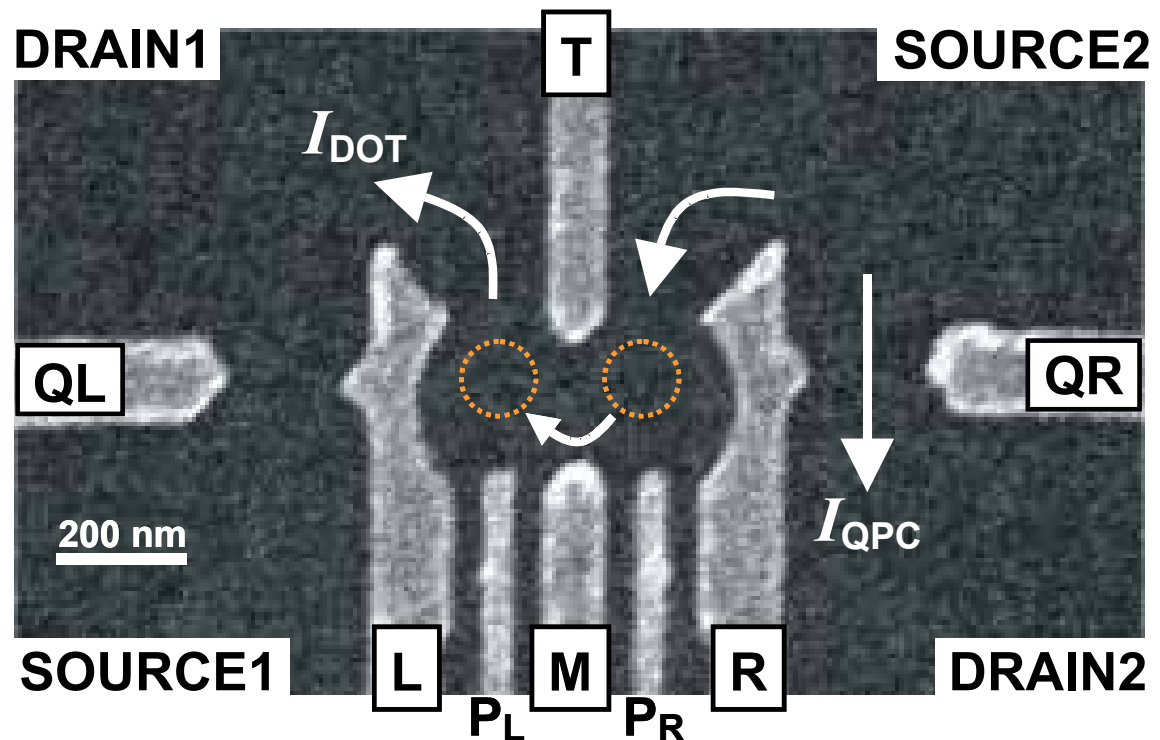
Ciorga '99

QPC charge
detector



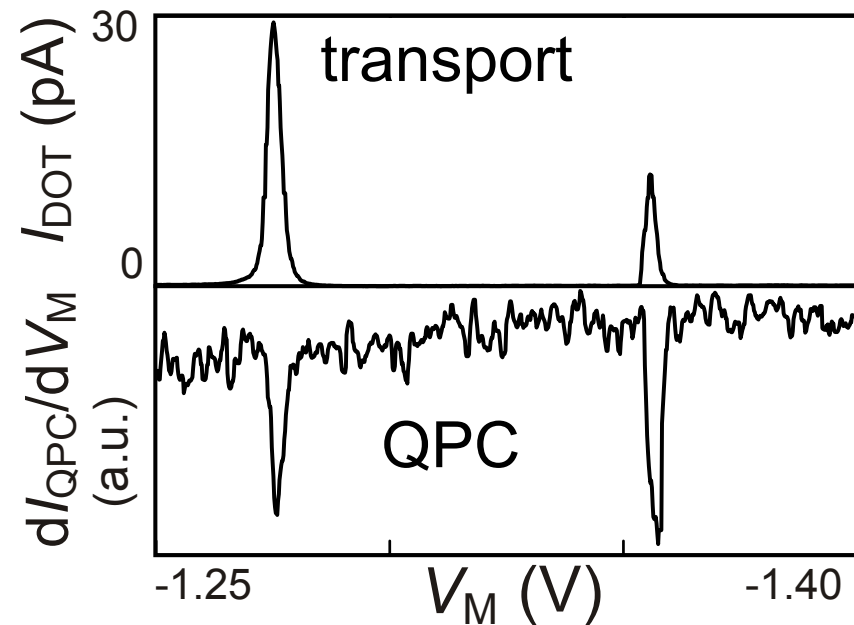
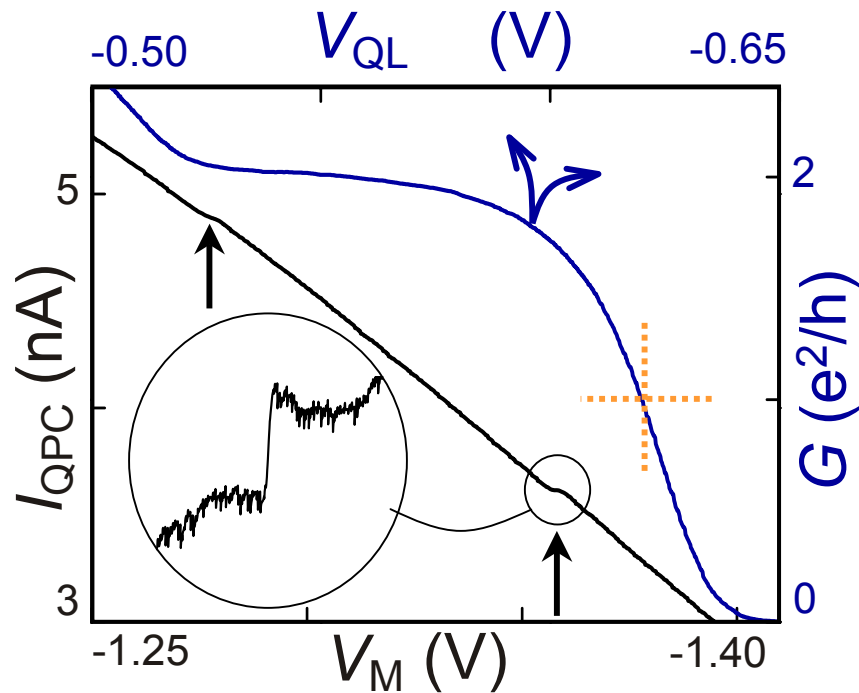
Field '93
Sprinzak '01

GaAs/ $\text{Al}_{0.27}\text{Ga}_{0.73}\text{As}$ heterostructure
2DEG 90 nm deep, $n_s = 2.9 \times 10^{11} \text{ cm}^{-2}$



Design: barriers don't close as DQD is depleted

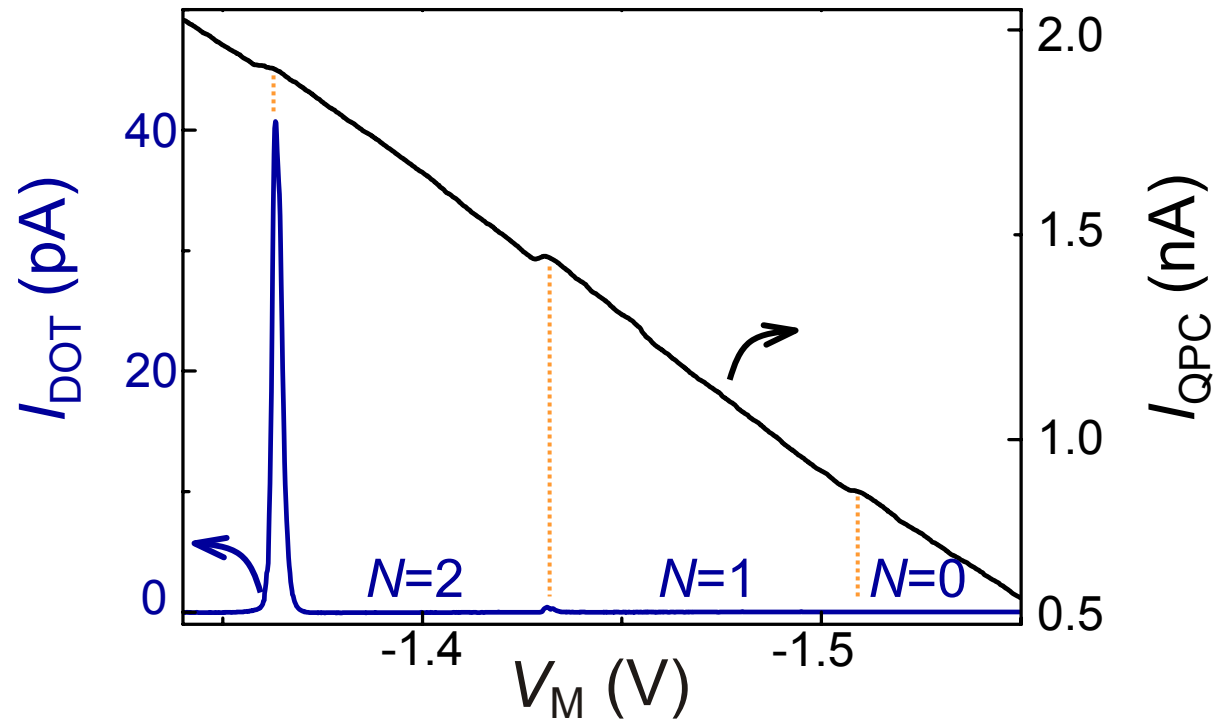
QPC - detector for single dot



- Tune to steepest point ($G \sim e^2/h$)
- Jumps in I_{QPC} ♥ change in electron number

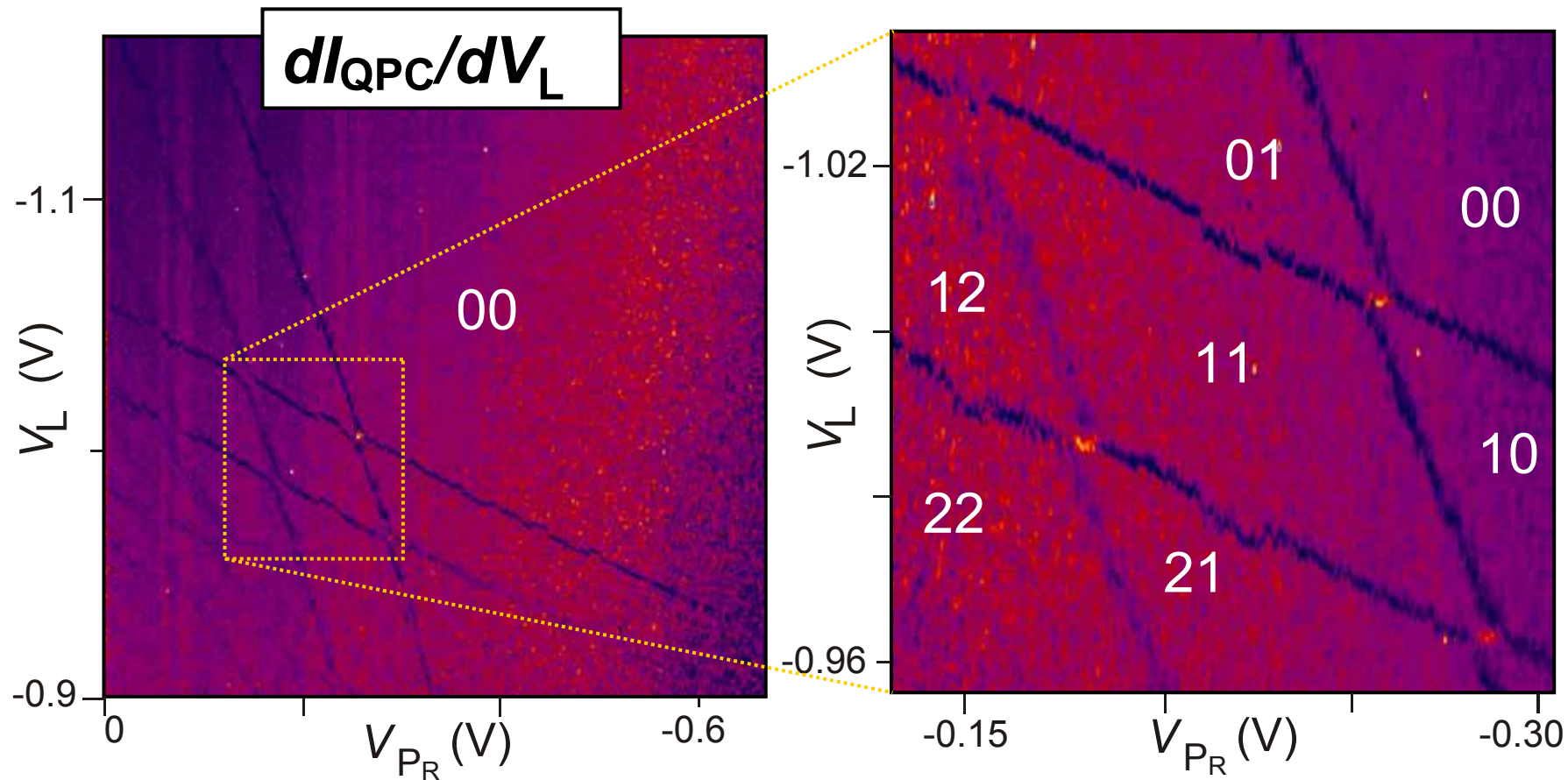
- Modulate V_M with lock-in
 - ♥ measure dI_{QPC}/dV_M
- **Dips in QPC-signal coincide with Coulomb peaks in transport!**
- Sensitivity $\sim 0.1e$ (at 17 Hz)

QPC - detector for isolated single dot



QPC can detect charge transitions in *isolated* QD!

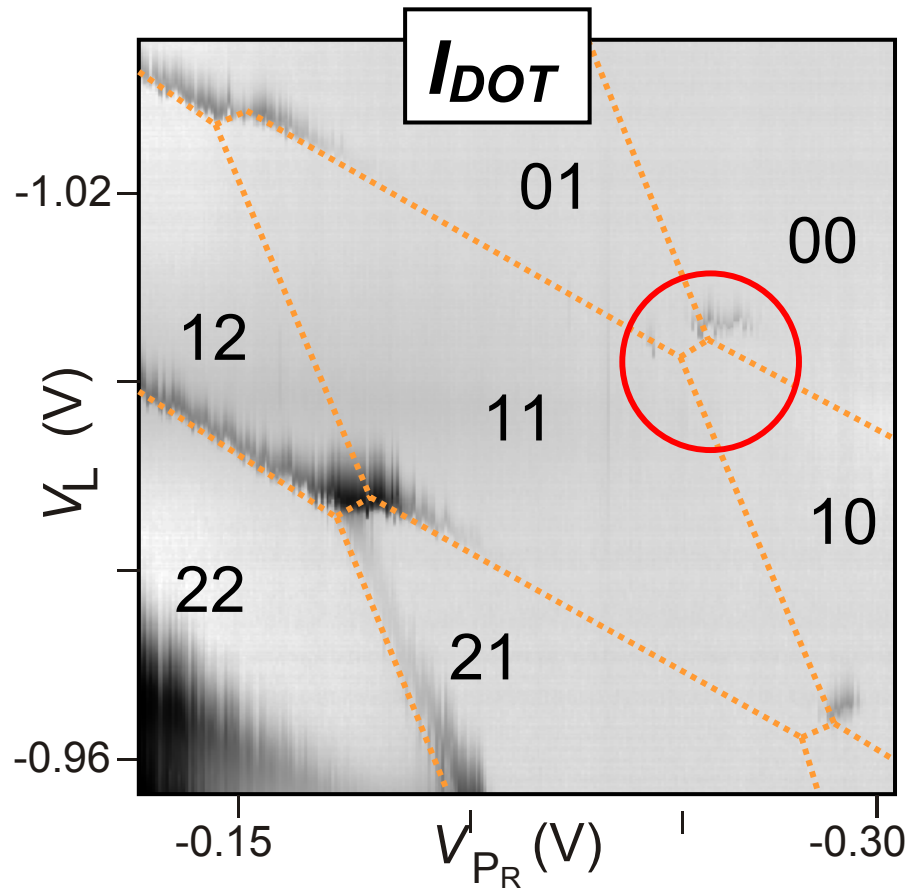
QPC - detector for double dot



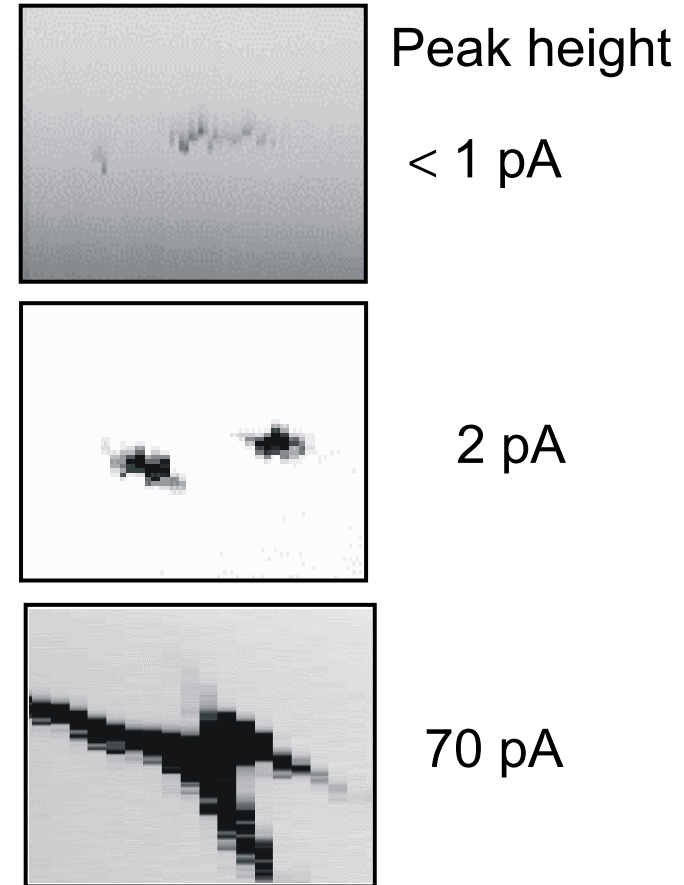
- **Double dot emptied completely !**
- $E_C = 4.5$ meV

QPC detects *all* charge transitions, also between dots

Tunable few-electron double dot

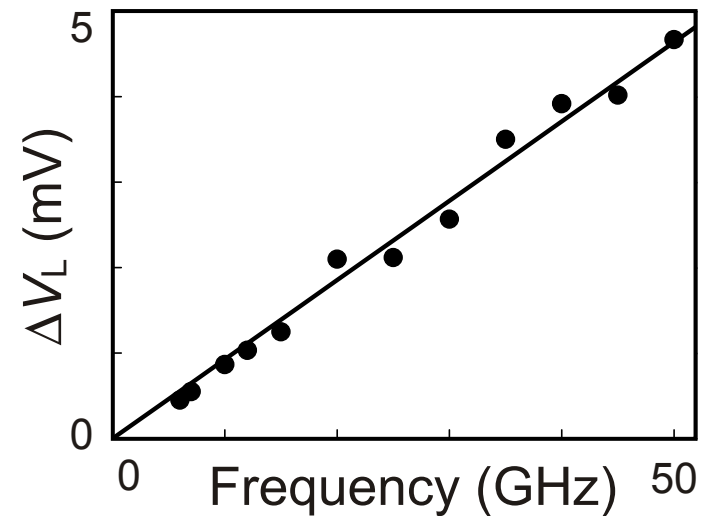
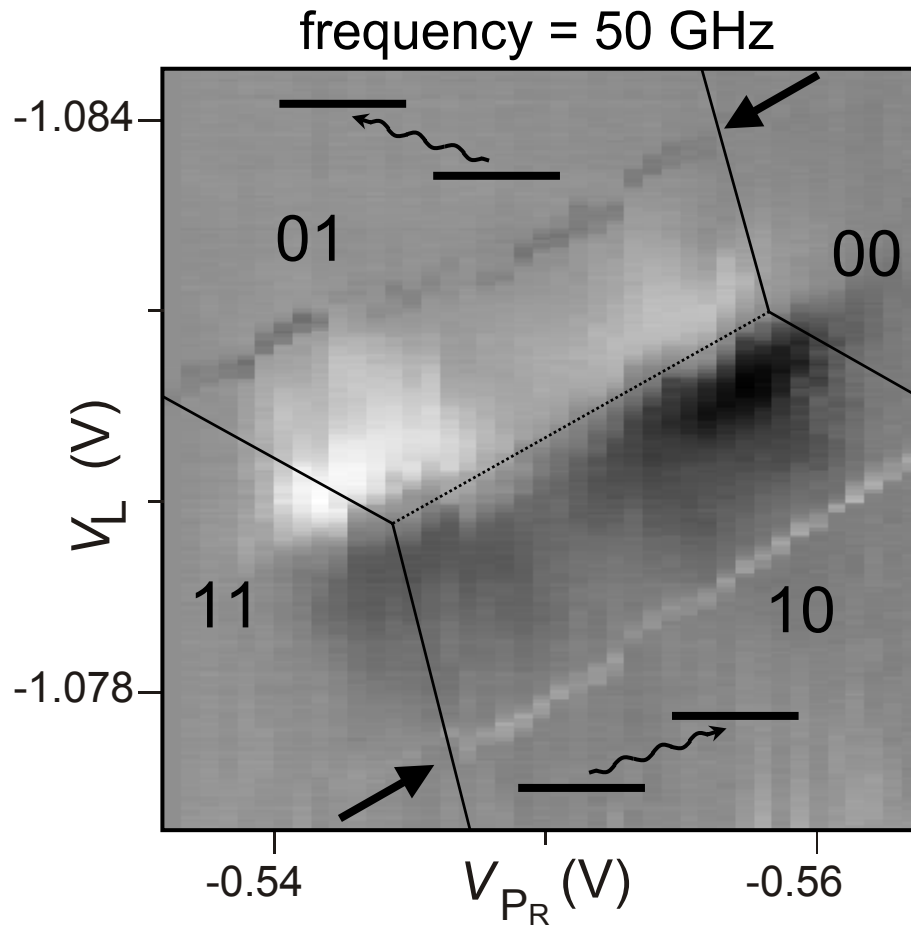


All triple points visible
♥ *barriers still open*



Double dot
fully tunable!

Photon-assisted tunneling



- Photon-assisted current visible for $f > 6$ GHz
- Always linear ♥ coherent coupling between two dots not visible

Summary...



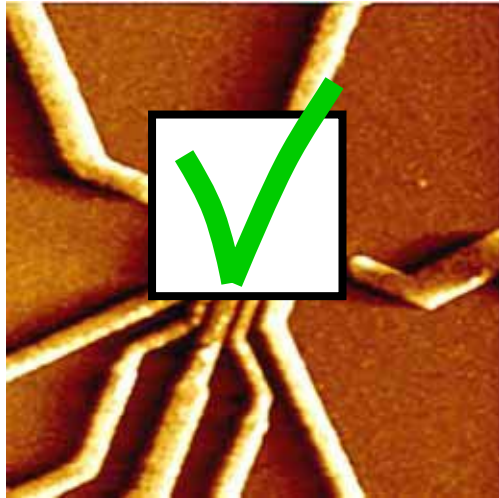
We can:

- **isolate *single electron spin* in (double) quantum dot**
- **study it using transport or charge detection**

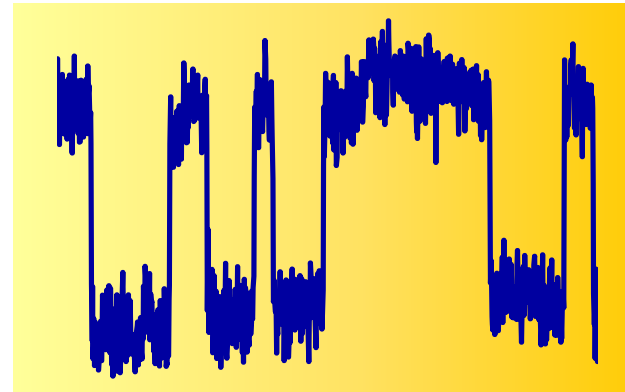
PRB 67, 161308(R) (2003)

We need...

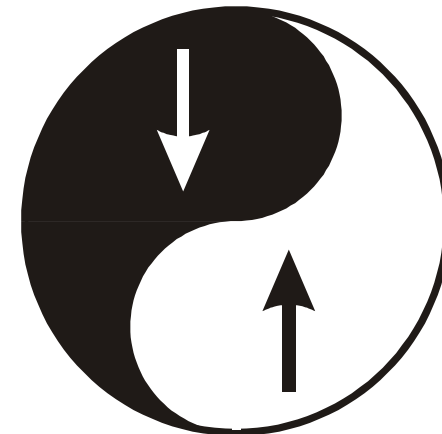
one-electron
double dots...



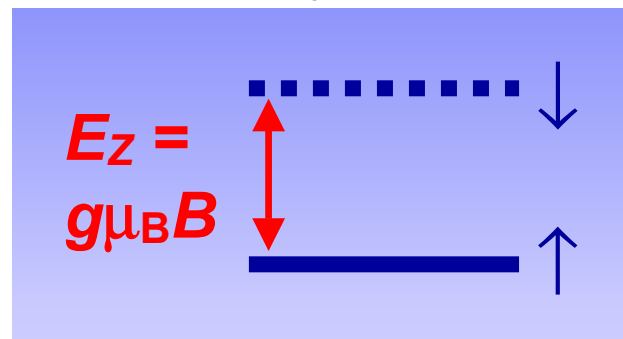
...fast charge
detection...



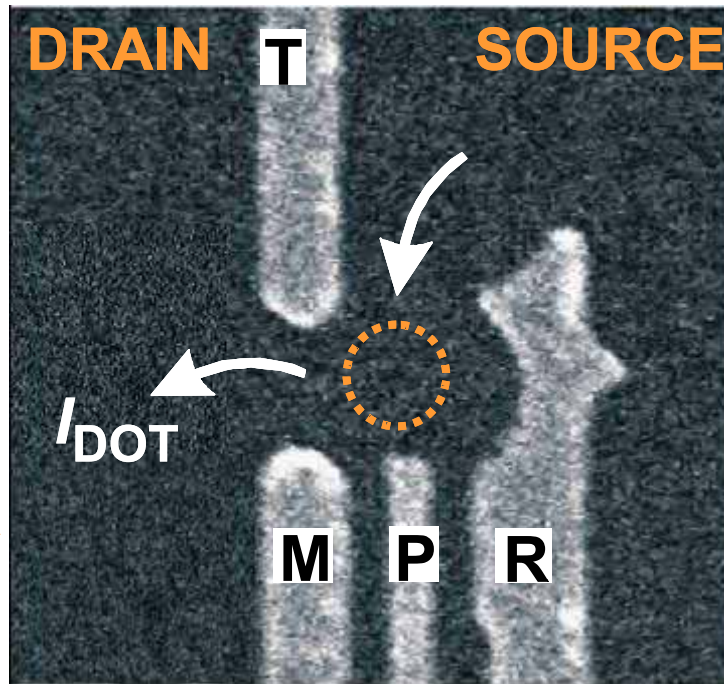
....single spin
measurement!



...two-level
system...



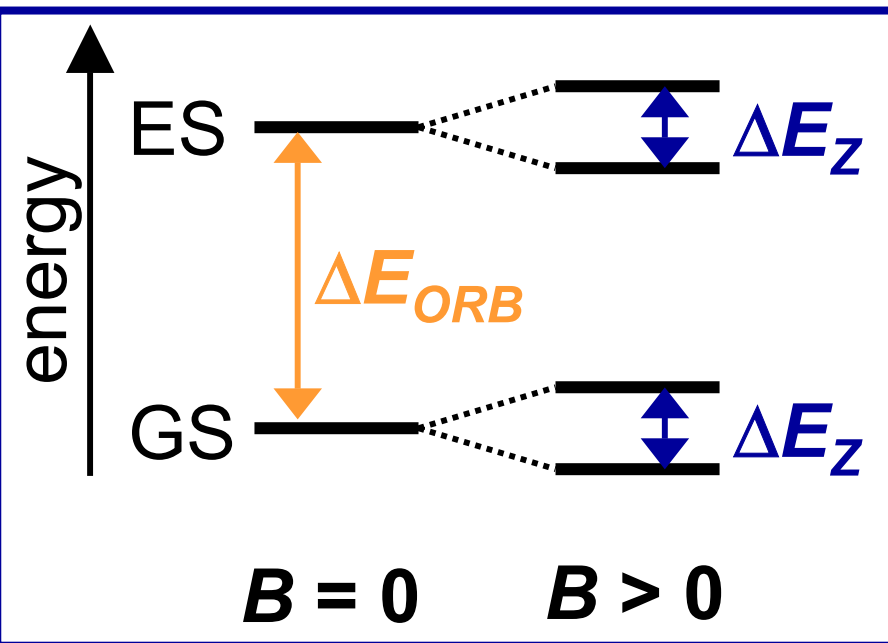
Zeeman splitting in an artificial Hydrogen atom



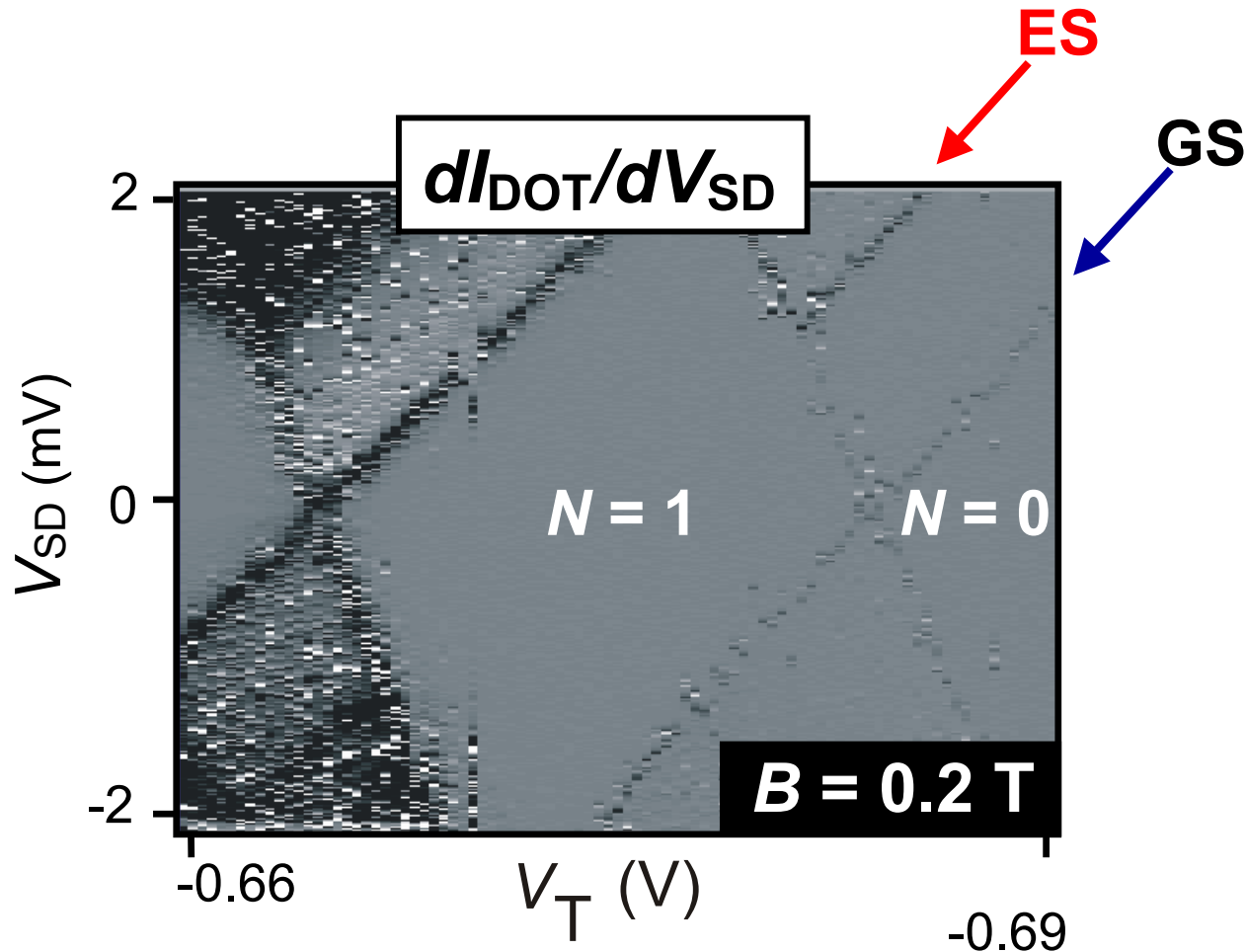
$B_{||}$

magnetic field *parallel*
to 2DEG

Similar double dot device:
other gates grounded

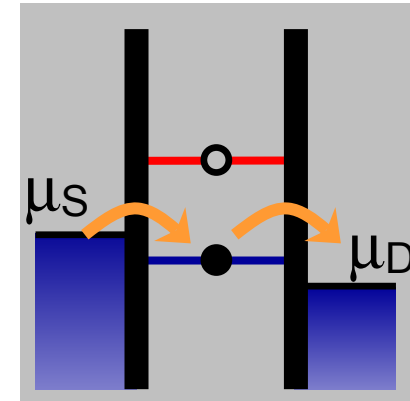


$N = 1$ Coulomb diamond

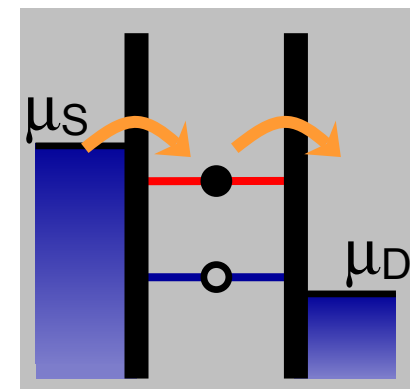


Large-bias spectroscopy reveals excited states

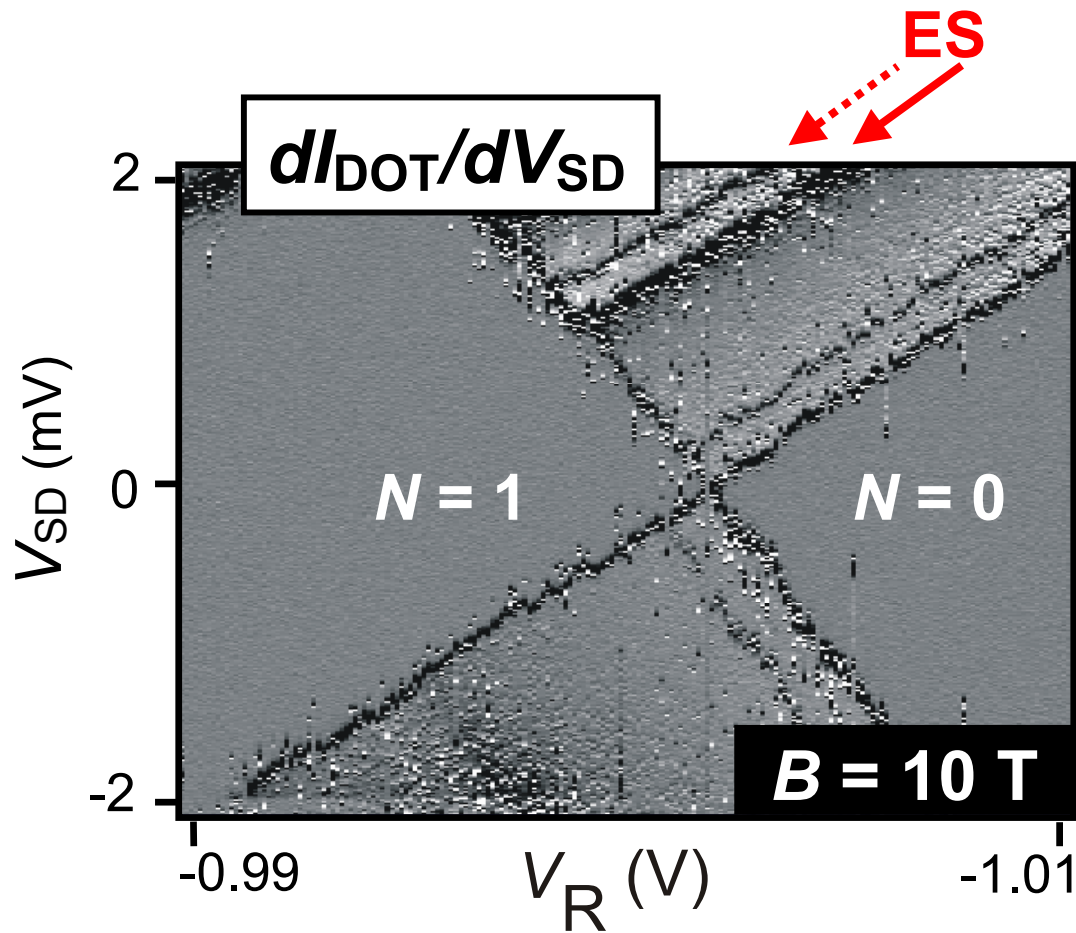
$N = 1$ orbital ground state



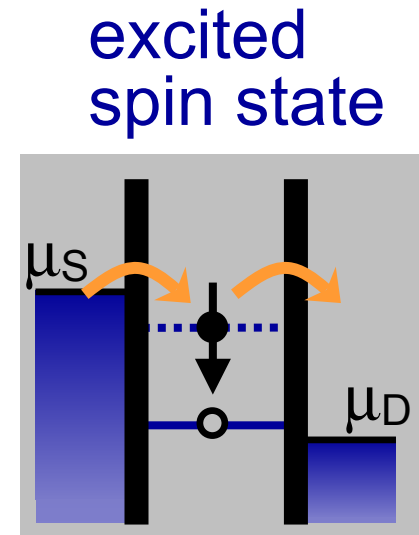
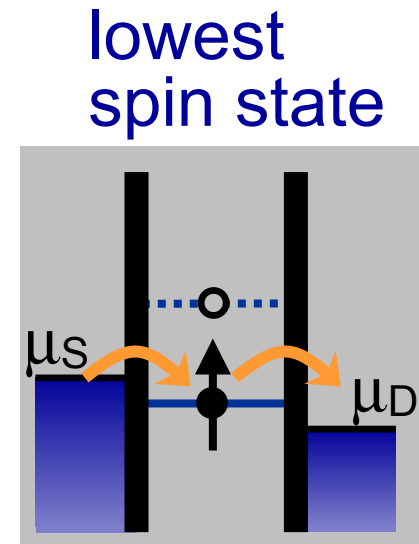
$N = 1$ orbital excited state



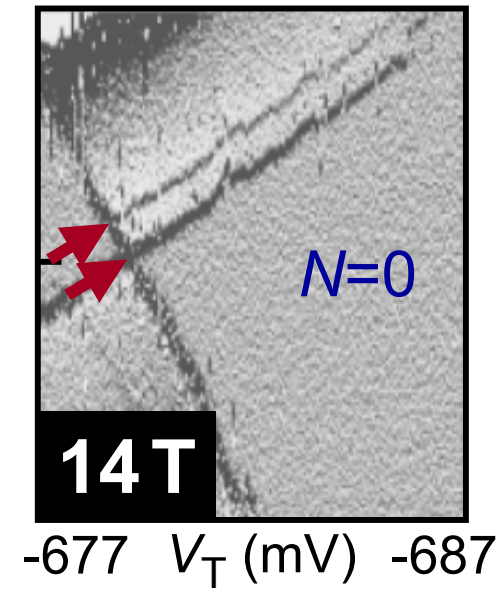
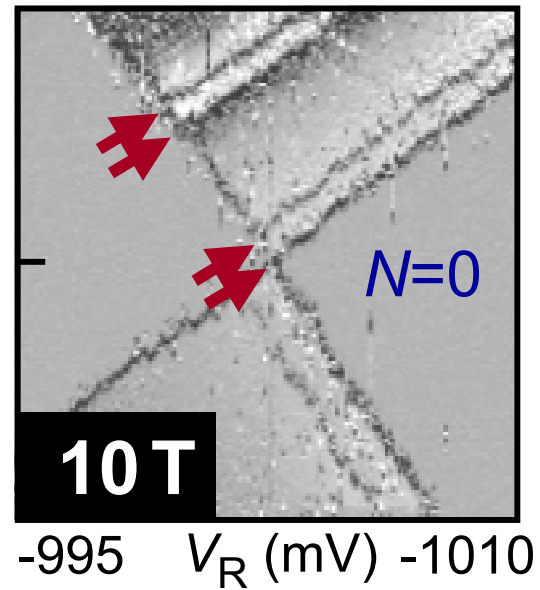
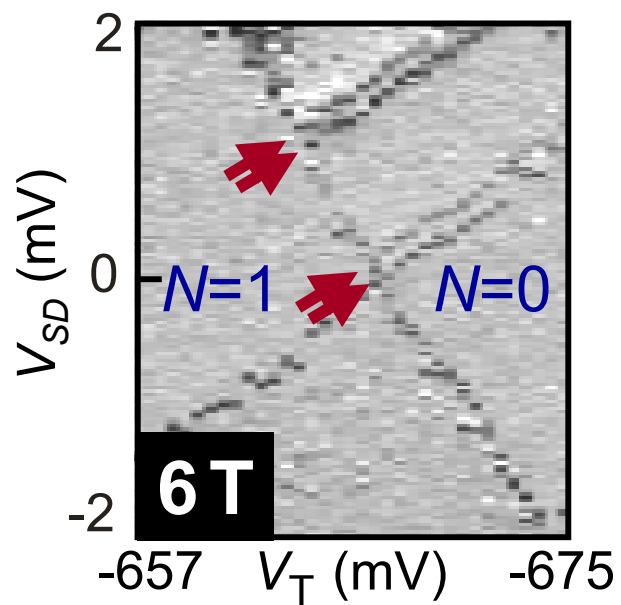
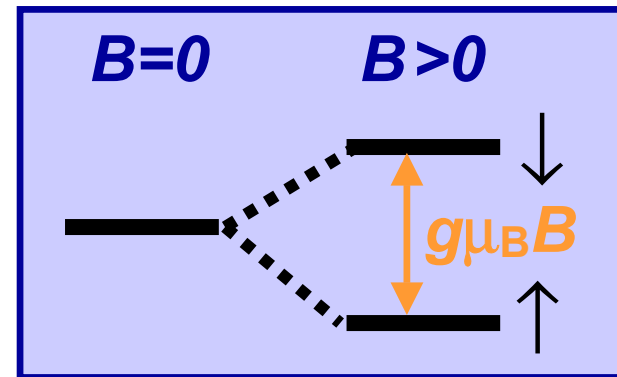
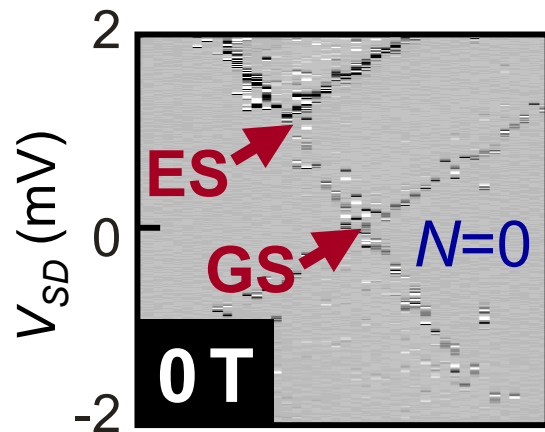
$N = 1$ Zeeman splitting in B_{\parallel}



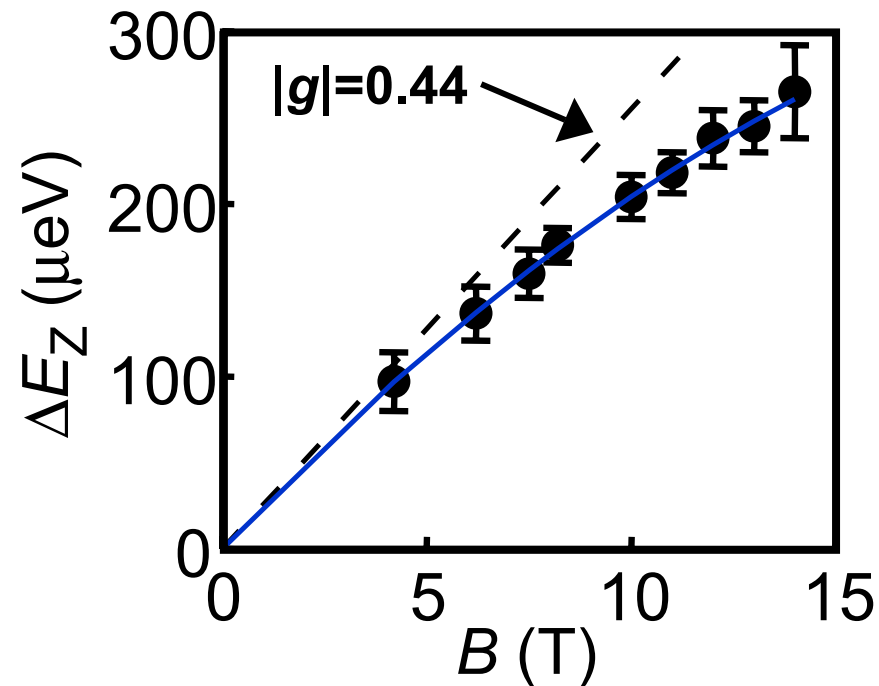
Zeeman splitting clearly visible for GS and ES !



$N = 1$ Zeeman splitting in B_{\parallel}



Spectroscopy of qubit 2-level system

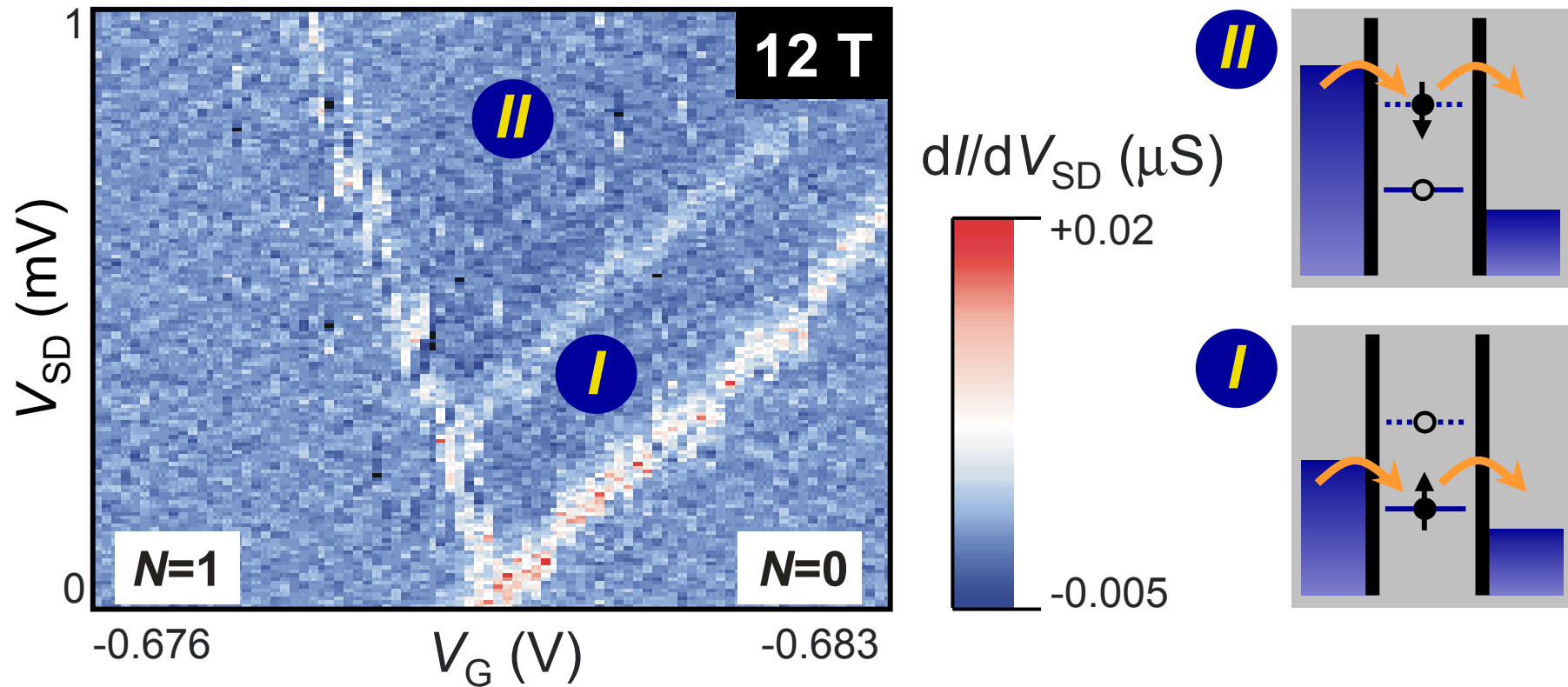


$$E_z = g \mu_B B_{||}$$

$$|g| = 0.43 - 0.008B$$

$N = 1$ dot as filter for spin-up

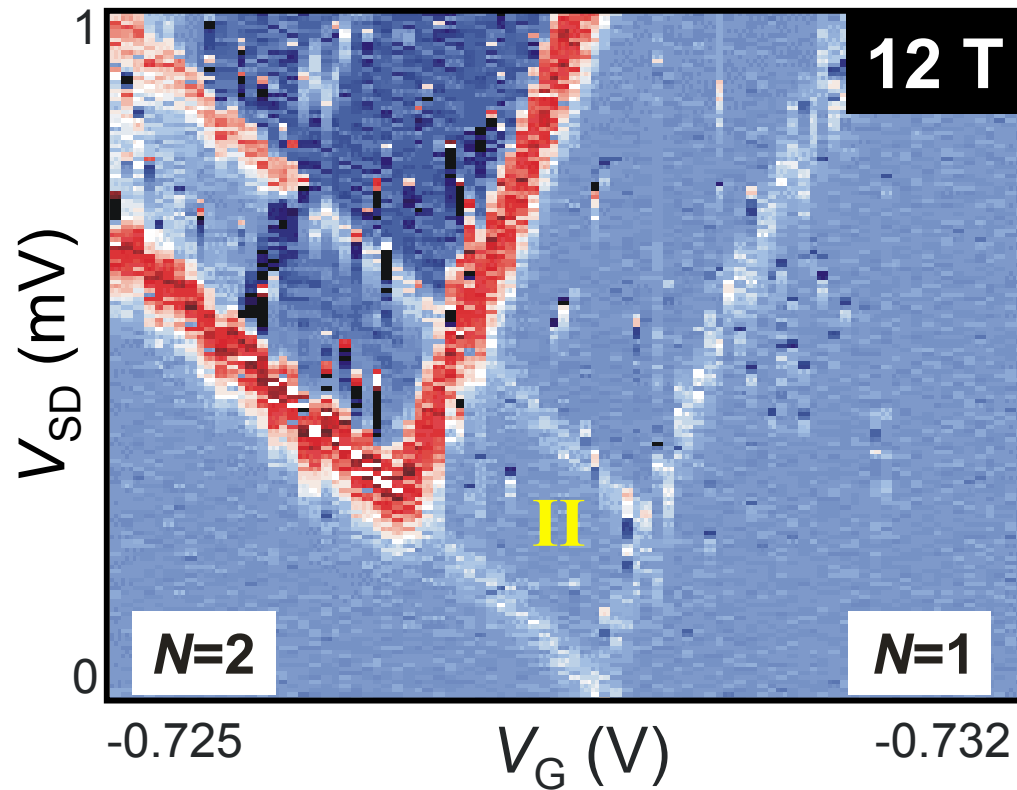
theory: Recher et al., PRL 85,1962 (2000)



Quantum dot in region I acts as *spin filter*; only spin-up electrons can pass through

$N = 2$ dot as filter for spin-down

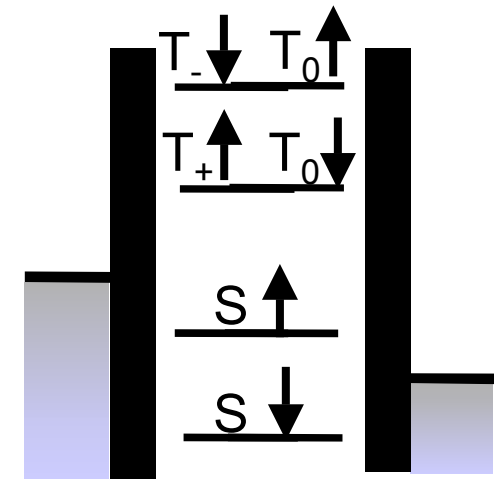
theory: Recher et al., PRL 85,1962 (2000)



dI/dV_{SD} (μS)

+1.0

-0.1



Quantum dot in region II acts as *spin filter*; only **spin-down** electrons can pass through

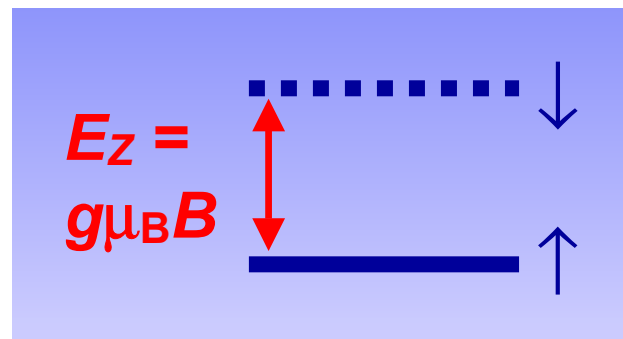
Summary...

We have:

- **identified (stable) two-level system**
- **($T_1 > 50 \mu\text{s}$)**
- **bipolar spin filter**

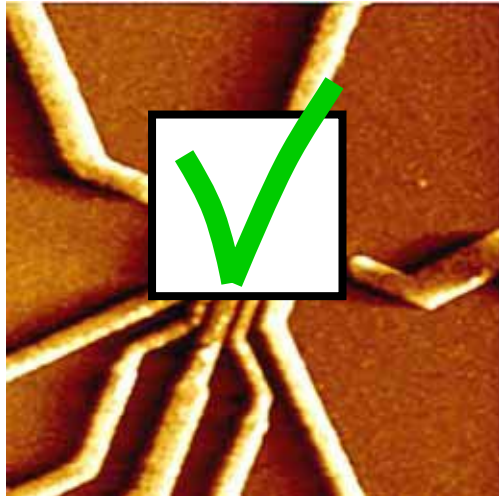
Hanson *et al.*, PRL 91, 196802 (2003)

Cond-mat/0311414

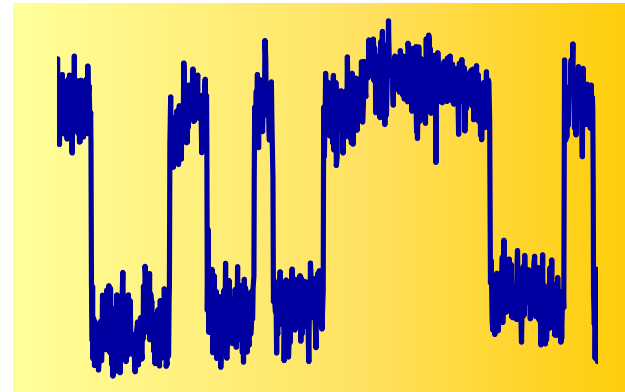


We need...

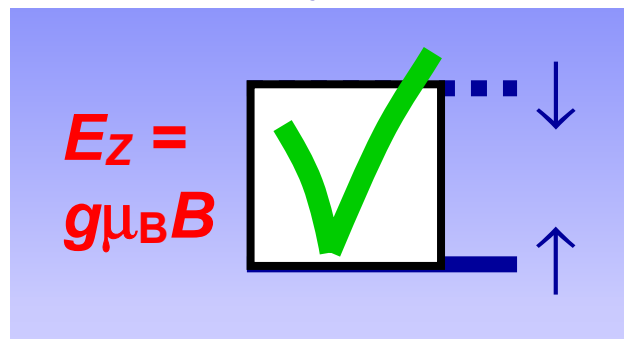
one-electron
double dots...



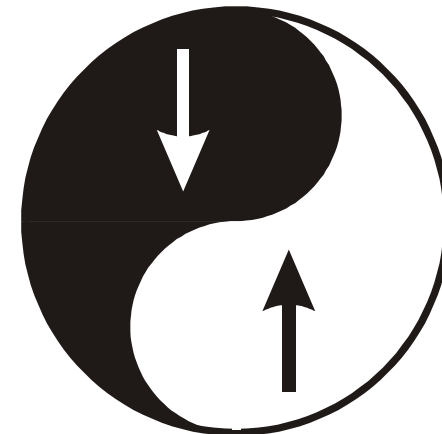
...fast charge
detection...



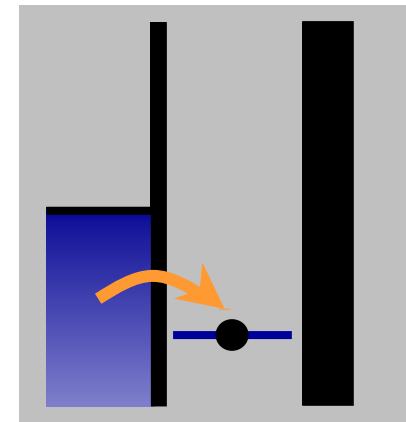
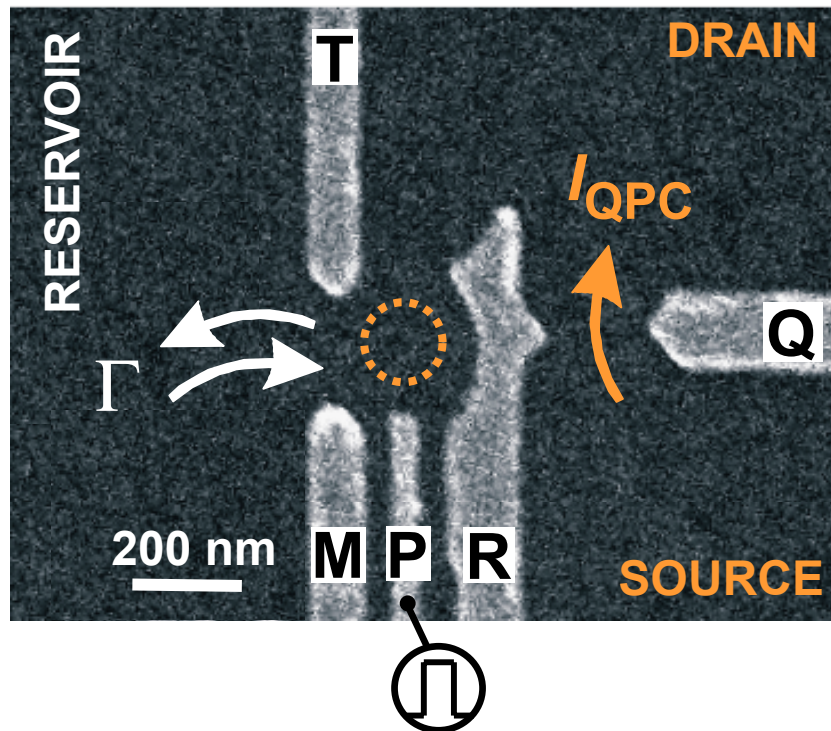
...two-level
system...



....single spin
measurement!



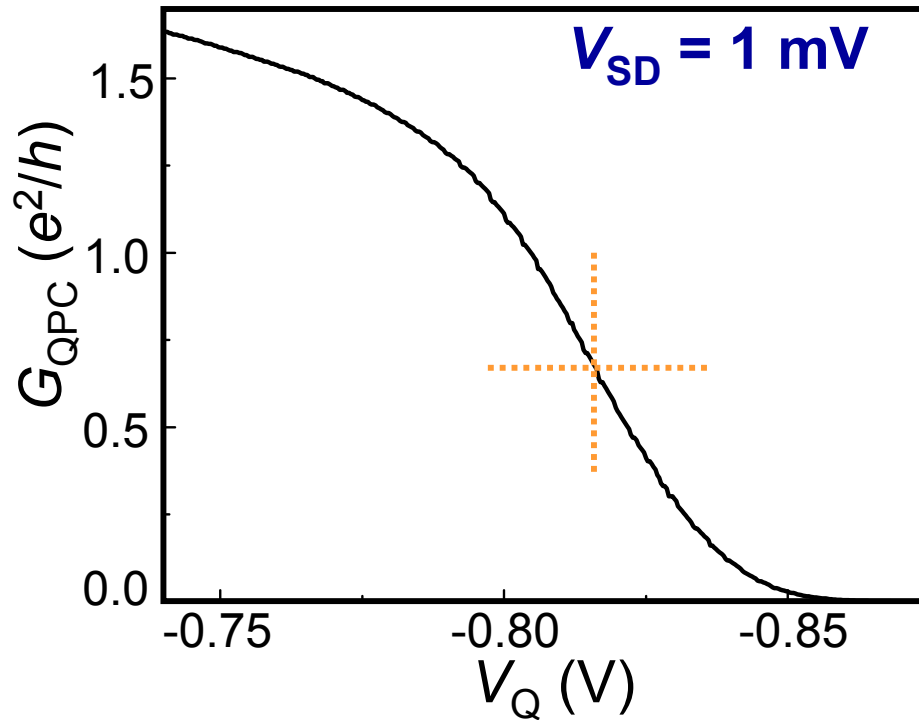
Quantum Point Contact as a fast charge detector



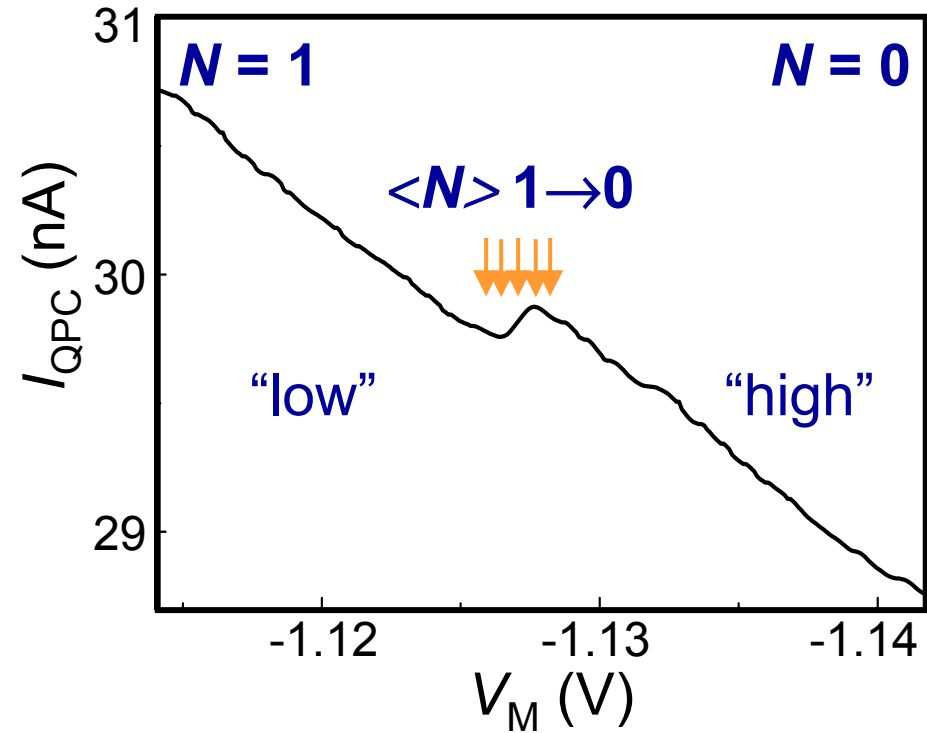
- Tunnel barrier to QPC-channel closed completely
- QD weakly connected to reservoir
- Detect individual tunnel events

- Fast IV-converter:
100 kHz, $0.8 \text{ nV/Hz}^{1/2}$
- Fast ISO-amp:
300 kHz
- Operating bandwidth:
40 kHz
- Shot noise limit:
100 MHz

QPC average charge detection (dc)

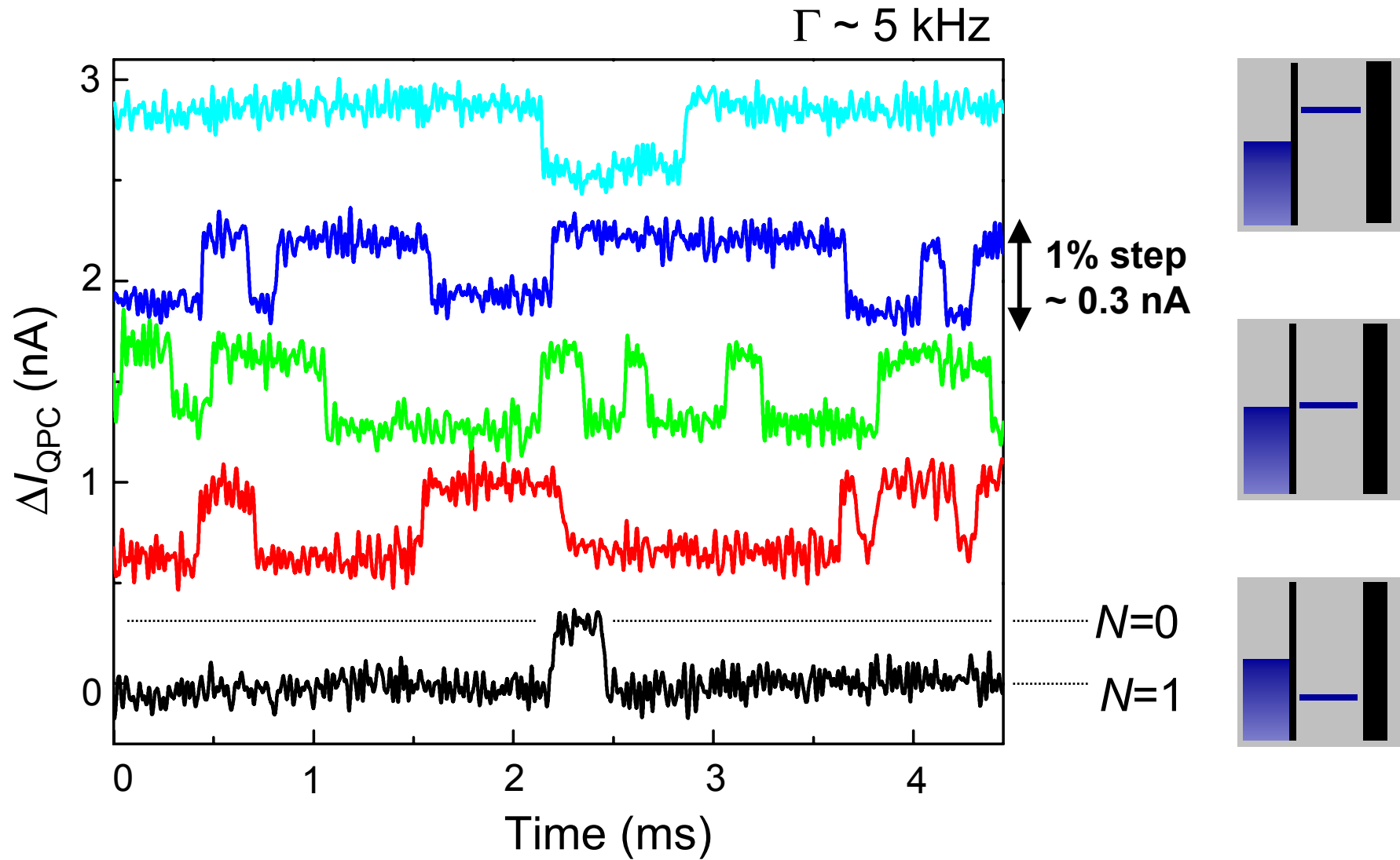


- $V_{\text{SD}} = 1$ mV
- $G_{\text{QPC}} \sim 0.5 - 1.0 e^2/h$
- $I_{\text{QPC}} \approx 30$ nA

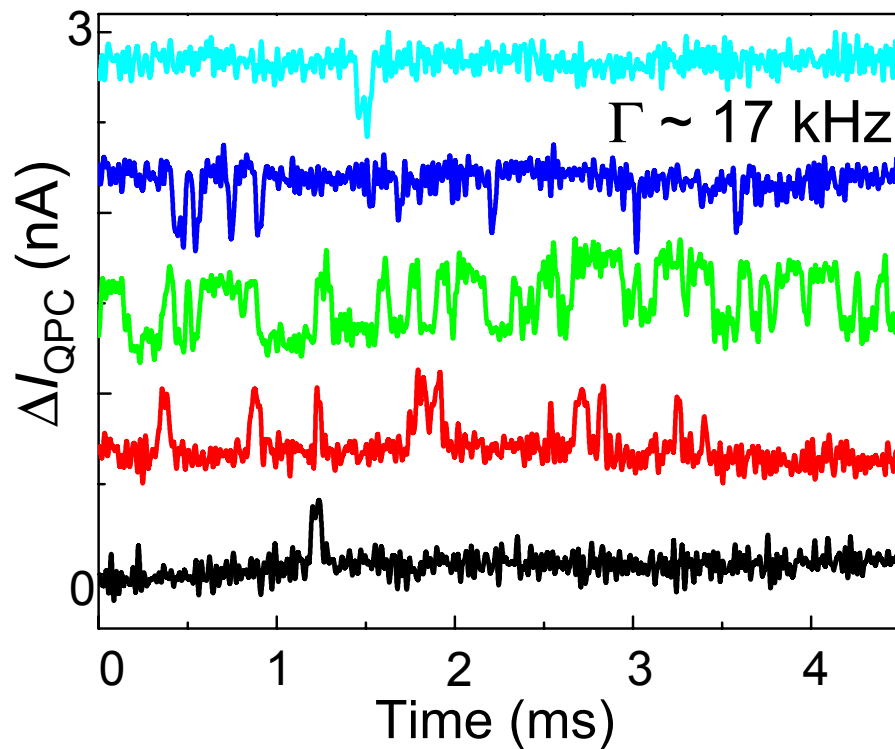


- Sweep dot-gate (V_M)
- I_{QPC} increases ($\sim 1\%$) when $\langle N \rangle$ from 1 to 0

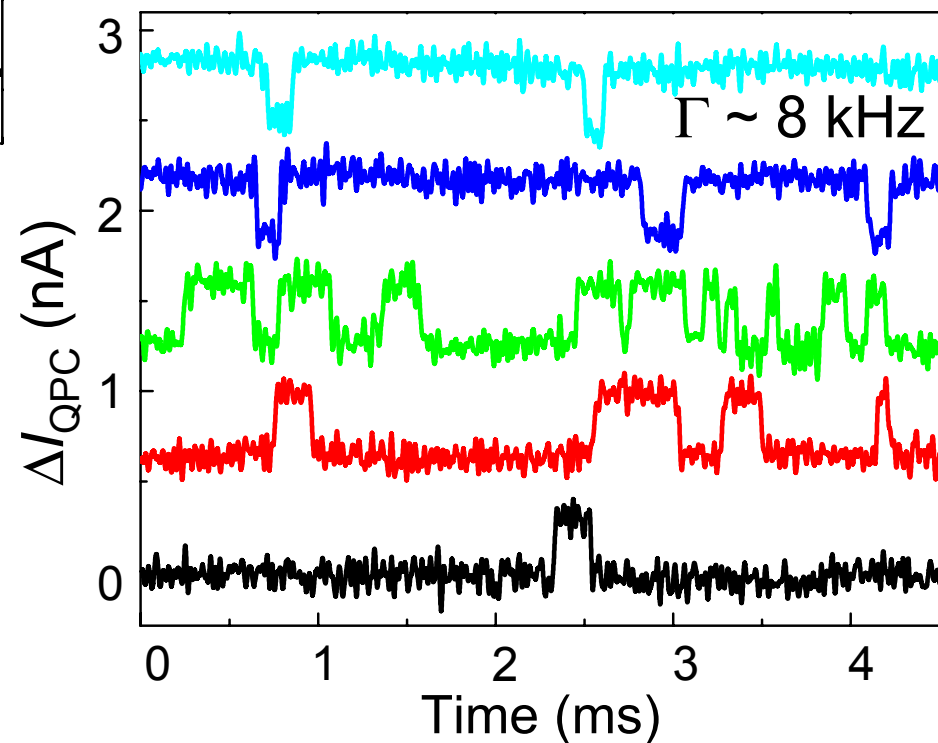
Real-time single-electron tunneling



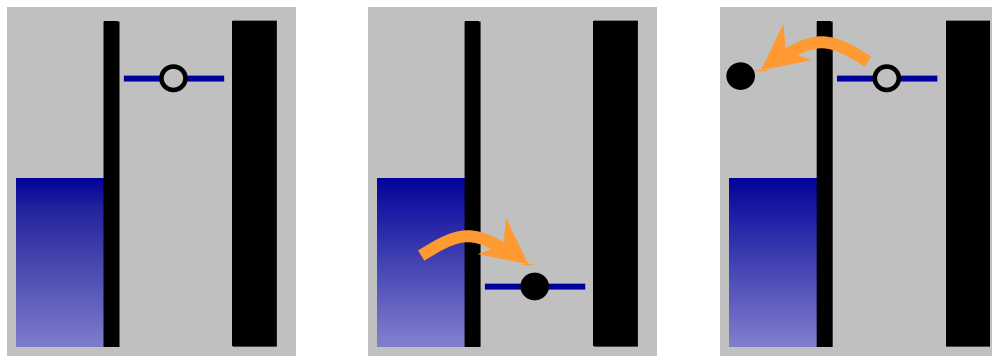
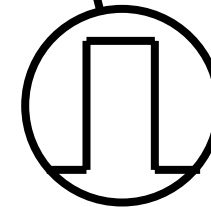
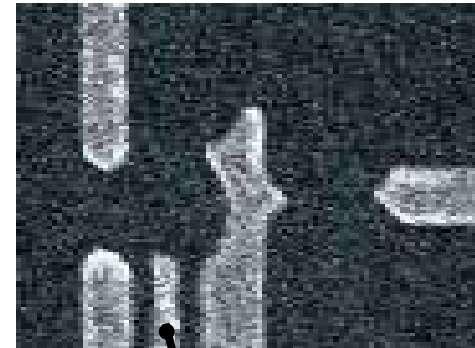
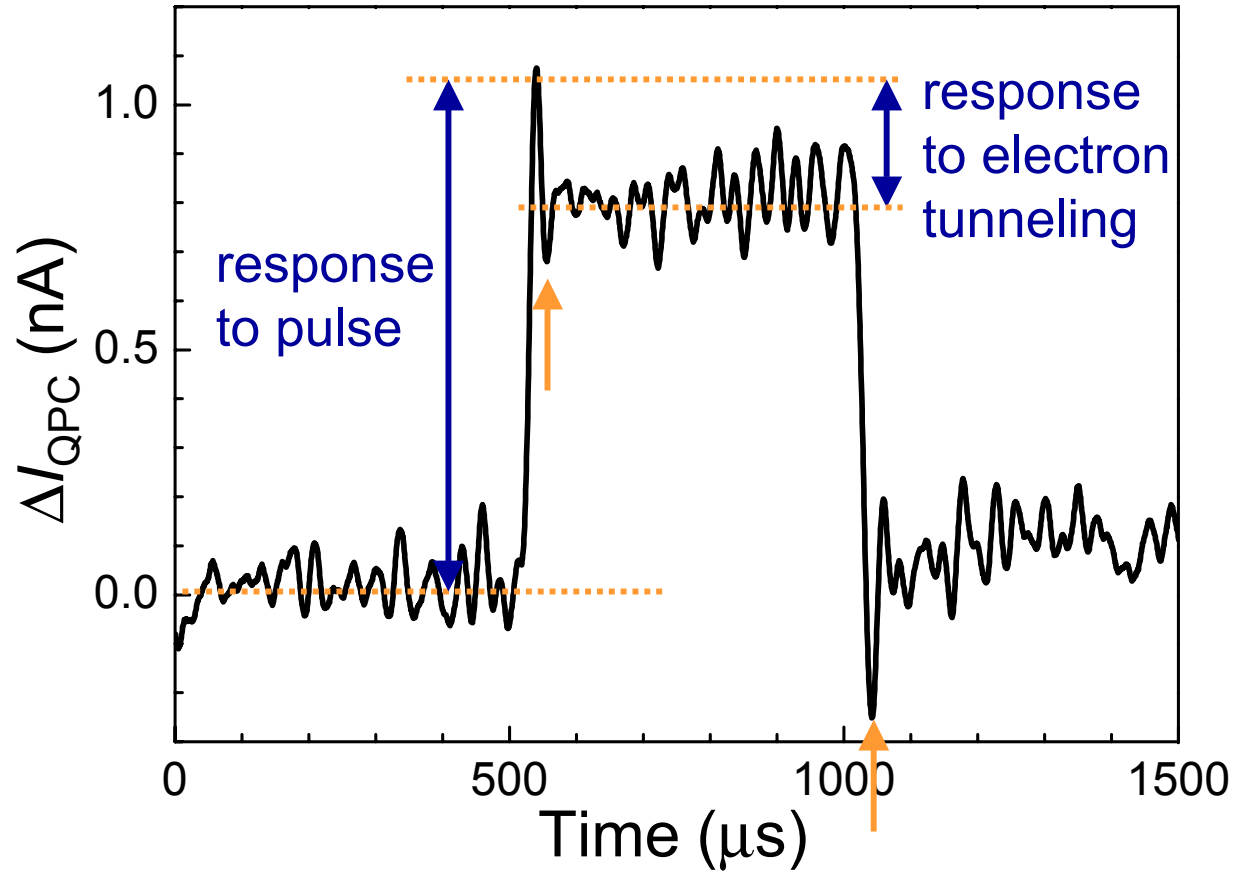
Real-time single-electron tunneling



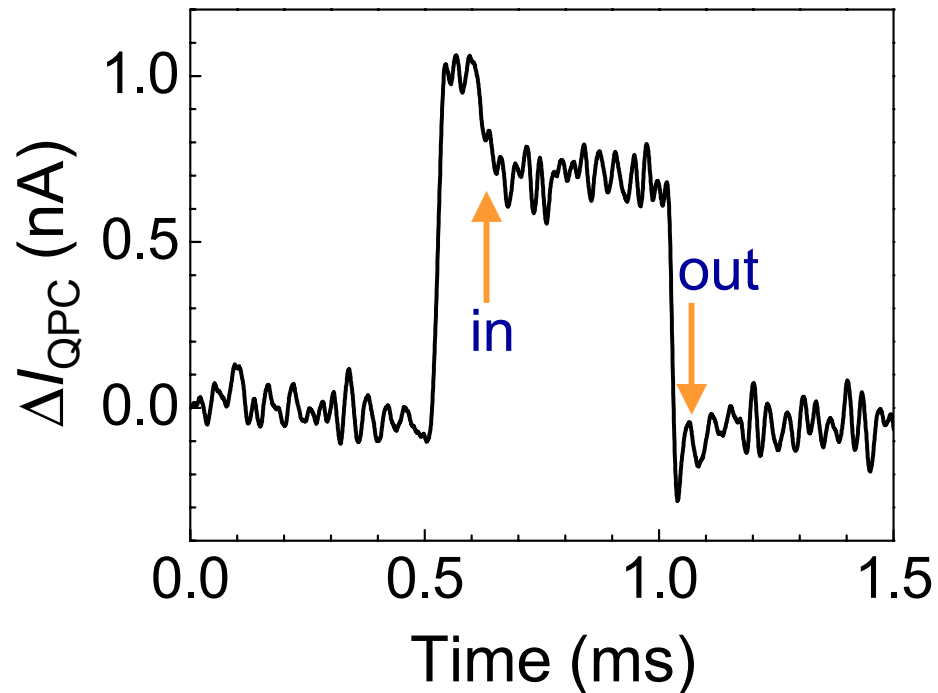
- $V_{\text{SD}} = 1$ mV
- $I_{\text{QPC}} \sim 30$ nA
- $\Delta I_{\text{QPC}} \sim 1\%$
- speed ~ 10 μs
- sensitivity $\sim 10^{-3} e (\text{Hz})^{-1/2}$



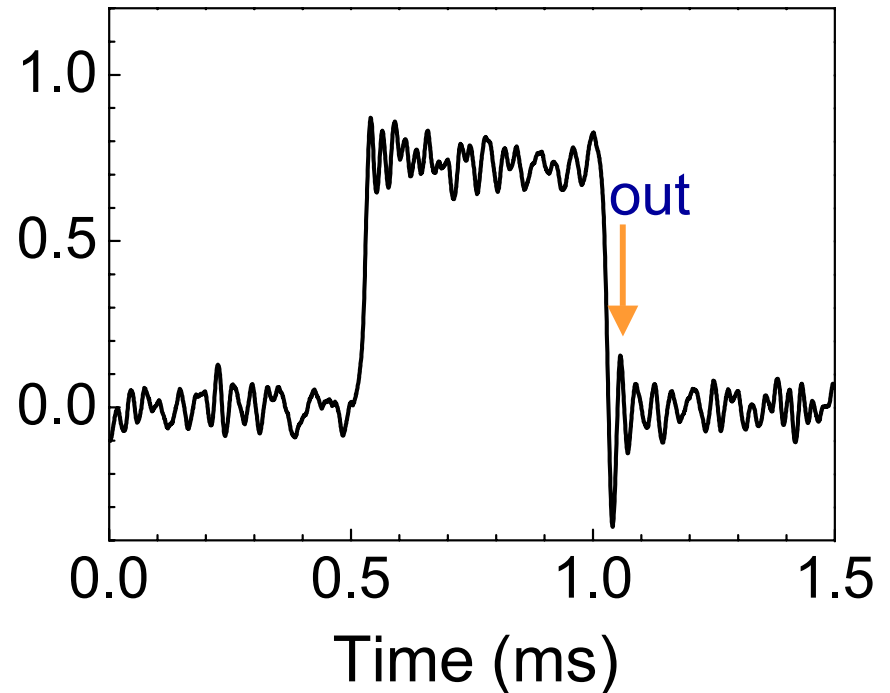
Tunneling induced by pulse



Tunnel-time is stochastic

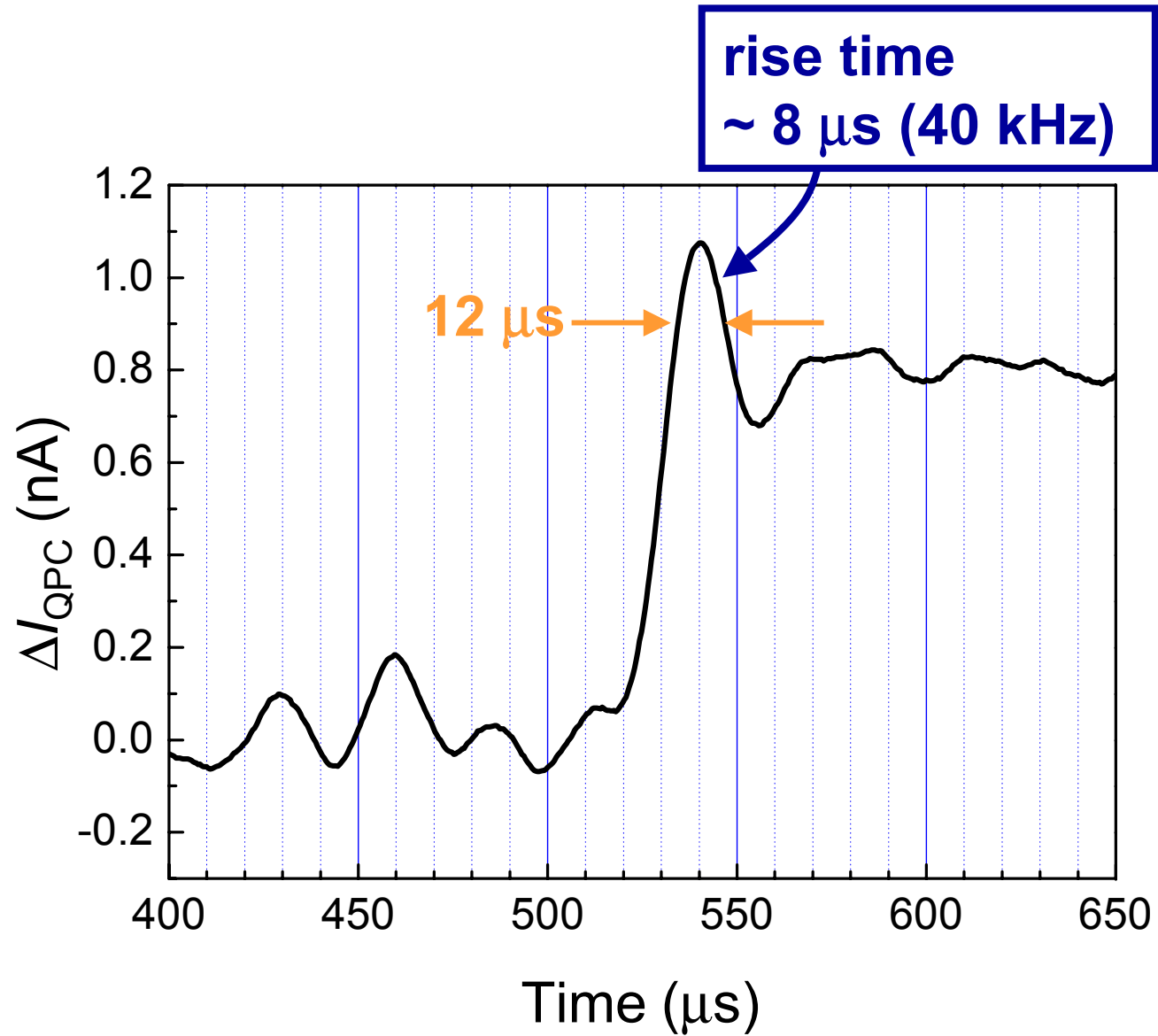


- Tunnel-in event visible
- Tunnel-out event very fast

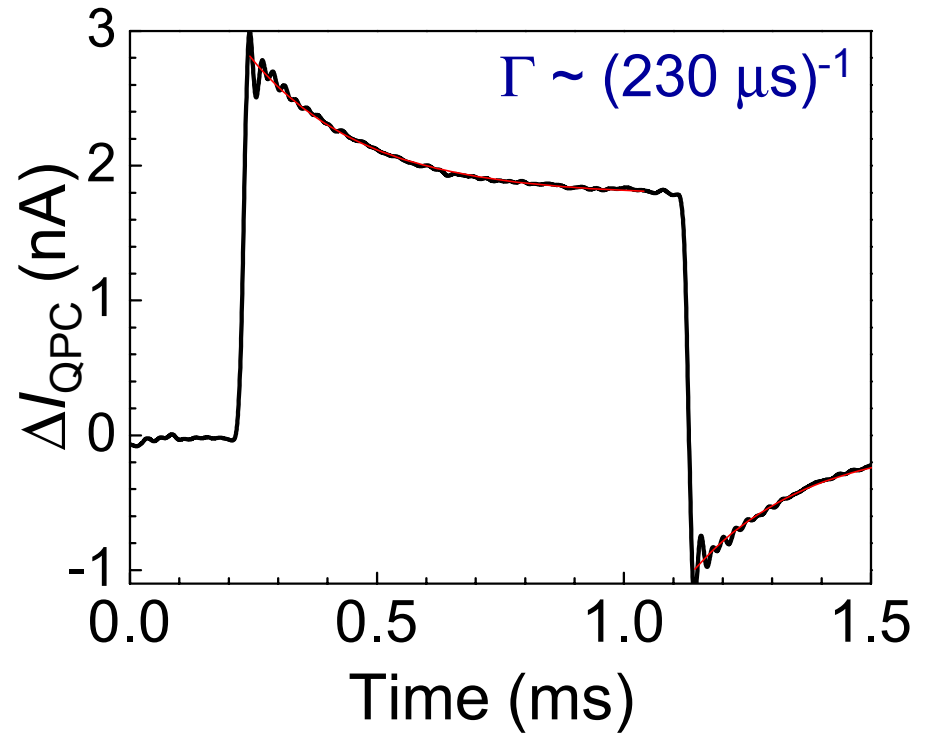
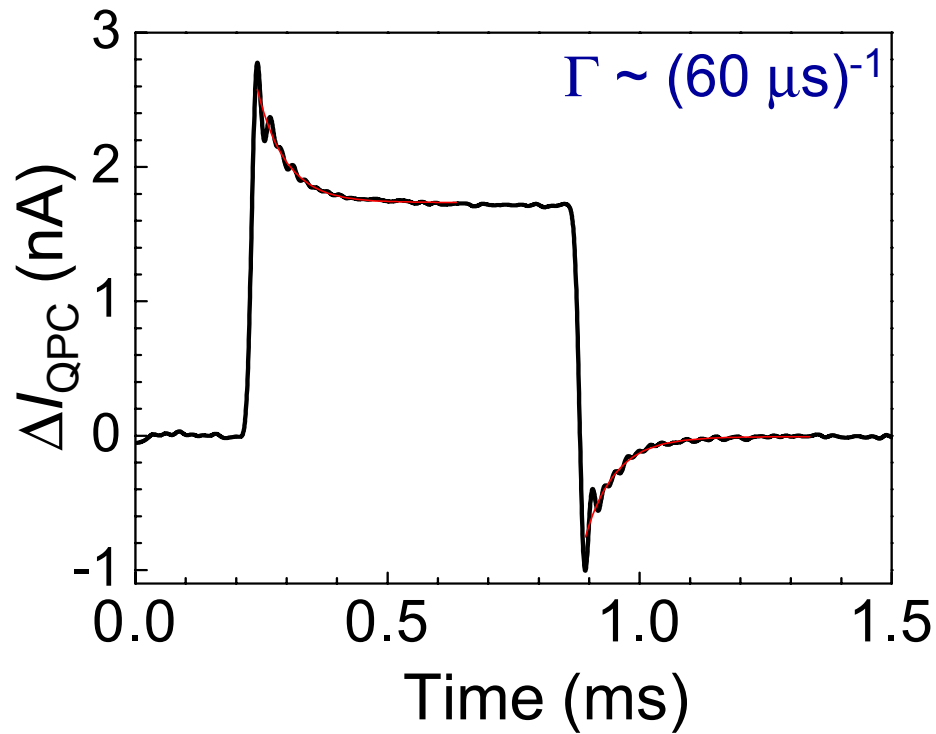


- Tunnel-in event too fast
- Tunnel-out event visible

Fastest tunnel events



Histograms tunnel time

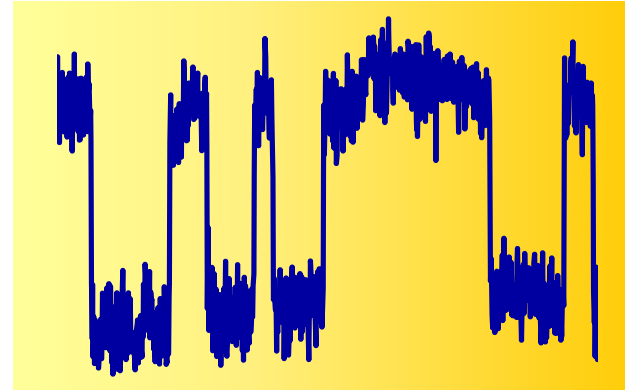


increase tunnel barrier

Summary...

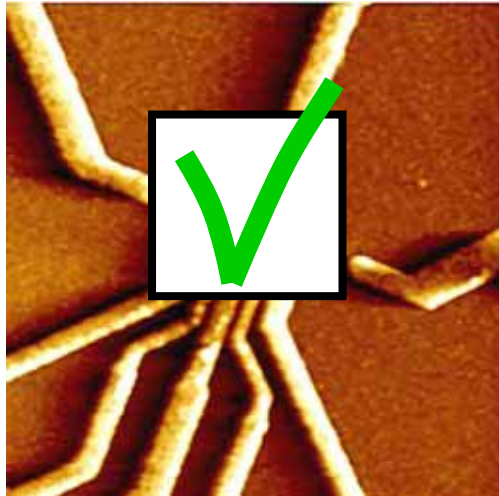
We can:

**measure single-electron
tunneling in *real-time***
($\sim 10 \mu\text{s}$)

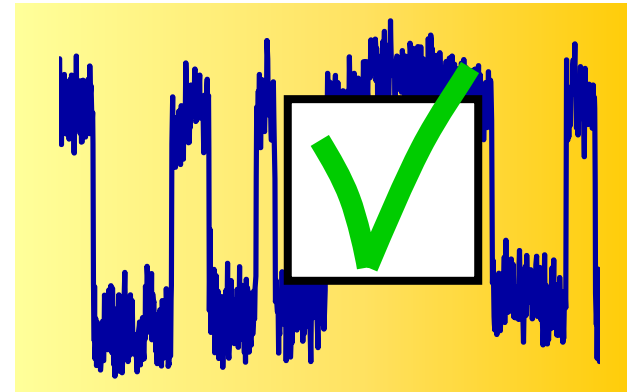


We need...

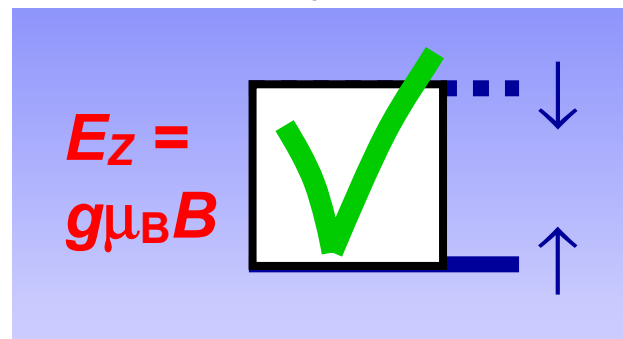
one-electron
double dots...



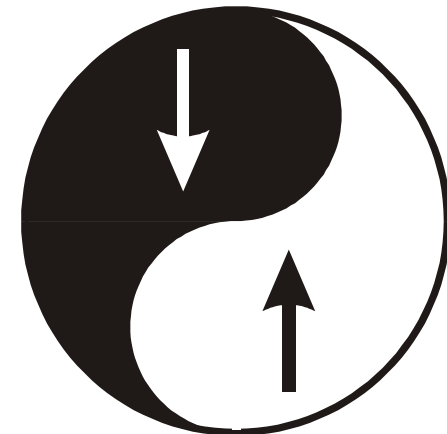
...fast charge
detection...



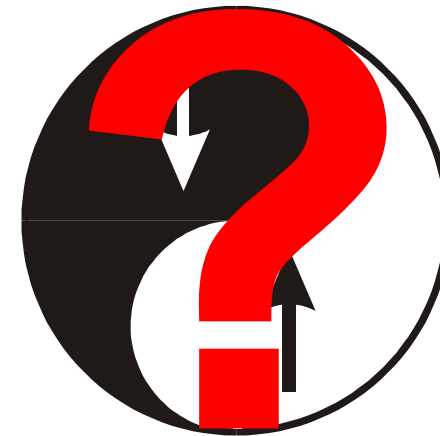
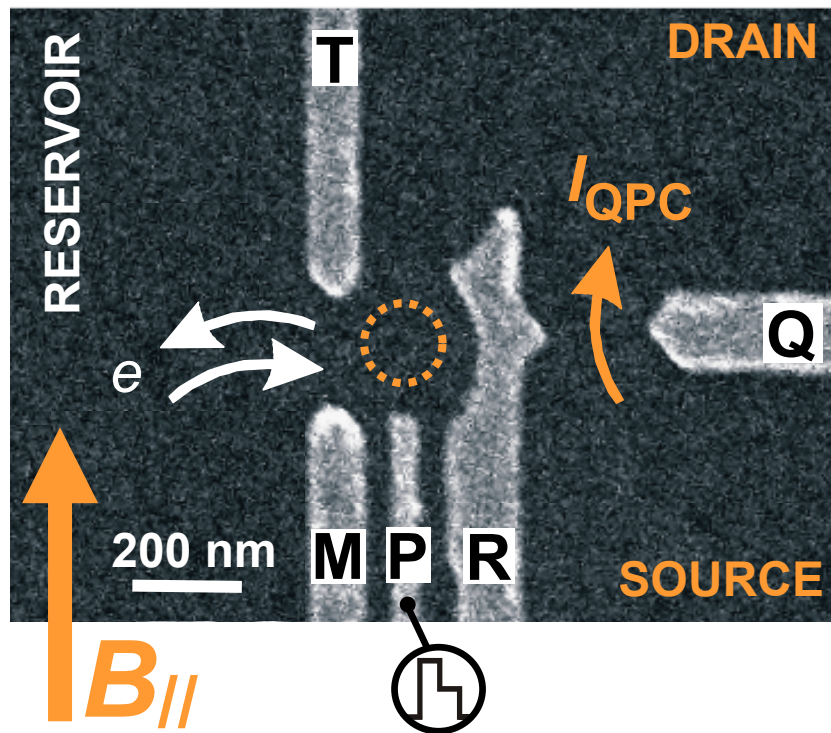
...two-level
system...



....single spin
measurement!

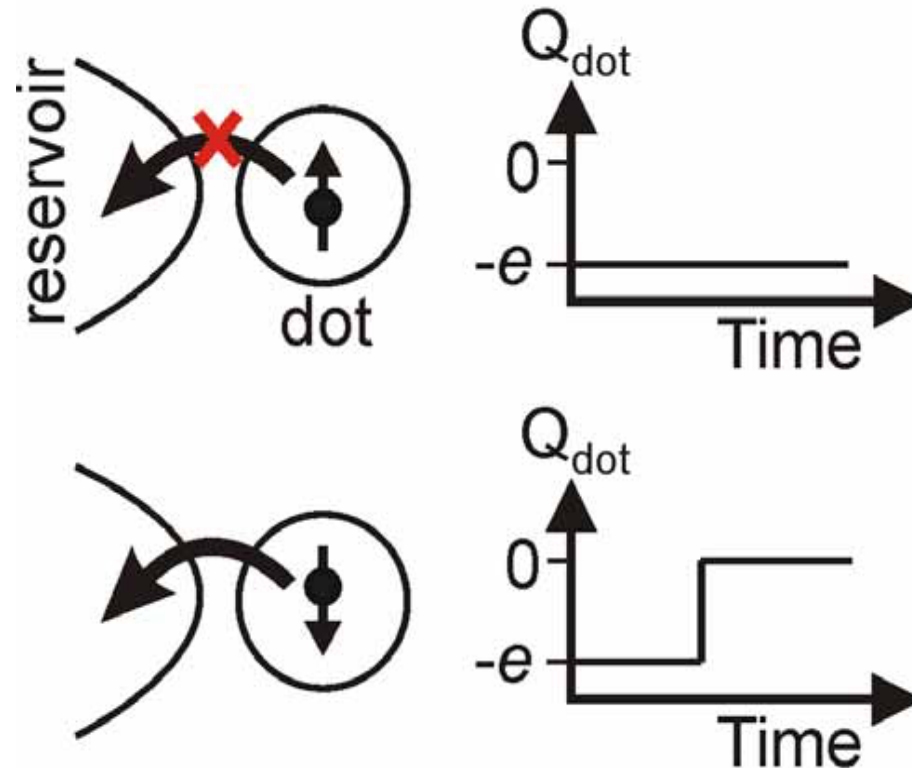


Single-shot measurement of a single electron-spin

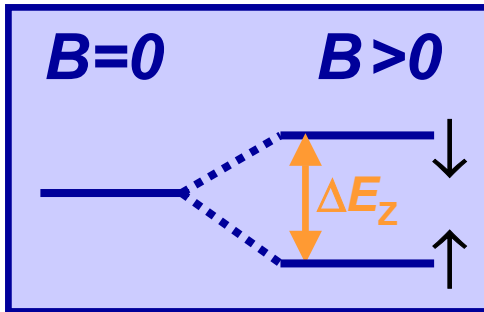


Single-spin measurement concept

- Spin magnetic moment: $\mu_B = 9.27 \times 10^{-23} \text{ A m}^2$ very small!
- But: spin attached to electron (which has charge)
- So: correlate spin orientation to electron's position
- Then measure charge

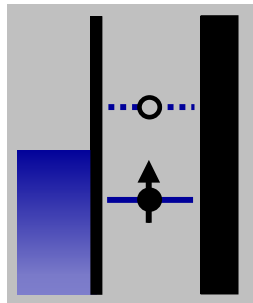


Spin-to-charge conversion



Use Zeeman splitting $\Delta E_Z = g\mu_B B$

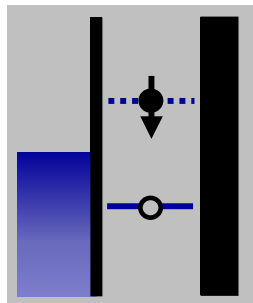
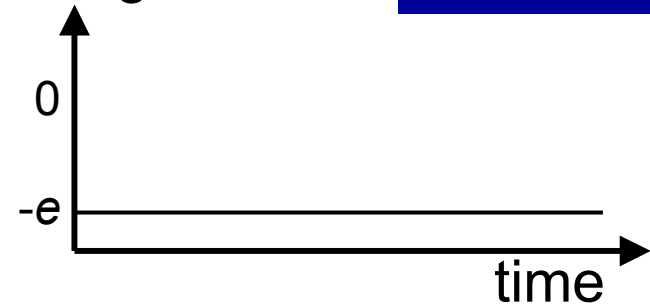
- Spin-to-charge conversion (within $T_1 > 50 \mu\text{s}$)
- Fast charge read-out



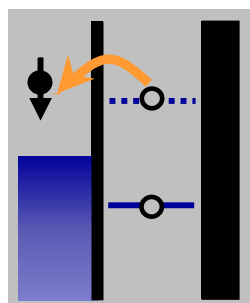
$N = 1$

charge

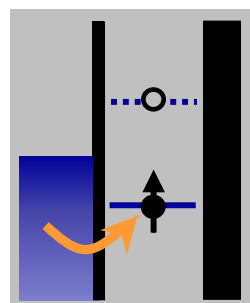
SPIN UP



$N = 1$



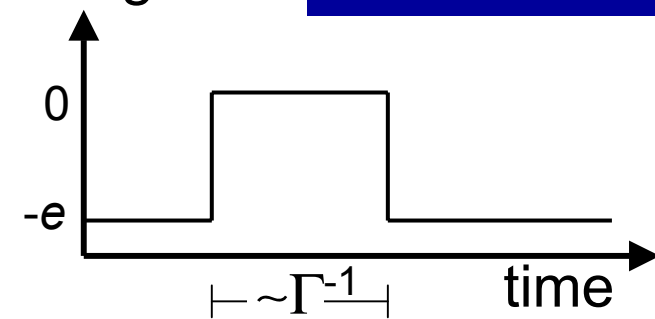
$N = 0$



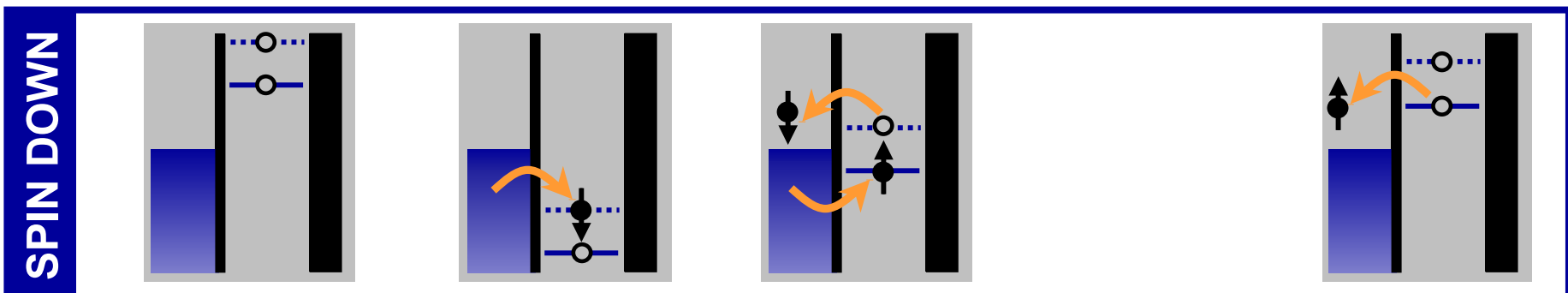
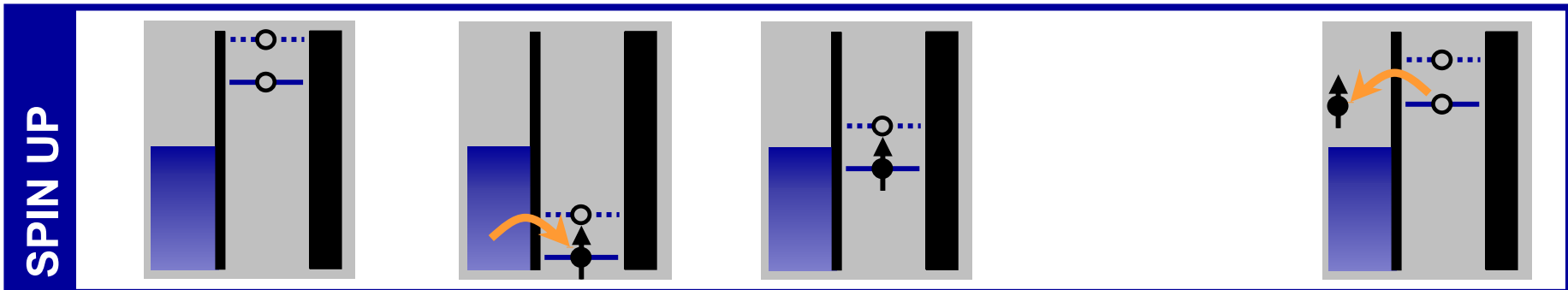
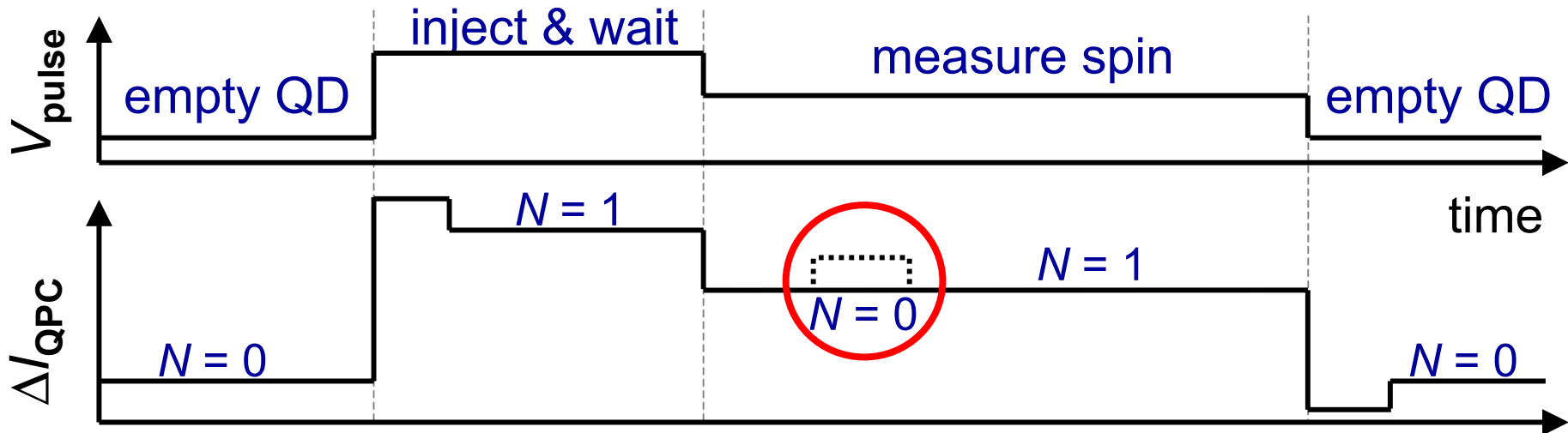
$N = 1$

charge

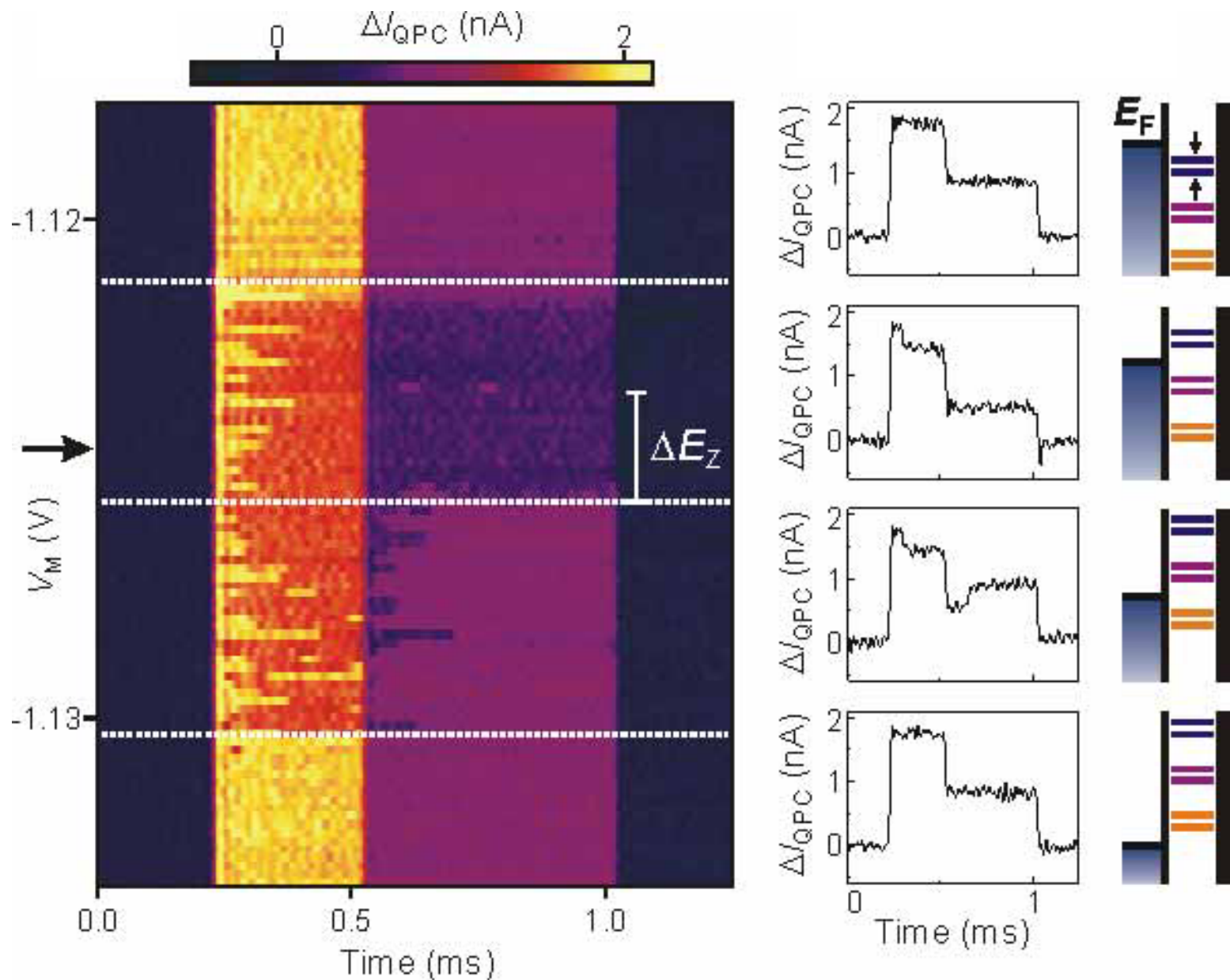
SPIN DOWN



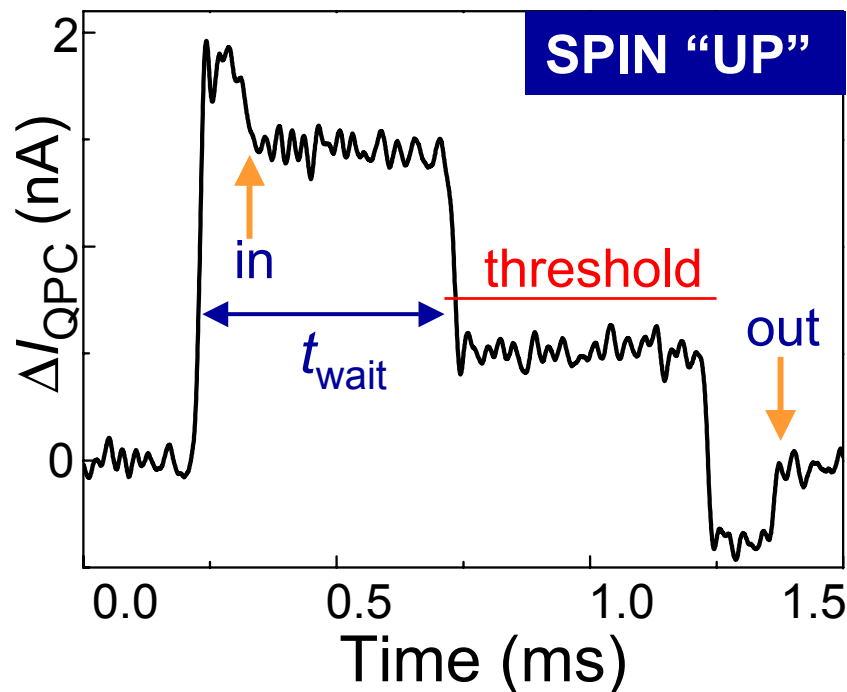
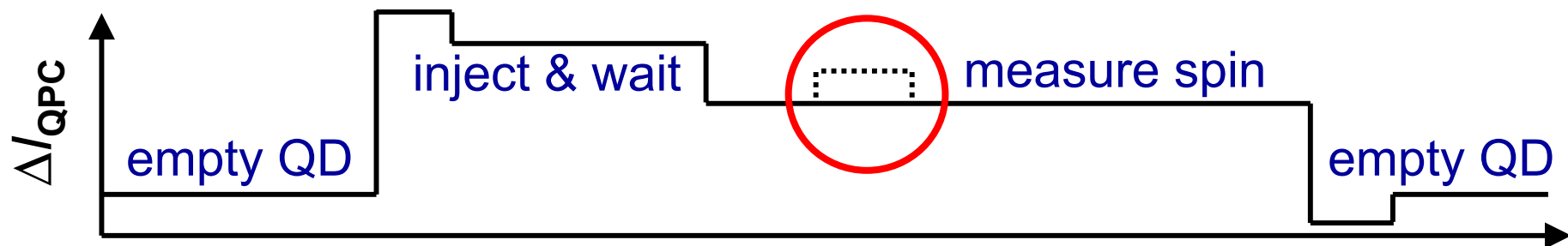
Spin read-out procedure



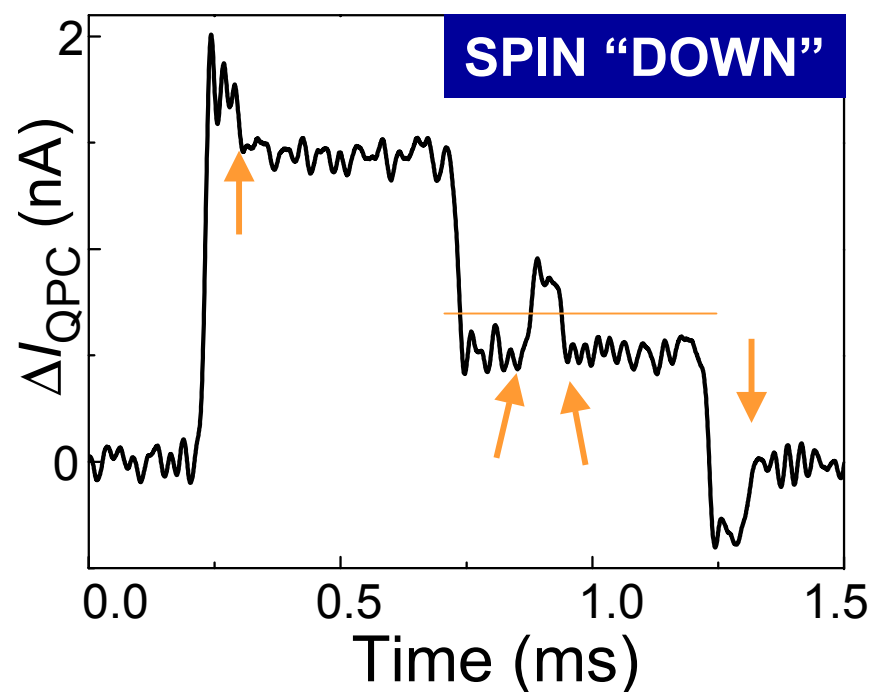
Finding the spin read-out regime



Single-shot spin read-out results

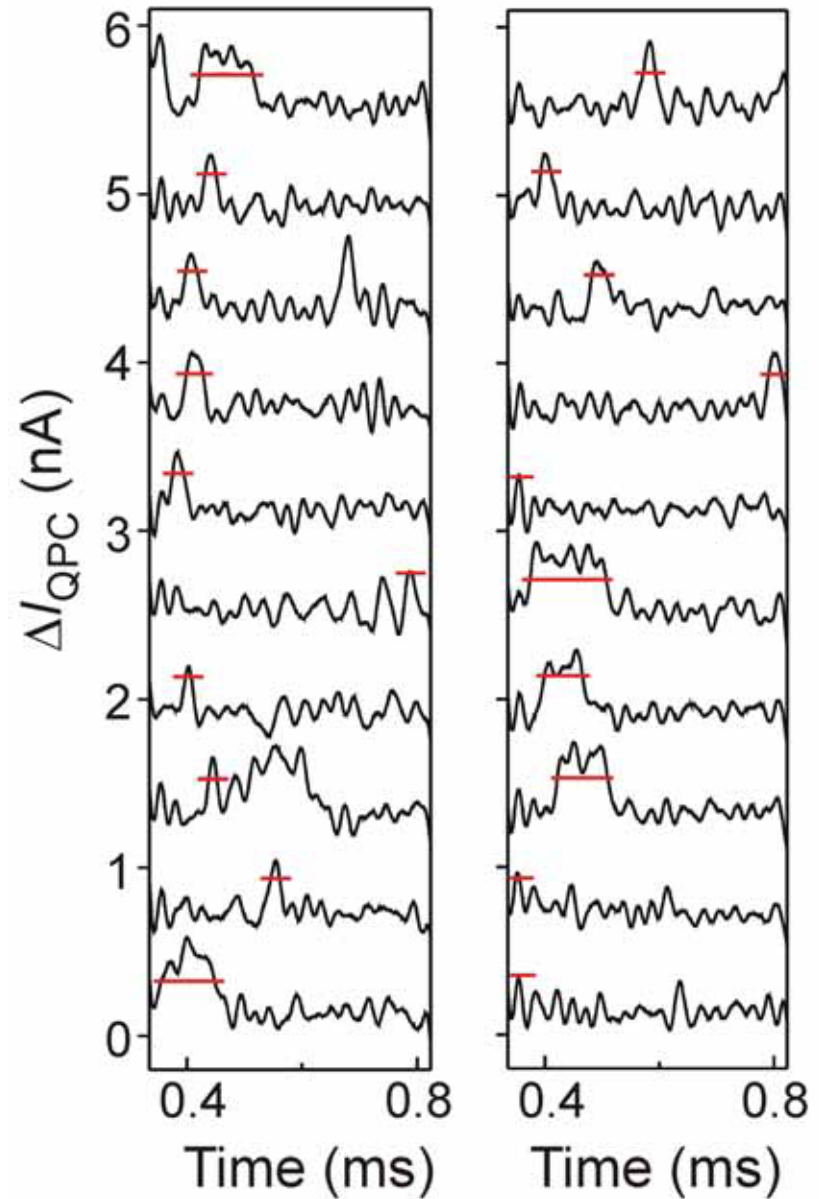
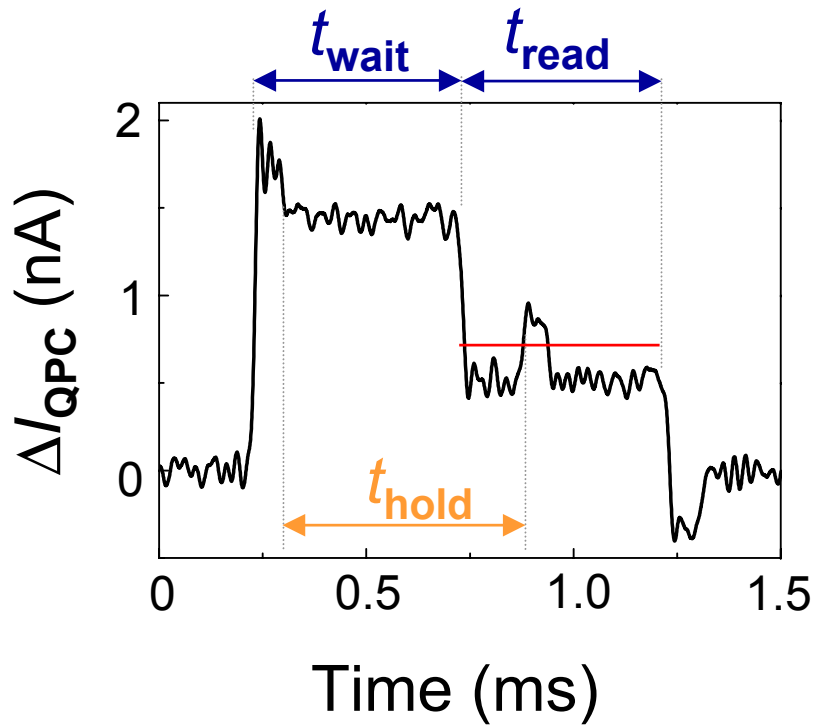


Flat signal in read-out region
♥ spin is "up"

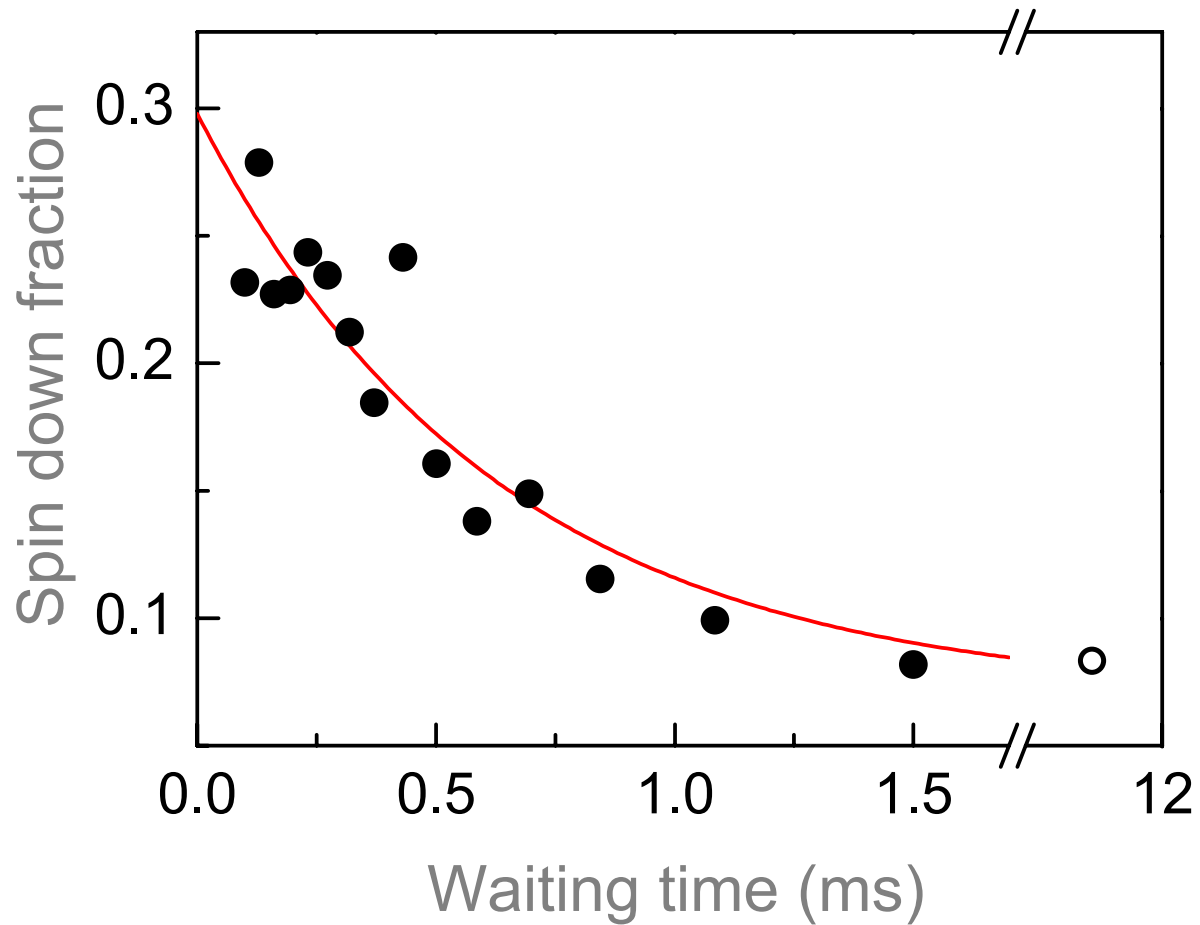


Step during read-out region
♥ spin is "down"

More spin-down traces

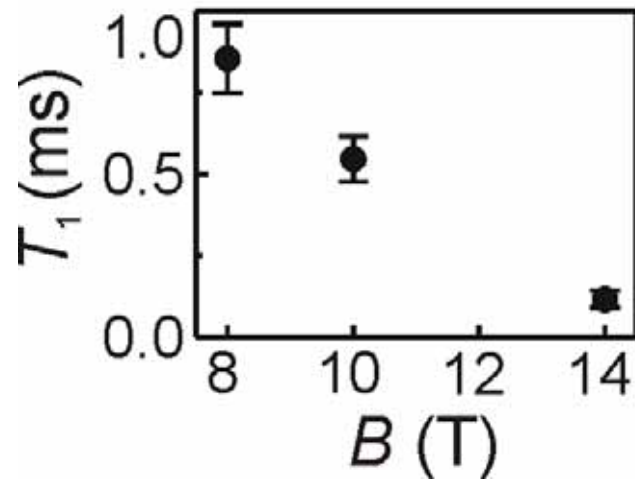


Verification spin read-out

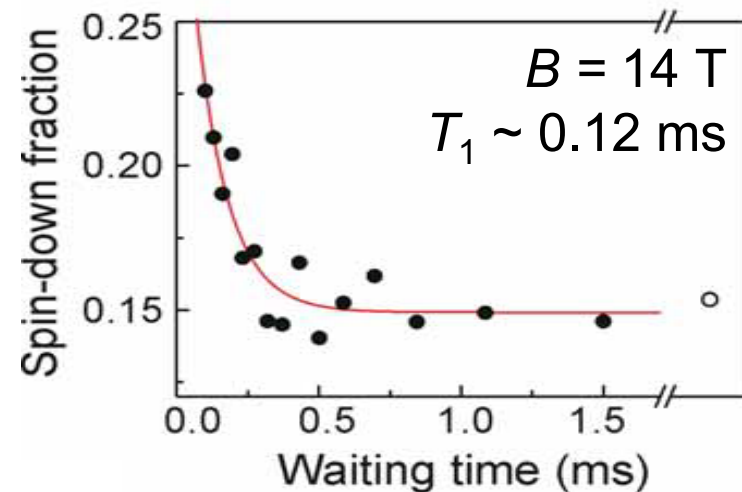
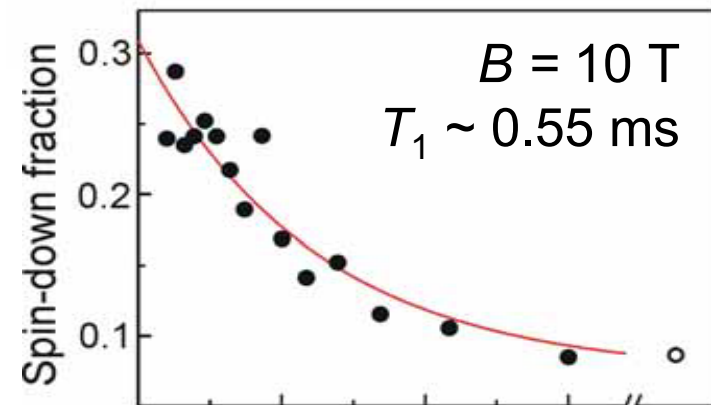
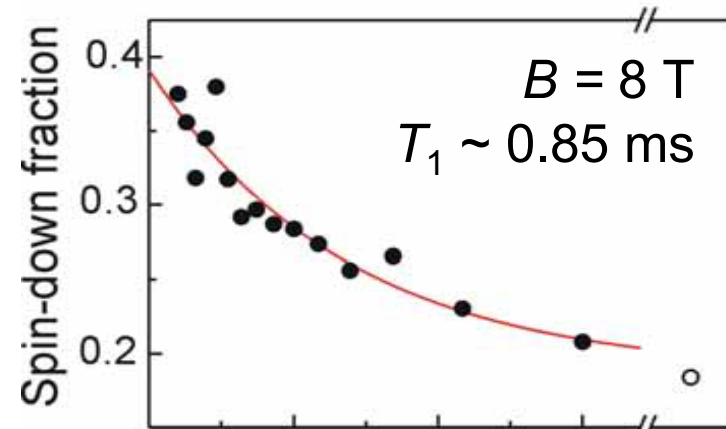


Measurement of T_1

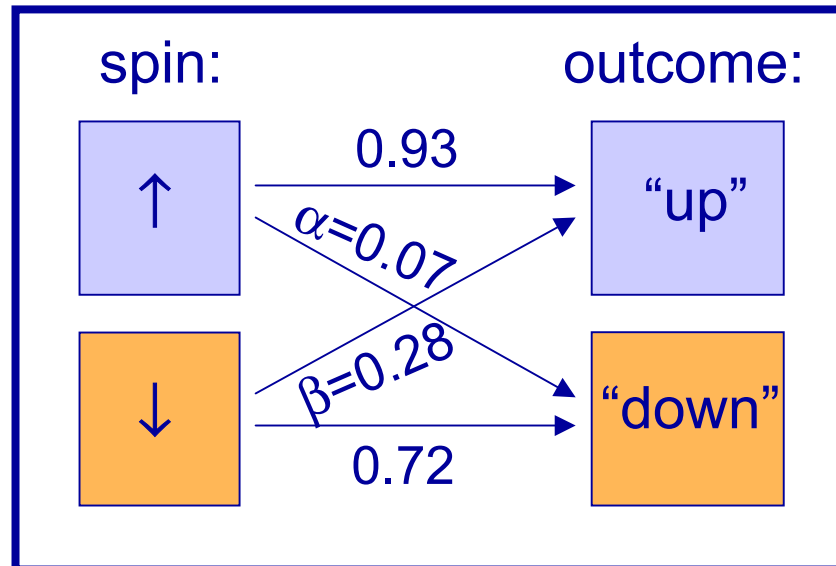
Elzerman *et al.*, to appear in Nature



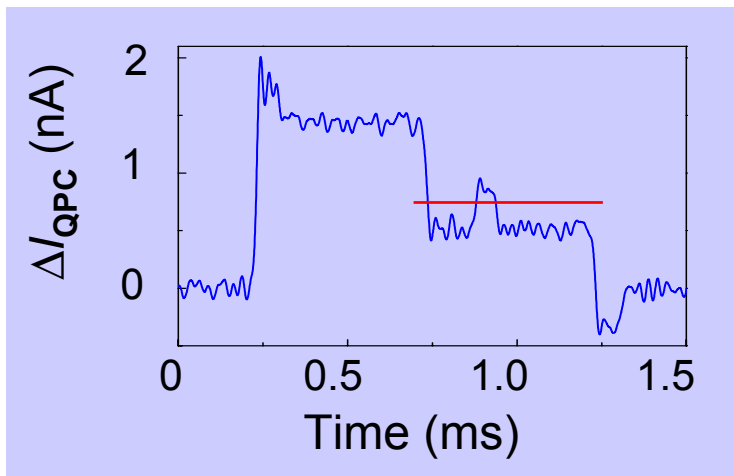
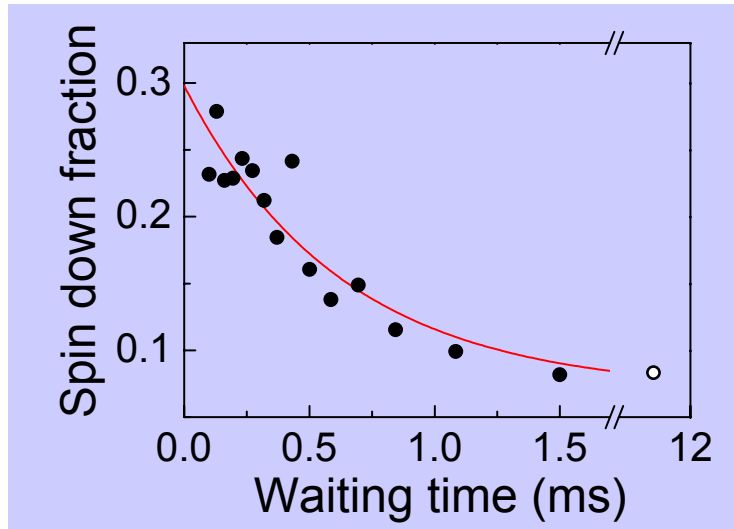
- Surprisingly long T_1
- T_1 goes up at low B



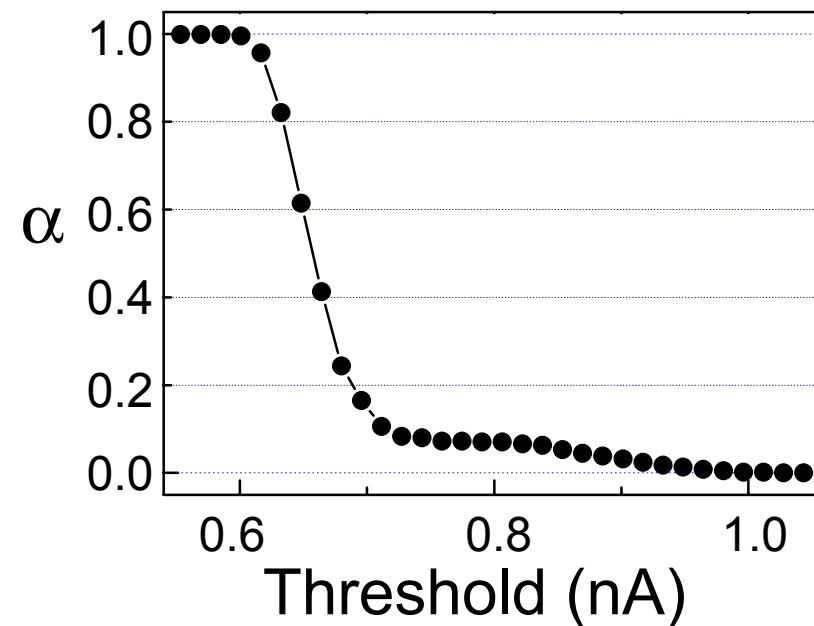
Read-out characterization



Characterization $\alpha = P$ (“down” if \uparrow)



$$p(1 - \beta - \alpha) \exp(-t / T_1) + \alpha$$

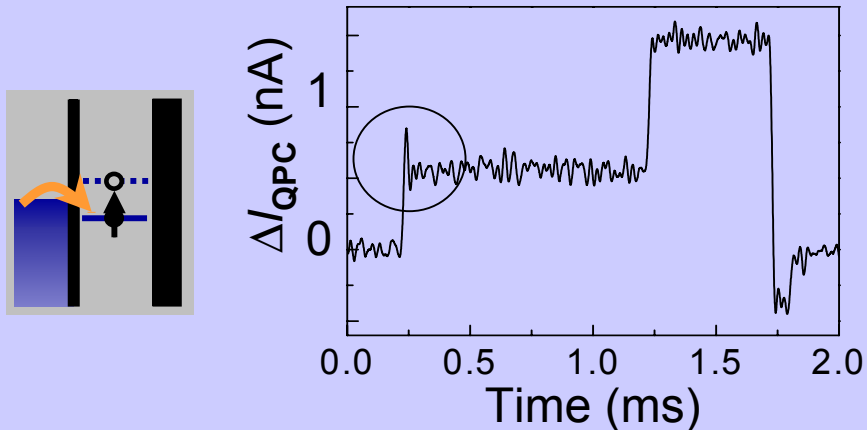


Characterization $\beta = P$ ("up" if \downarrow)

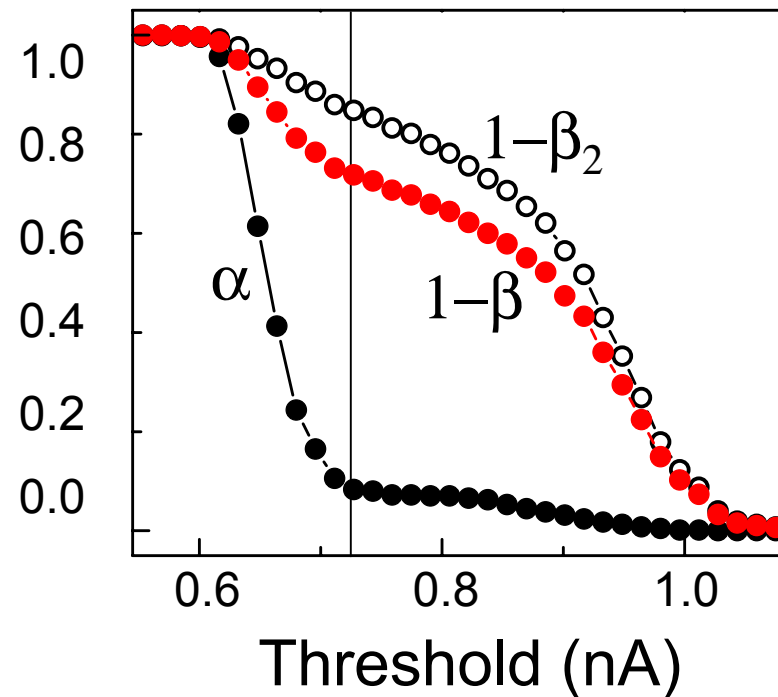
$\beta_1 = P$ (flips before tunnelling)

$$\frac{1/T_1}{1/T_1 + \Gamma_{\downarrow}} = \frac{1}{1 + \Gamma_{\downarrow} T_1}$$

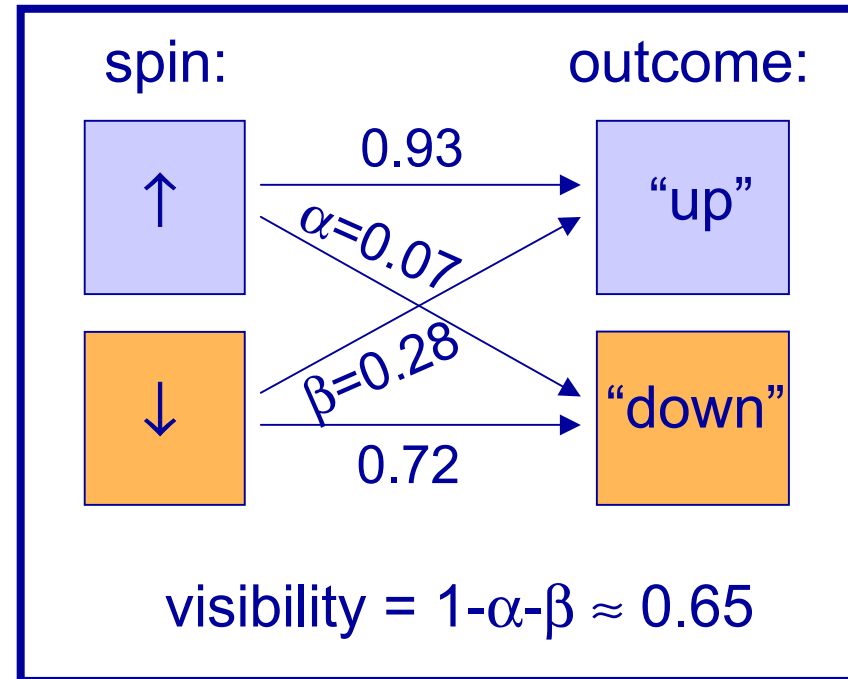
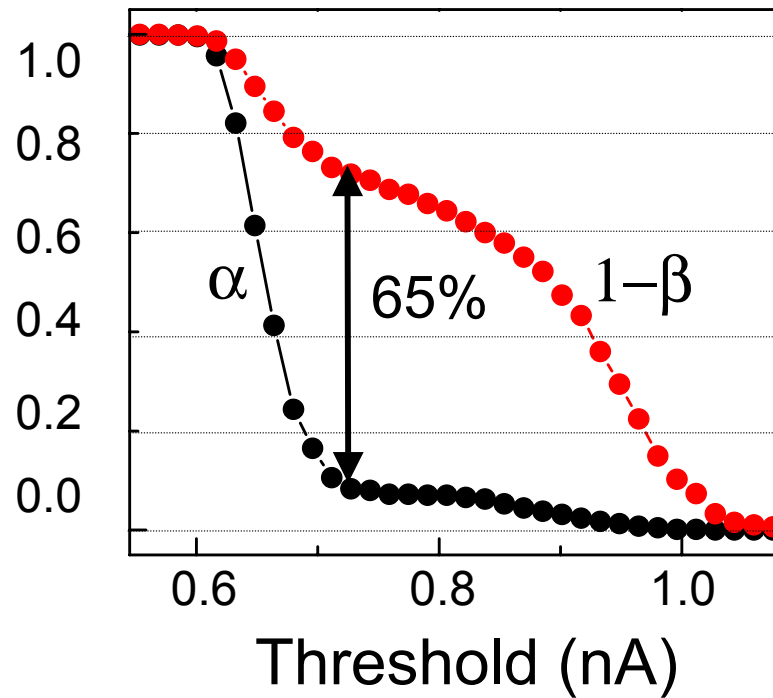
$\beta_2 = P$ (miss step)



$$1 - \beta = (1 - \beta_1)(1 - \beta_2) + \alpha\beta_1$$



Summary single-shot read-out fidelity



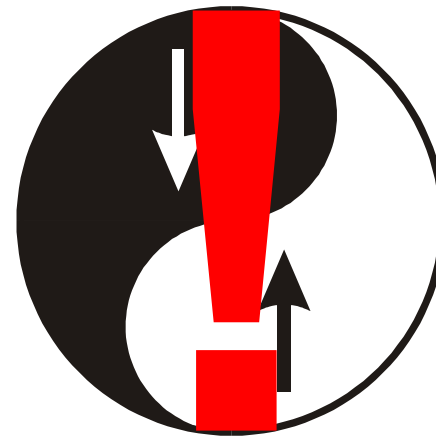
Future improvements:

- α : lower T_{el}
- β : faster charge detection

Summary...

We can:

**perform single-shot
measurement of one
electron-spin**

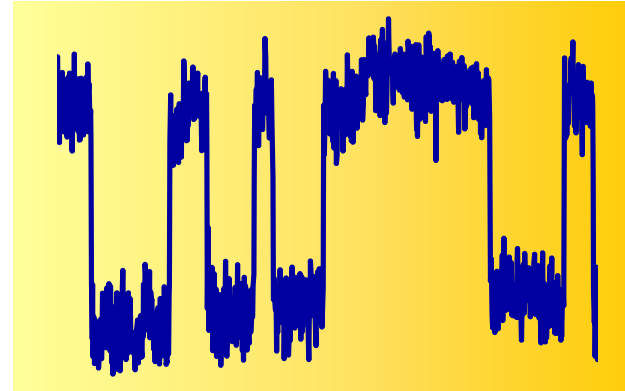


We have...

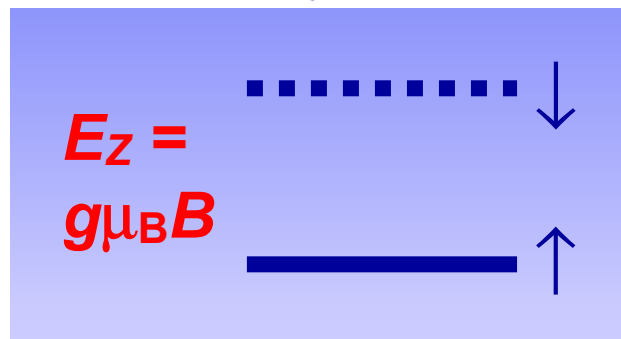
one-electron
double dots...



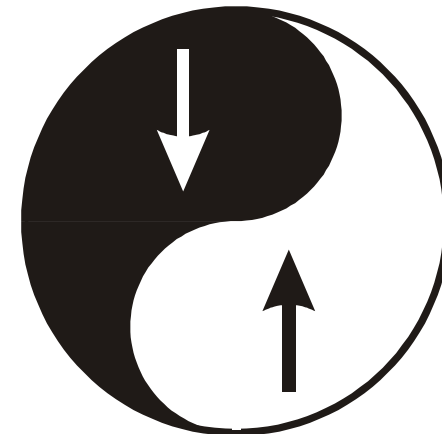
...fast charge
detection...



...two-level
system...



....single spin
measurement!



Part IV: Outlook

Coherent spin manipulation (ESR)

Two-qubit operations (SWAP^{1/2})

Bell's inequalities for massive particles

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Quantum simulation?

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Quantum computation??