Many-body Anderson localization in 1D

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Déjà vu?-- Anderson localization of a BEC (2009)

D. Delande, C. A. Müller, J. Zakrzewski, K. Sacha

Effective one body (EOB) approach

This work - full many body solution

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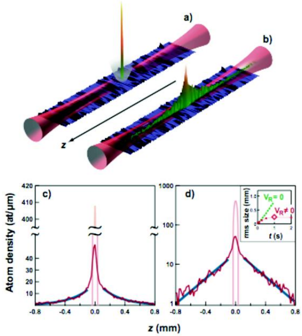
Scope:

- Anderson localization of a BEC – experiments.
- Bright solitons in a BEC.
- Anderson localization of bright solitons- EOB approach.
- Many-body Anderson localization.
- Conclusions.

Anderson localization of a BEC

Experiments

$$\left[-\frac{1}{2}\partial_z^2 + V(z) + g|\phi(z)|^2\right]\phi(z) = \mu\,\phi(z), \qquad \langle\phi|\phi\rangle = N$$



Experiments in the non-interacting limit...

J. Billy et al., Nature **453**, 891 (2008) G. Roati et al., Nature **453**, 895 (2008)

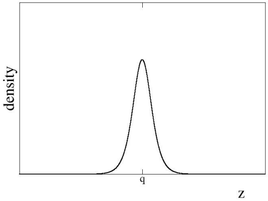
Bright solitons in a BEC

Gross-Pitaevskii equation:

$$\left[-\frac{1}{2}\partial_z^2 + g|\phi_0|^2\right]\phi_0 = \mu\phi_0, \qquad g < 0 \qquad \langle\phi|\phi\rangle = N$$

$$\phi_0(z-q) = \sqrt{\frac{N}{2\xi}}\,\frac{\exp(-i\theta)}{\cosh\left(\frac{z-q}{\xi}\right)},$$

$$\xi = \frac{-2}{Ng} \qquad \mu = -\frac{N^2 g^2}{8}$$



Yet full exact many-body solution - uniform

Bright solitons in a BEC

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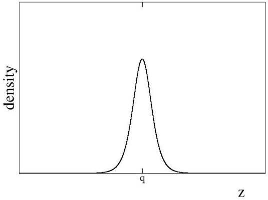

Yet full exact many-body solution - uniform
The position of the center of mass should be treated quantum mechanically.
q - position operator for N particle soliton.

Bright soliton in a BEC

The effective Hamiltonian

If we add **weak disorder** potential $V(z)$, the effective Hamiltonian describing center of mass motion reads:

$$\hat{H}_{\text{eff}} \approx \frac{p^2}{2N} + \int dz\ V(z)\ |\phi_0(z-q)|^2 .$$

- Simple arguments:

  Substitution of a time-dependent bright soliton solution $\sim e^{ipz}\phi_0(z-q)$ to the energy functional

$$E = \int dz \left[\frac{1}{2}|\partial_z\phi|^2 + \frac{g}{2}|\phi|^4 - \mu|\phi|^2 + V(z)|\phi|^2\right] = \frac{p^2}{2N} + \int dz V|\phi_0(z-q)|^2$$

- Bogoliubov theory:
  (J. Dziarmaga, (2004) for $V = 0$)

$$\hat{H} \approx \sum_{n, E_n > 0} E_n \, \hat{b}_n^\dagger \, \hat{b}_n + \frac{\hat{p}^2}{2N} + \int dz \; V(z) \, |\phi_0(z-q)|^2 .$$

$$N \; V_0 \;\ll\; E_1 = |\mu| = \frac{N^2 g^2}{8},$$

Bright solitons in a BEC

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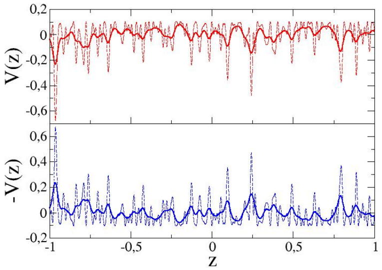
Weak perturbation cannot populate internal excited states of the soliton.
Shape preserved

Bright solitons in a BEC

Anderson localization

- $V(z)$ is optical speckle potential with correlation length $\sigma_0 \ll \xi$.
- To the second order (Born approximation) in the potential strength V_0, inverse localization length, valid for $\gamma(k) \ll k$,

$$\gamma(k) \approx \frac{N^2}{k^2} \pi \sigma_0 V_0^2 \left(\frac{N\pi k\xi}{\sinh(\pi k\xi)} \right)^2$$



Bright solitons in a BEC

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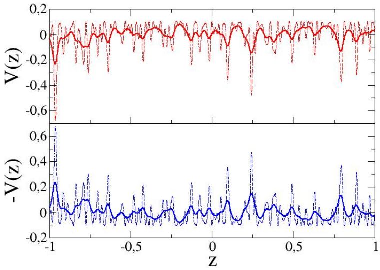

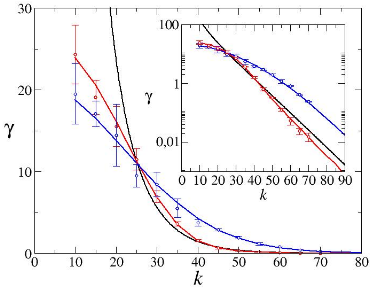

$NV_0 = 10^{-2}|\mu|, \; \sigma_0 = 0.3\xi, \; N = 100$

Bright solitons in a BEC

Proposal for an experiment

- Suppose we prepare soliton in a harmonic trap $\frac{1}{\sqrt{\omega_z}} \gg \xi$,

$$|\tilde{\psi}(k)|^2 = \frac{1}{\sqrt{\pi N \omega_z}} e^{-k^2/N\omega_z}$$

- Long-time density averaged over disorder realizations

$$\overline{|\psi(q)|^2} \; = \; \frac{1}{2} \int dk \; \gamma \; e^{-\gamma |q|} \; |\tilde{\psi}(k)|^2 \; \sim \; \frac{1}{|q|}$$

Bright solitons in a BEC

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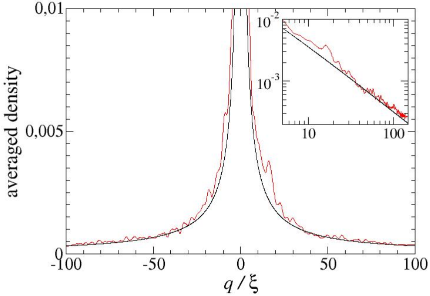


$\omega_\perp = 2\pi \cdot 5000$ Hz
$\omega_z = 2\pi \cdot 64$ Hz
N=100 Li^7 atoms
$\xi = 1\ \mu$m
$\sigma_0 = 0.27\ \mu$m
$NV_0 = 10^{-2}|\mu|$
$t = 5$ s

Any many body effect omitted in the former effective CM quantization approach could destroy the phase coherence and the wavefunction.
Need for a full many body test

$$\hat{H} = \int dz\ \hat{\psi}^\dagger(z) \left[-\frac{1}{2}\ \frac{\partial^2}{\partial z^2} + V(z) \right] \hat{\psi}(z) + \frac{g}{2} \int dz\ \hat{\psi}^\dagger(z)\ \hat{\psi}^\dagger(z)\ \hat{\psi}(z)\ \hat{\psi}(z)$$

Discretization (3-point kinetic energy) gives Bose-Hubbard Hamiltonian:

$$H = \sum_l \left[-J(a_l^\dagger a_{l+1} + h.c.) + \frac{U}{2} a_l^\dagger a_l^\dagger a_l a_l + V_l\ a_l^\dagger a_l \right]$$

$J = \frac{1}{2\delta^2}$, $U = \frac{g}{\delta}$ and $V_l = V(z_l)$, $z_l = l\delta$

B. Schmidt and M. Fleischhauer, Phys. Rev. A75 (2007)

Many-body approach to Anderson localization

Technicalities

- space restricted to $2K+1=1921$ points $[-K\delta, K\delta]$
- Matrix product state (MPS) cvariational representation

$$|\psi\rangle = \sum_{\alpha_1,\ldots,\alpha_M;\ i_1,\ldots,i_M} \Gamma^{[1],i_1}_{1\alpha_1}\lambda^{[1]}_{\alpha_1}\Gamma^{[2],i_2}_{\alpha_1\alpha_2}\ldots\Gamma^{[M],i_M}_{\alpha_{n-1}1}|i_1,\ldots,i_M\rangle$$

  $\Gamma^{[l],i_l}$ - site dependent tensors, $\lambda^{[l]}$ - bond vectors
- We find a quasi-exact many body ground state – bright soliton in a shallow trap (imaginary time propagation TEBD)
- Trap is removed, disorder turned on, real time propagation with TEBD
- Reliable calculations for $N=25$ particles

D. Delande et al., New. J. Phys. (2013)

Many-body approach to Anderson localization

Technicalities

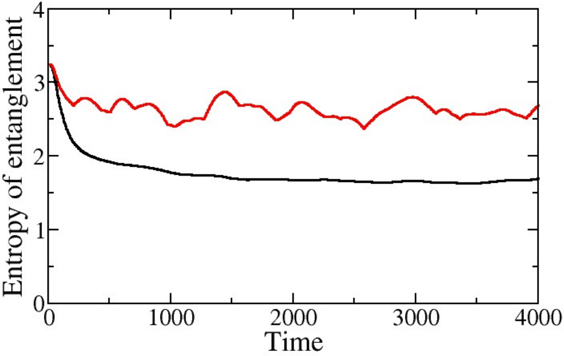
- Unit of length – soliton size ξ – Unit of time ξ^2
- Initial harmonic oscillator $\omega = 0.025/\xi^2$ (not to disturb the soliton shape yet to confine CM to a distance slightly larger than ξ)
- strength of the random potential comparable to soliton energy $\omega/4$ i.e. $V_0 = 2.5 \times 10^{-4}$
- correlation of the disorder $\sigma_0 = 0.4\xi$
- discretization $\delta = \xi/5$ tests on smaller...
- time step (Trotter errors!) $dt = 0.008\xi^2$
- $N_{max} = 14$ (needed!) despite $N\delta/2\xi = 2.5$ for $N = 25$
- $\chi = 30$ (small possible) dimension per site 450 (1921 sites)

Many-body approach to Anderson localization

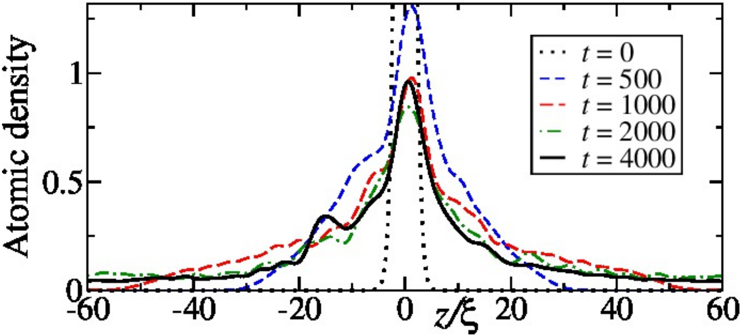
Tests

- χ, N_{max}, dt, ...
- entropy of entanglement growth

$$S = \sup_l S_l = \sup_l \left[-\sum (\lambda_\alpha^{[l]})^2 \ln (\lambda_\alpha^{[l]})^2 \right]$$



- Atomic density in time

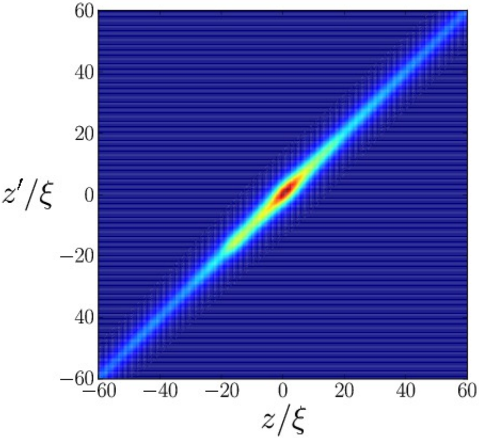


96 realizations of disorder

Many-body approach to Anderson localization

Results

- One body density matrix $\langle \psi^\dagger(z)\psi(z')\rangle$



Transverse width $\approx \xi$. Largest eigenvalue $=$ condensate fraction $=0.14$!

Many-body approach to Anderson localization

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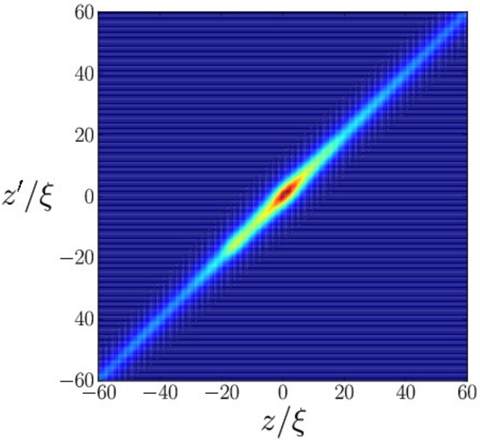
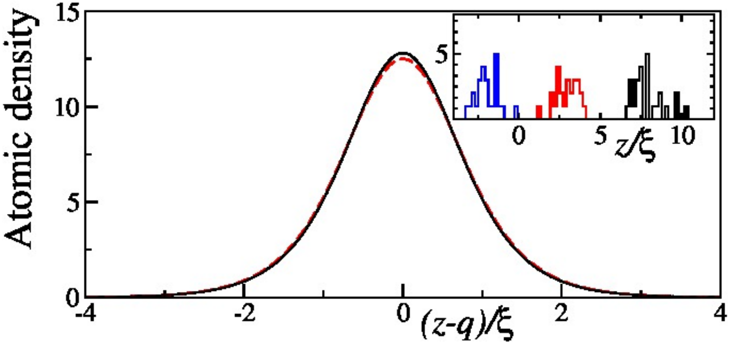


Transverse width $\approx \xi$. Largest eigenvalue $=$ condensate fraction $=$0.14.

Many-body approach to Anderson localization

Simulation of the measurement

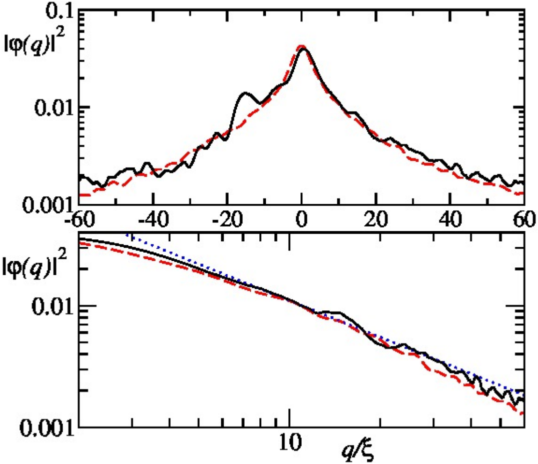
- From MPS repesentation $\rightarrow \rho^{[l]}$ by contraction of tensors..
- Choose n_l according to the statistical distribution
- Project MPS on subspace with that n_l on l site and normalize
- Repeat scanning other sites till reaching N



Many-body approach to Anderson localization

Comparison with effective one body of 2009

- Let us compare CM densities coming from both approaches



96 versus 10 000 realizations of disorder for EOB. Dotted 1/q.

Conclusions:

- AL for attractive interactions in 1D disorder
- Excellent agreement between full many body and EOB description
- Full simulation of the experiment including the measurements possible.