Taming quantum chaos in the many-body Wannier-Stark system

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Complex Dynamics in Quantum Systems

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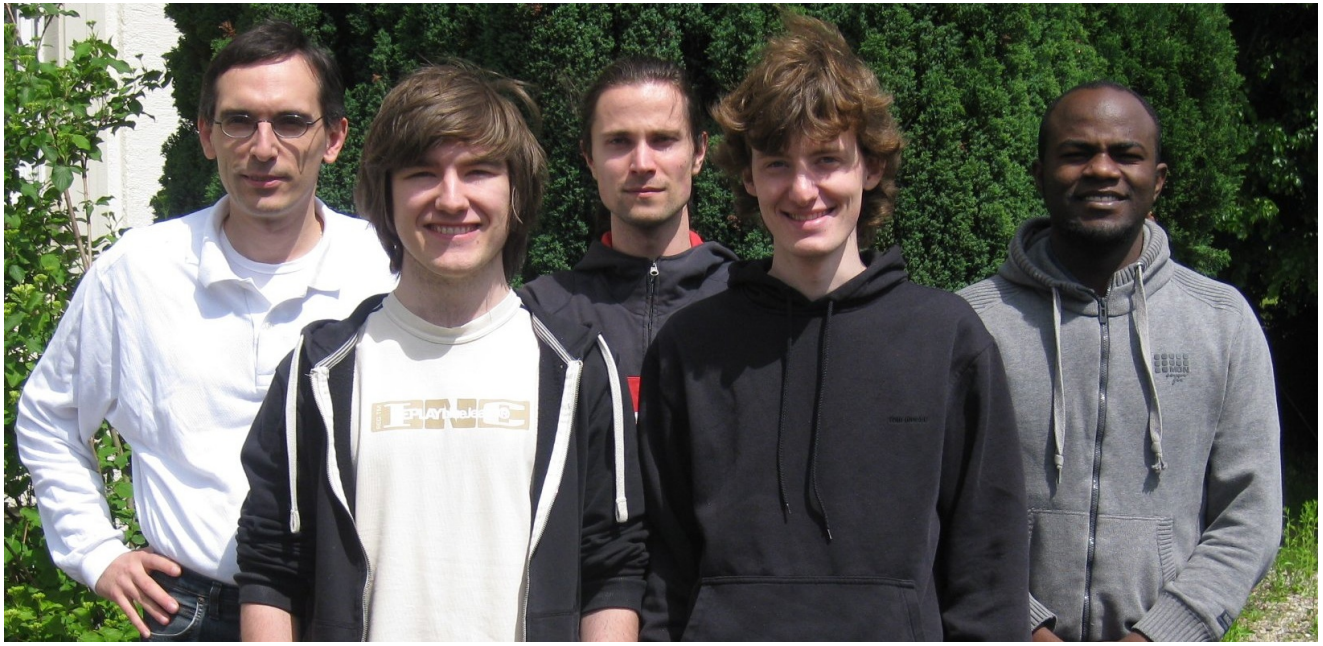
Center for Quantum Dynamics

FOR760: Scattering Systems with Complex Dynamics

Many thanks to



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Collaborators

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- Andrey Kolovsky (Krasnoyarsk, T)
- Peter Schlagheck (Liège, T)

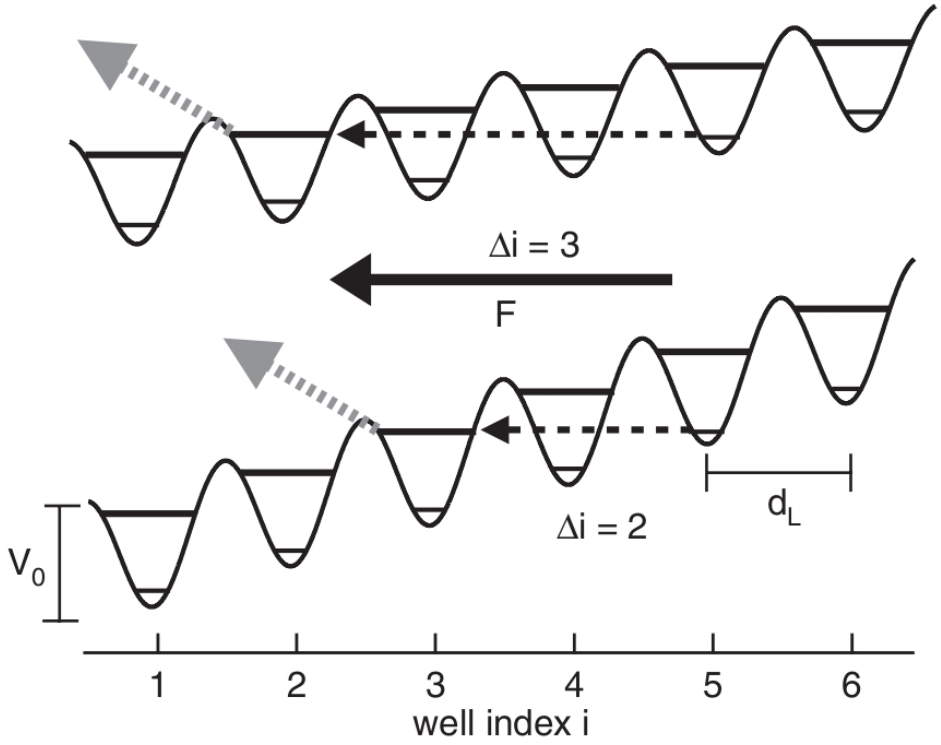
Program

1. Hors-d'œuvre
2. Wannier-Stark: Single-particle – Mean-field – Many-body
3. Perspectives

The Wannier-Stark problem

Single-particle quantum transport in a 1D spatially periodic lattice:

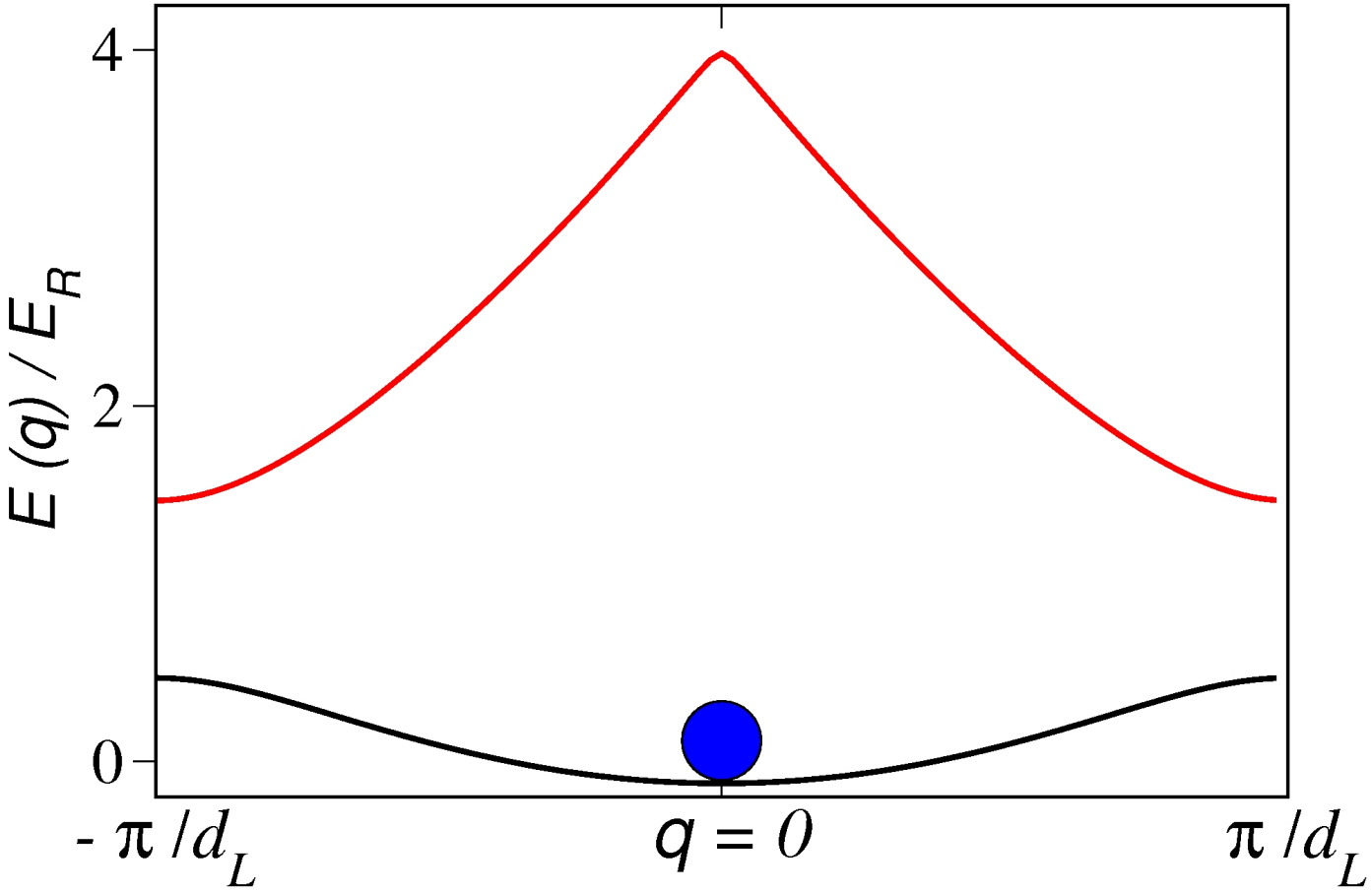
$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[-\frac{\hbar^2}{2M}\frac{\partial^2}{\partial x^2} + V\,\sin^2\left(\frac{\pi x}{d_L}\right) + Fx\right]\psi(x,t)$$



review: Arimondo & SW in *Dynamical Tunneling*, CRC-Press, 2011

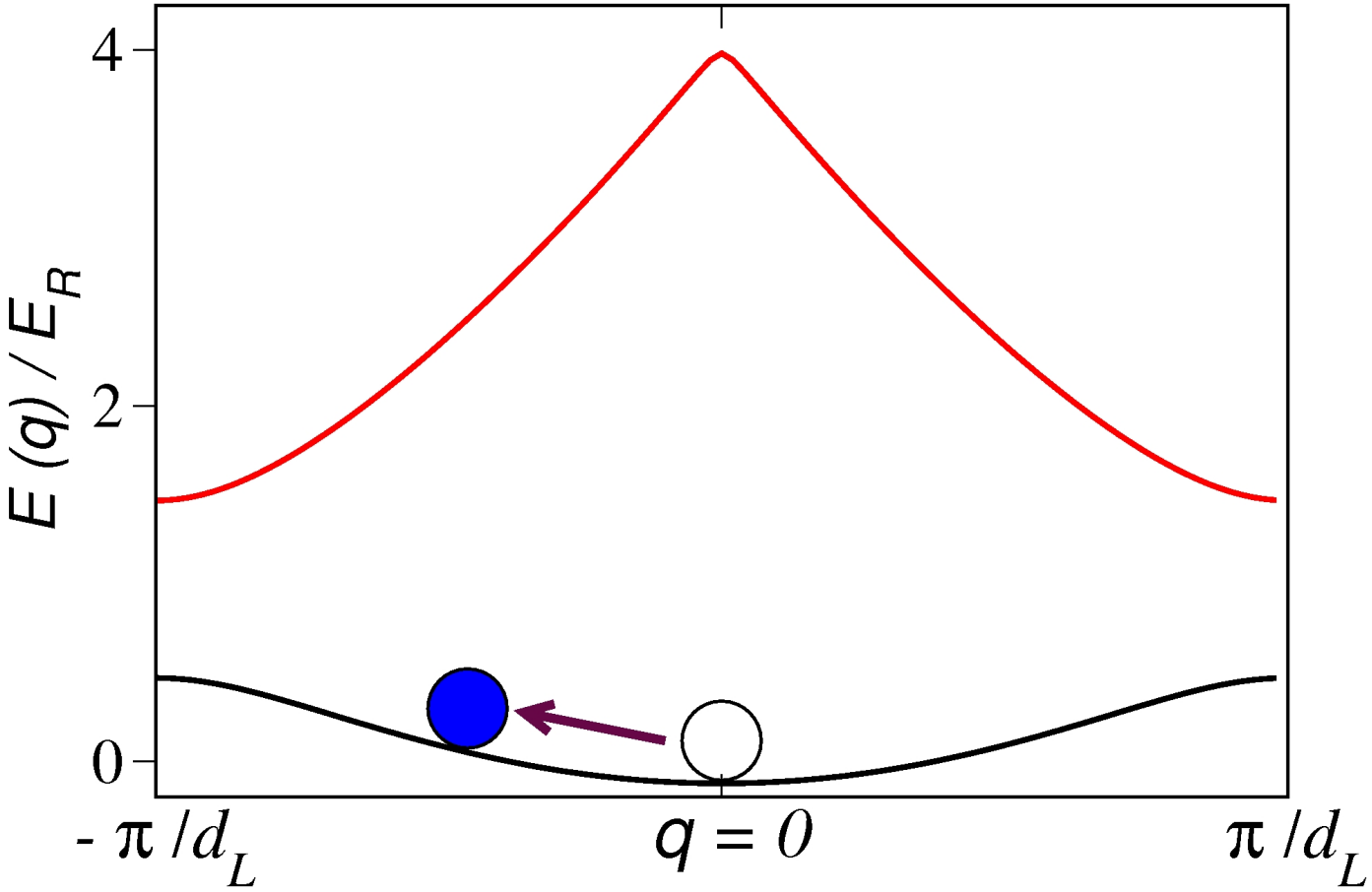
The Wannier-Stark problem – interband dynamics

Band structure of unperturbed system ($F = 0$):



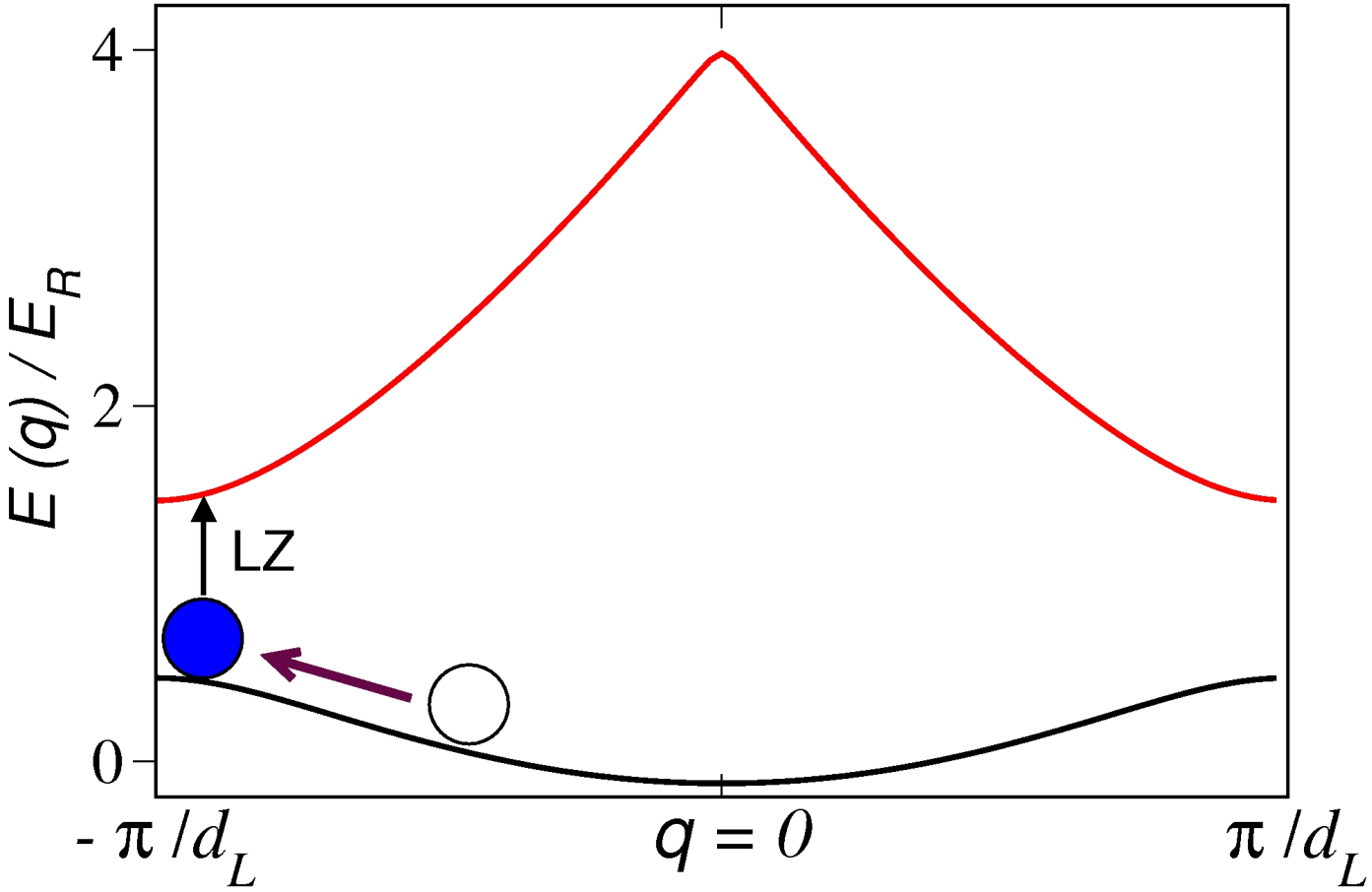
The Wannier-Stark problem – interband dynamics

With Wannier-Stark force ($F \neq 0$):



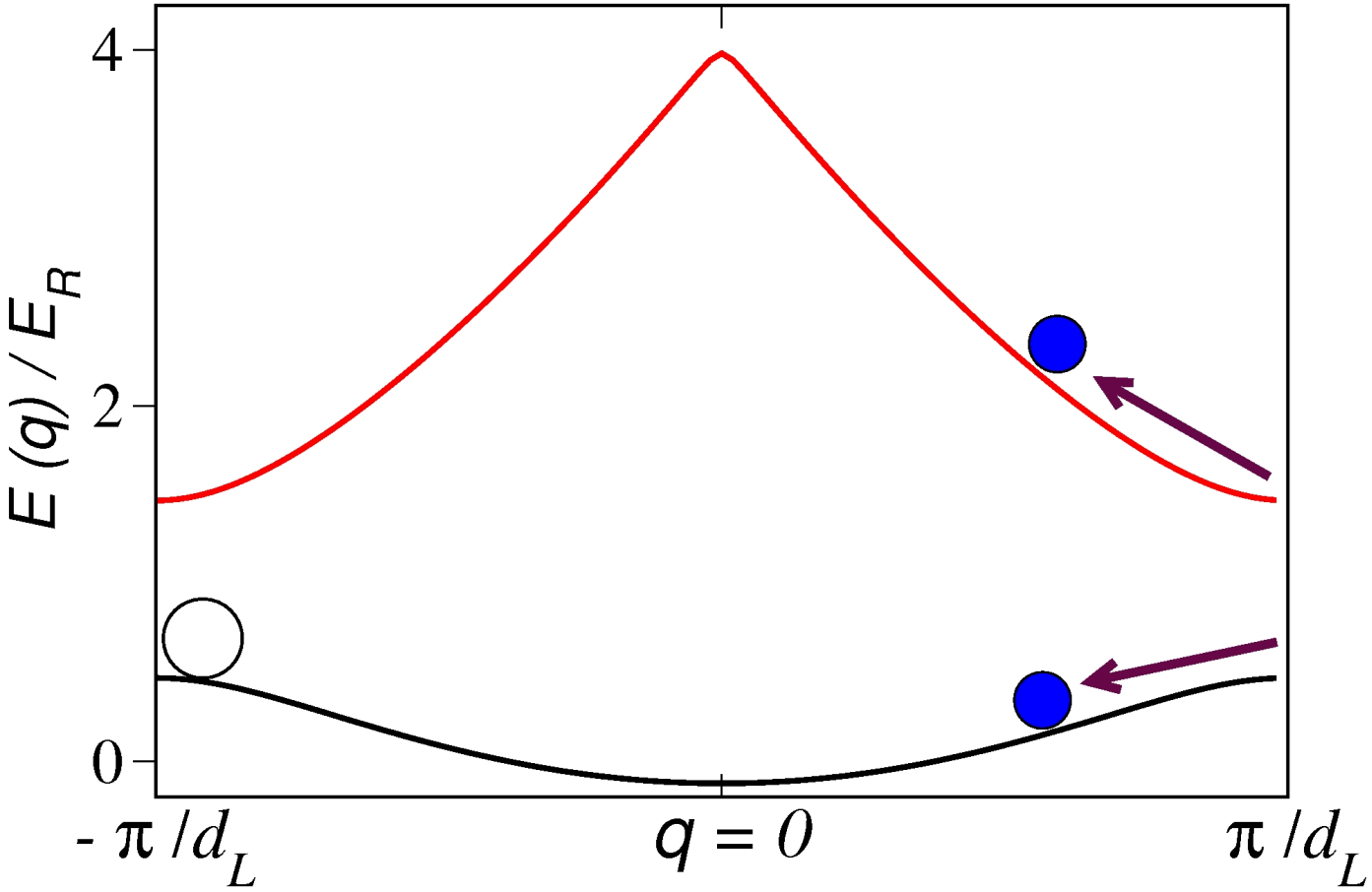
The Wannier-Stark problem – interband dynamics

Bloch oscillations and Landau-Zener (LZ) tunnelling:



The Wannier-Stark problem – interband dynamics

Landau-Zener tunnelling between two bands:



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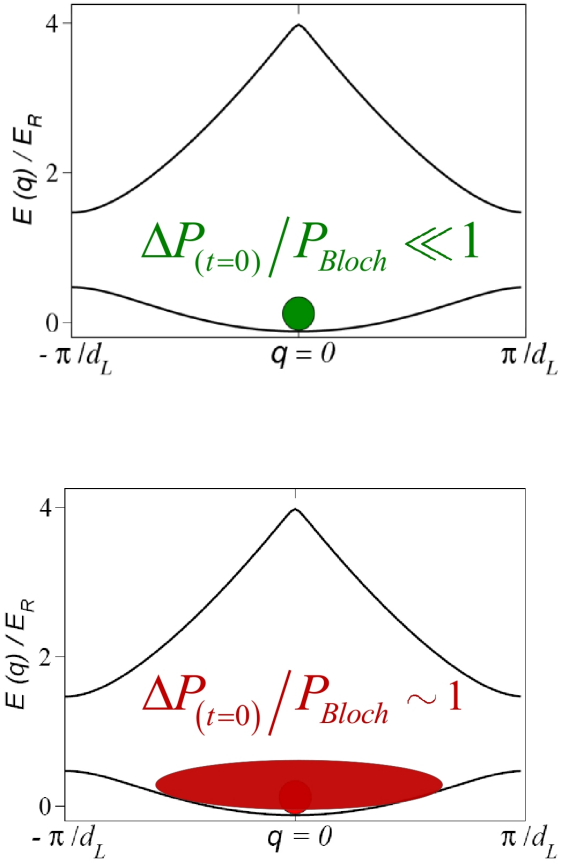
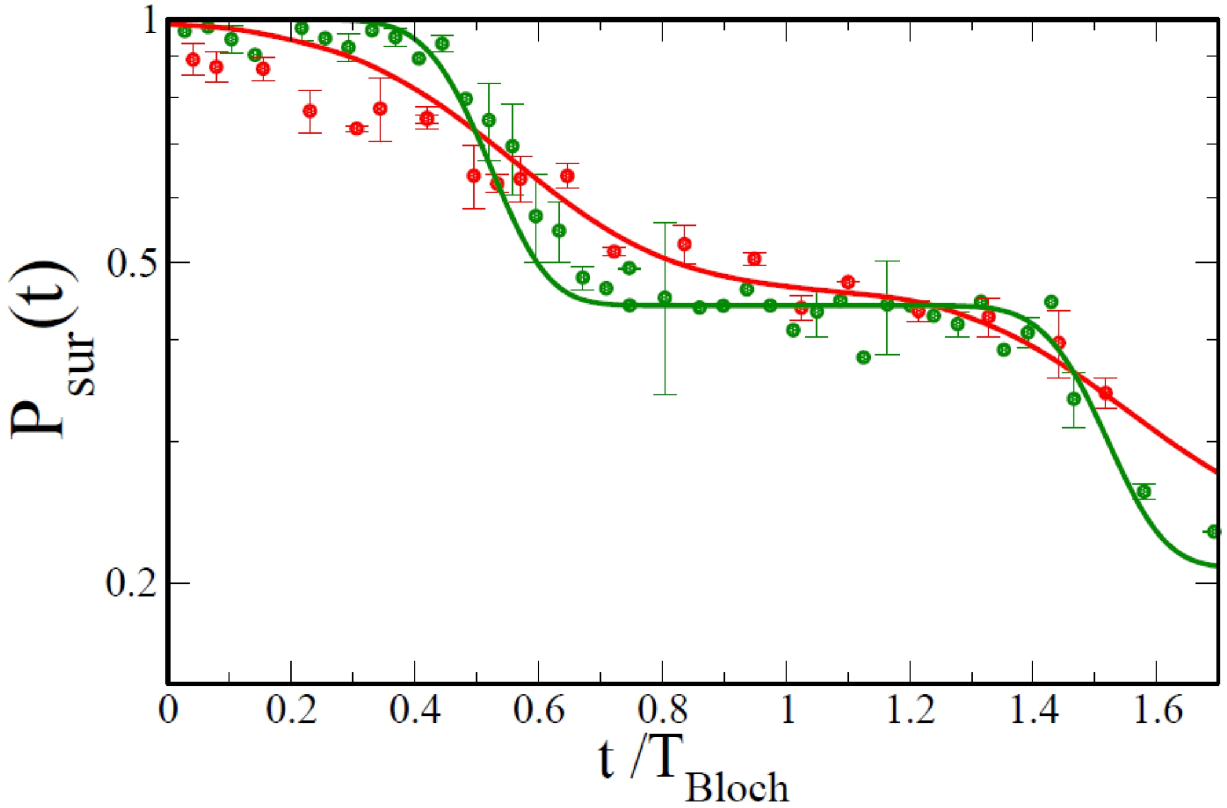
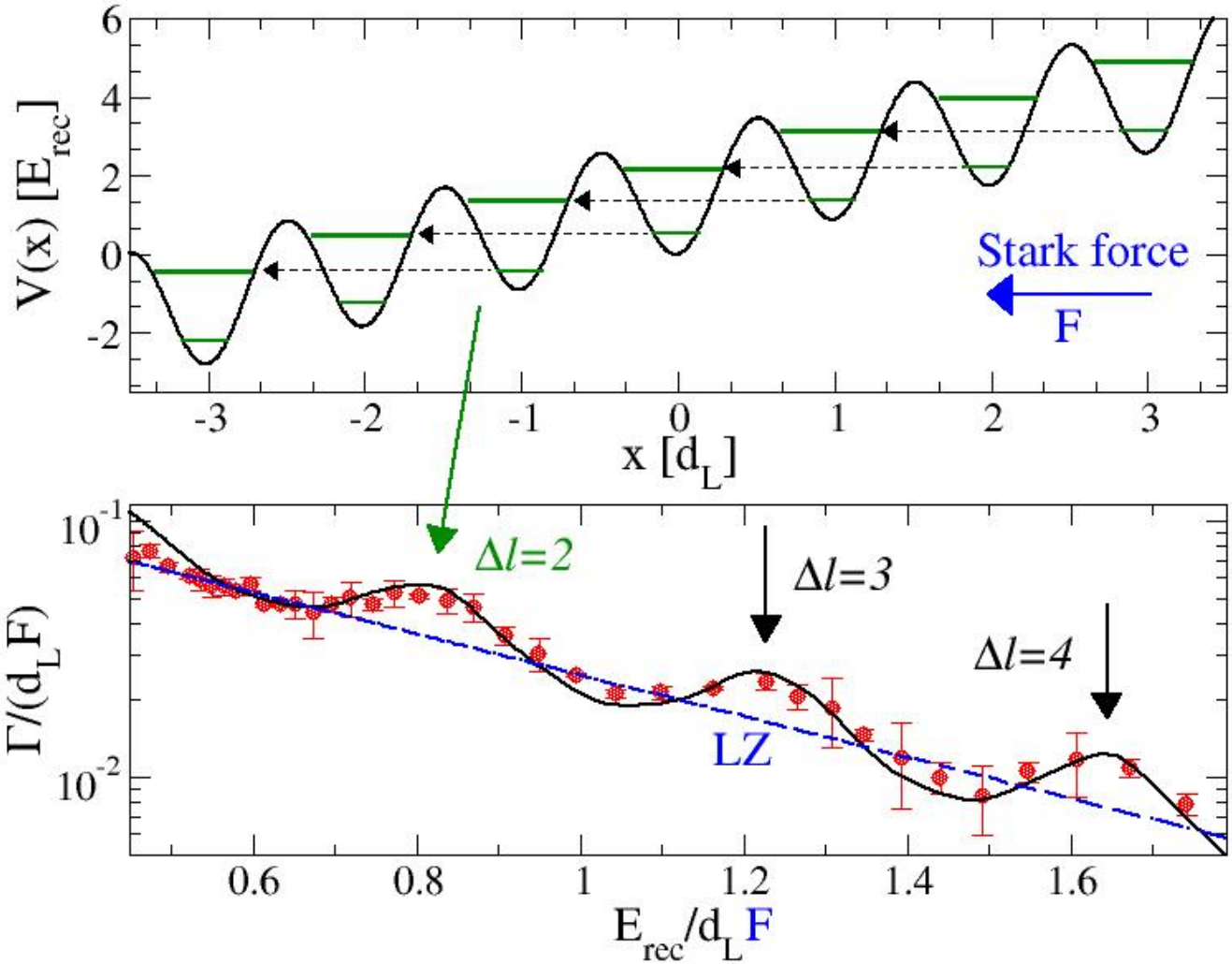
Survival probability: time resolved structure





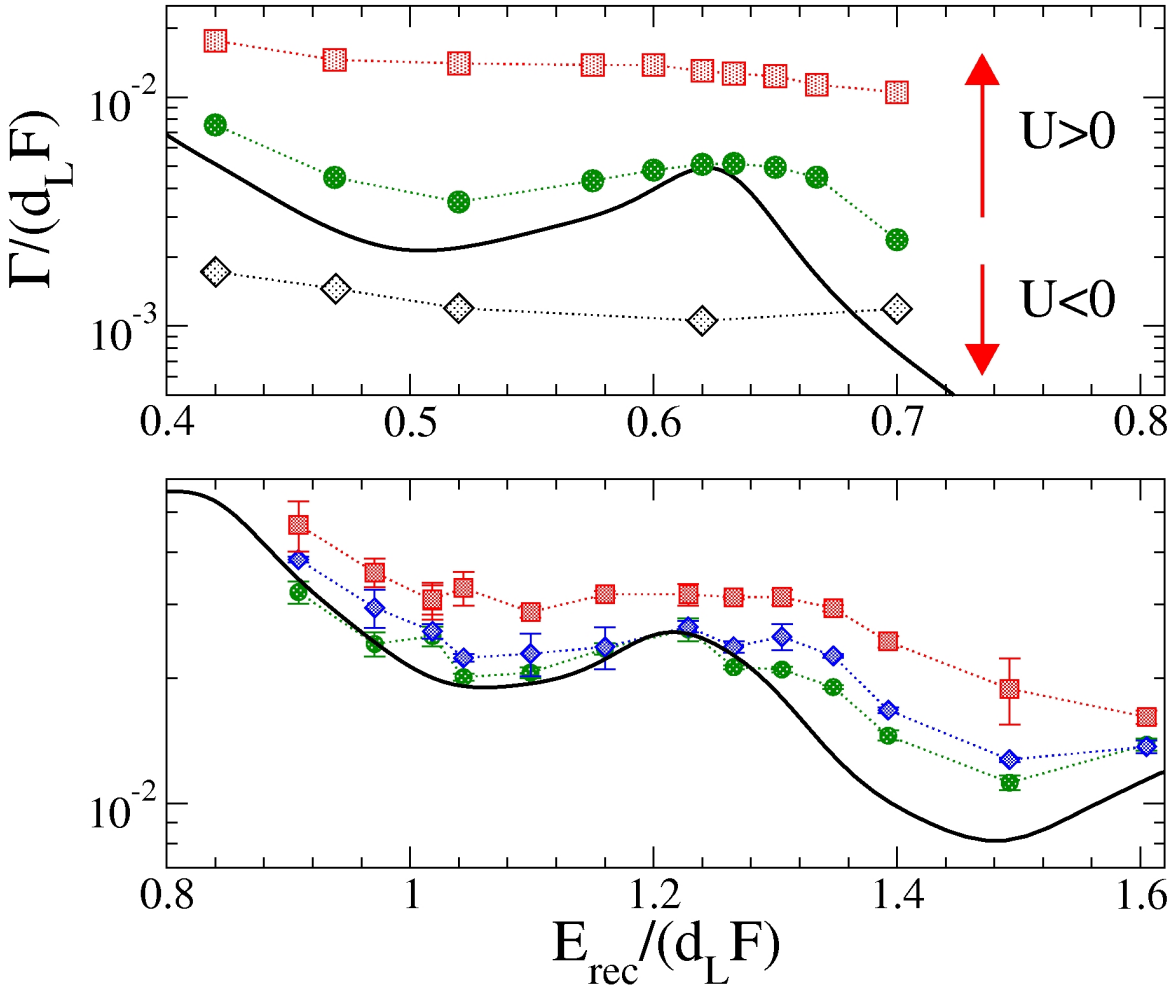
Zenesini, Lignier, Tayebirad, Radogostowicz, Ciampini, Mannella, SW, Morsch, Arimondo PRL 2009

Resonantly Enhanced Tunnelling (RET)



Sias, Zenesini, Lignier, SW, Ciampini, Morsch, Arimondo PRL 2007

Nonlinear resonant tunnelling – effect of interactions



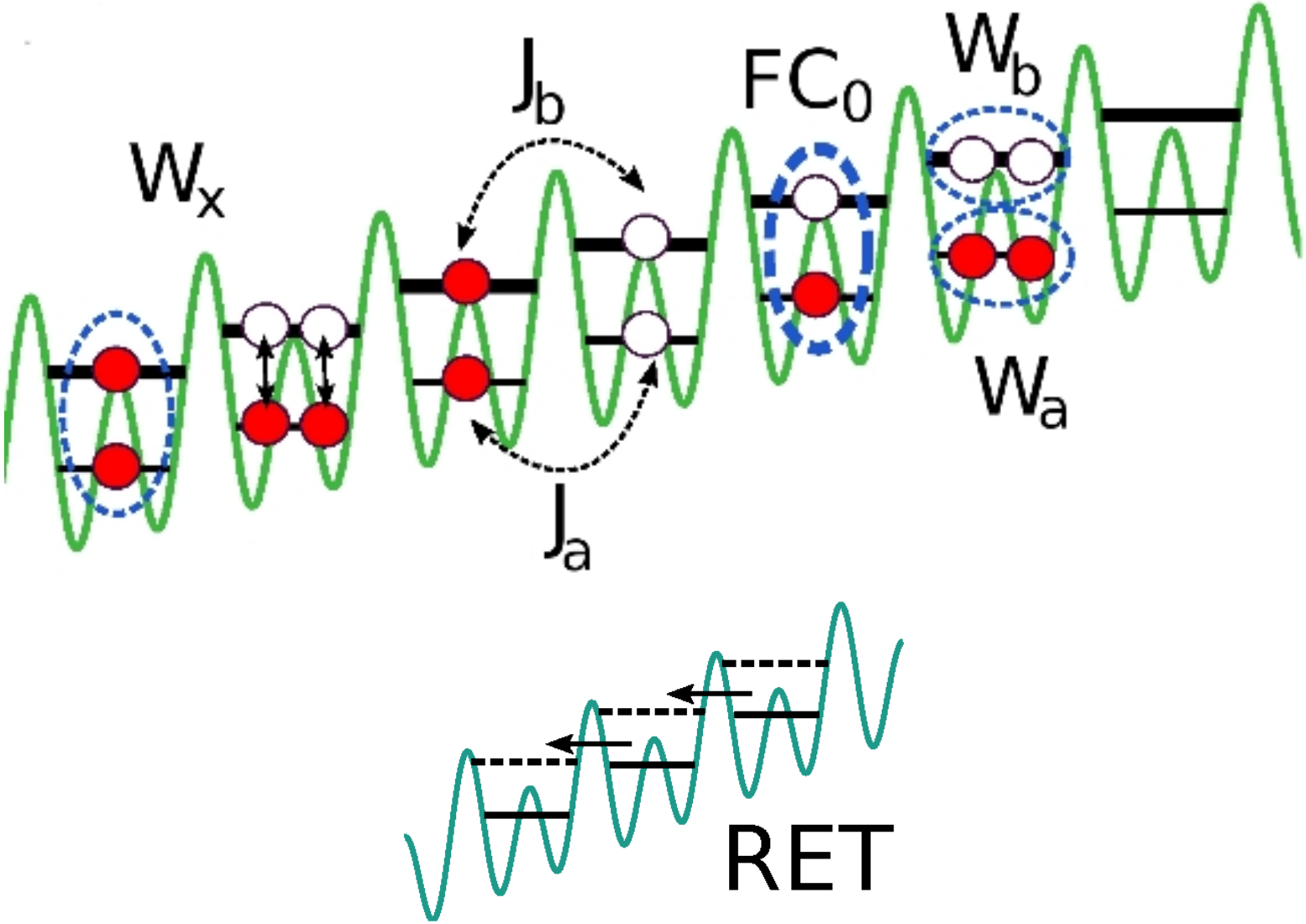
- ⋆ theoretical prediction ($V/E_{\mathrm{rec}} = 5$)

  SW, Mannella, Morsch, Arimondo, Kolovsky, Buchleitner PRA 2005

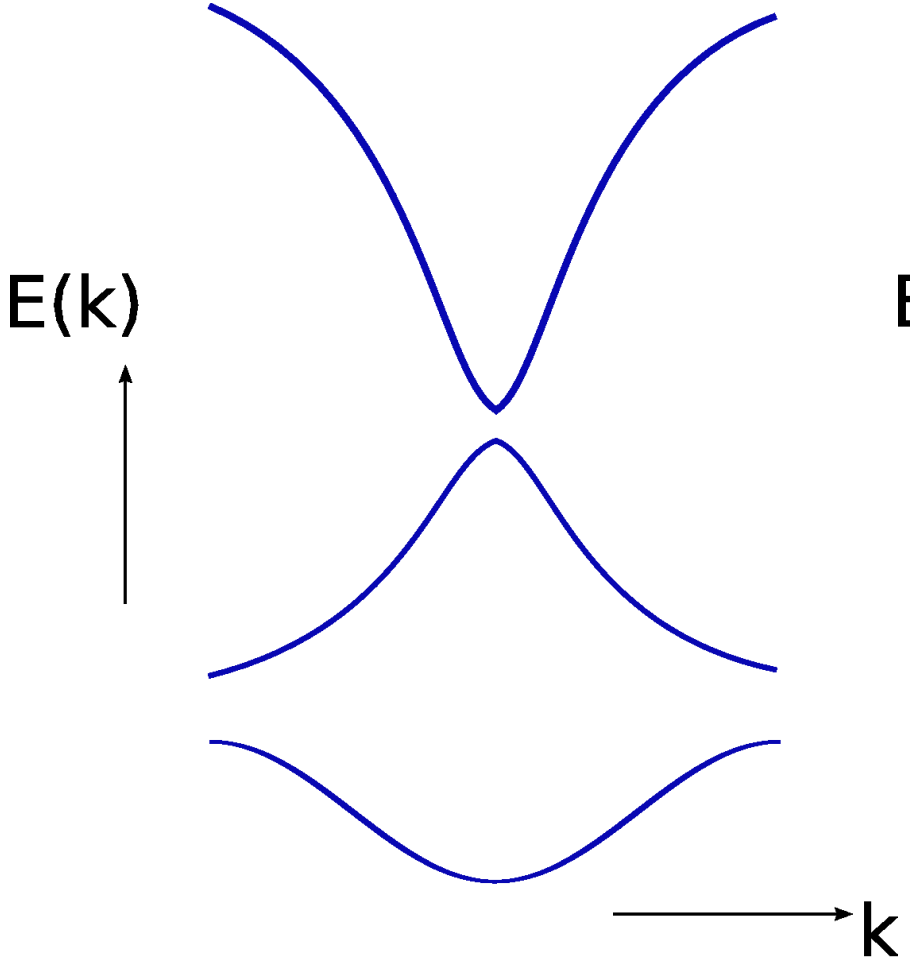
- ⋆ experimental results ($V/E_{\mathrm{rec}} \approx 2.5$)

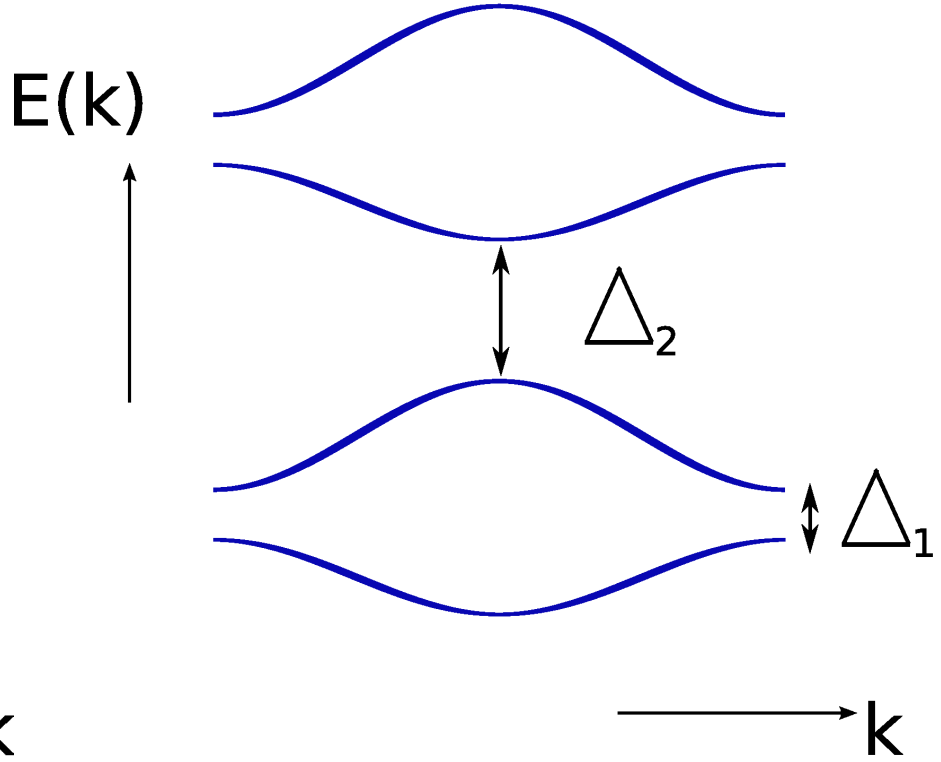
  Sias, Zenesini, Lignier, SW, Ciampini, Morsch, Arimondo PRL 2007

Two-band model for doubly periodic lattice



Two-band model for doubly periodic lattice





Two-band Bose-Hubbard model

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{\mathrm{opt}}(x) + \boxed{Fx}$$

with interactions $\Downarrow$ many-particle model

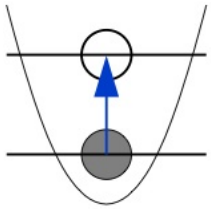
$$\begin{aligned}\hat{H} = &\sum_{\alpha=1}^{2}\left[-\sum_l \frac{J_\alpha}{2}\left(a_{l+1}^{(\alpha)\dagger}a_l^{(\alpha)} + \mathrm{h.c.}\right) + \frac{W_\alpha}{2}\sum_l n_l^{(\alpha)}\left(n_l^{(\alpha)} - 1\right)\right] \\ &+ 2W_x\sum_l n_l^{(1)}n_l^{(2)} + \frac{W_x}{2}\sum_l\left(a_l^{(2)\dagger}a_l^{(2)\dagger}a_l^{(1)}a_l^{(1)} + h.c.\right) \\ &+ \triangle_{\mathrm{gap}}\sum_l n_l^{(2)} + \sum_l\sum_s \tilde{F}_s\left(a_{l+s}^{(2)\dagger}a_l^{(1)} + h.c.\right) + \boxed{F\sum_l l\left(n_l^{(1)} + n_l^{(2)}\right)}\end{aligned}$$

Mapping onto Quantum Ising Model: Plötz, Schlagheck, SW EPJD 2011

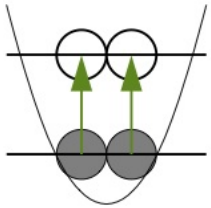
Microscopic model for many-body Landau-Zener decay

From two-band model follow two dominating decay mechanisms:

$$H_1 = F \cdot D_{\mathrm{x}}(V) \cdot \left(\sum_l \hat{a}_l^{\dagger\,(2)} \hat{a}_l^{(1)} + \mathrm{h.c.} \right) \quad \Rightarrow \quad \Gamma_1(F, V)$$



$$H_2 = W_{\mathrm{x}}(V) \cdot \left(\sum_l \hat{a}_l^{\dagger\,(2)} \hat{a}_l^{\dagger\,(2)} \hat{a}_l^{(1)} \hat{a}_l^{(1)} + \mathrm{h.c.} \right) \quad \Rightarrow \quad \Gamma_2(V)$$



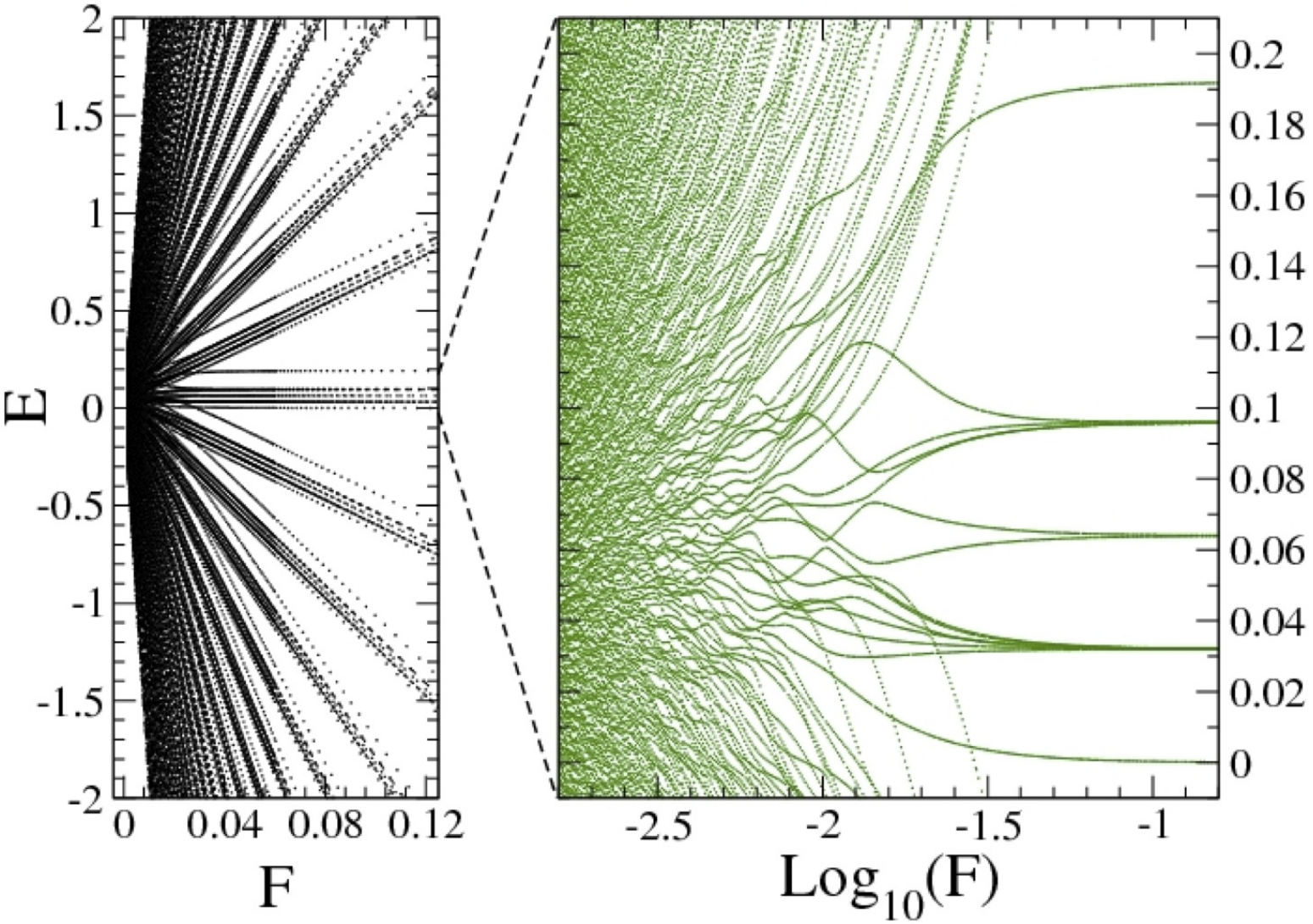
Effective Hamiltonian for ground band:

$$H_{\mathrm{BH,1B}} \to H_{\mathrm{BH,1B}} - \frac{\mathrm{i}}{2} \left[\Gamma_1(F, V) + \Gamma_2(V) \right] ,$$

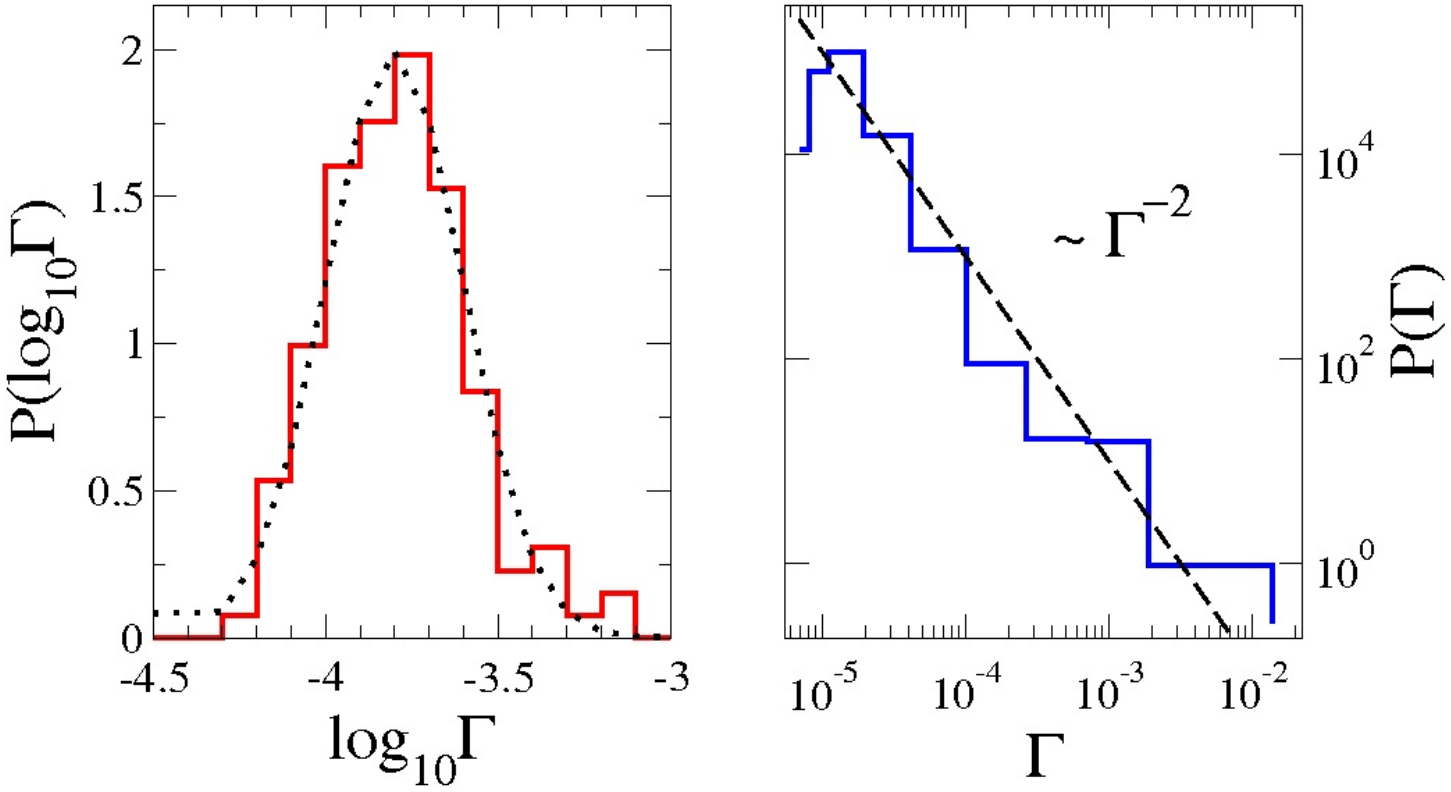
range of validity: $F \cdot D_{\mathrm{x}}, W_{\mathrm{x}}(V) \ll$ band gap energy.

Understanding complex quantum spectra?

Parameter scan for many-body Wannier-Stark system:



From Stark localisation to diffusive dynamics

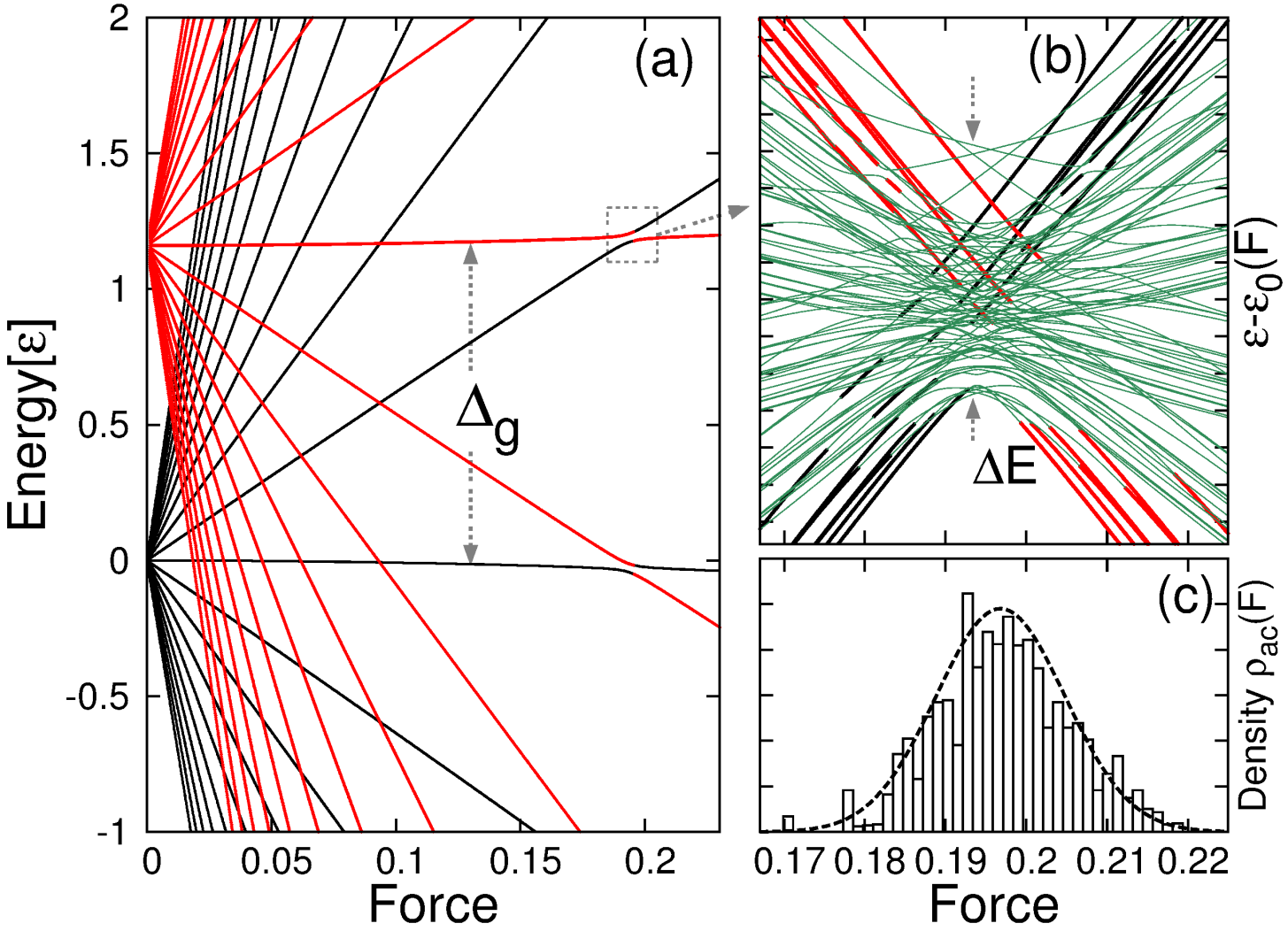


⇒ Stark/Anderson localisation: log-normal distribution!

⇒ microscopic quantum chaos/diffusive transport: power-law!

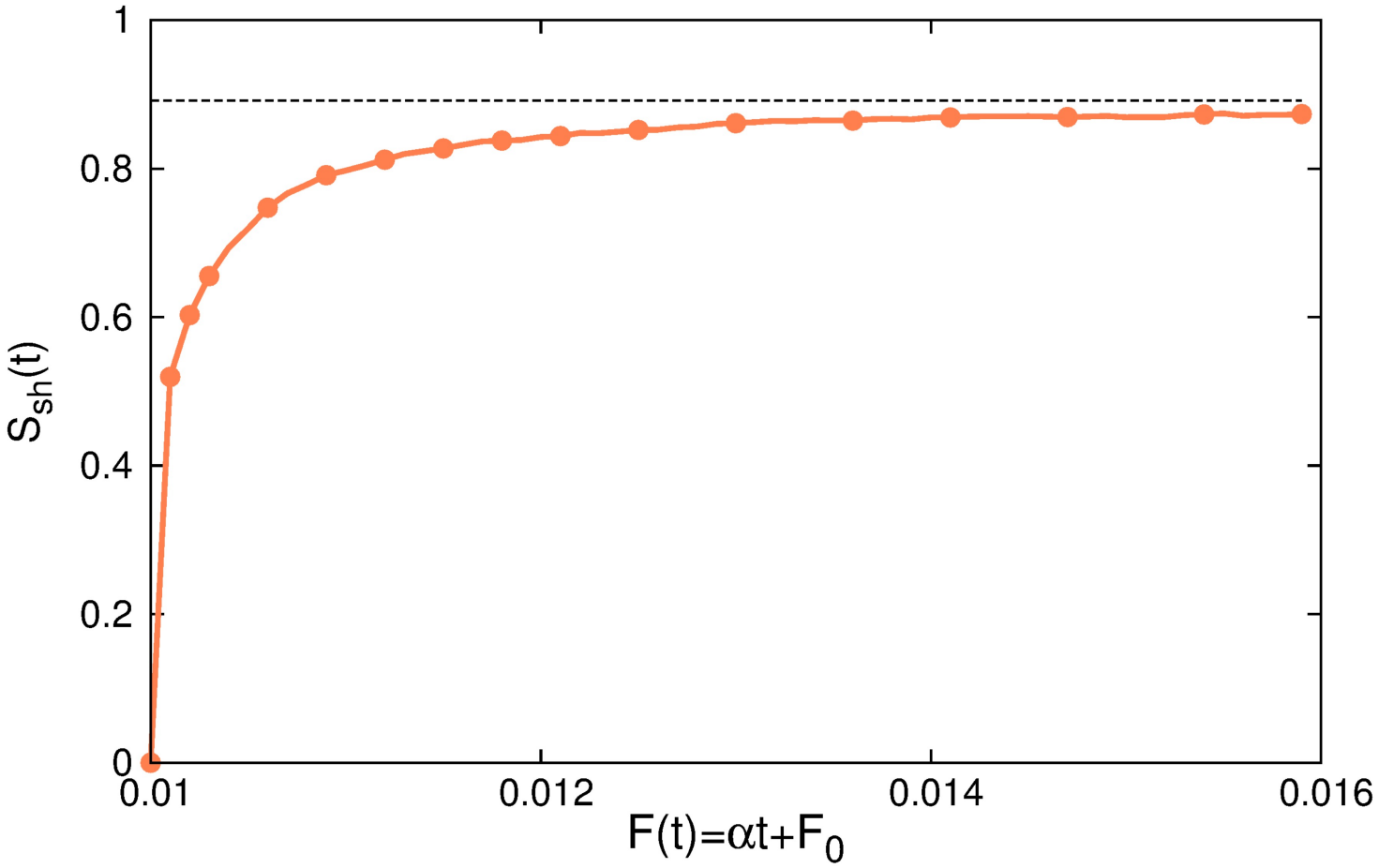
Tomadin, Mannella, SW PRL 2007

Spectral study of full two-band system



Parra-Murillo, Madroñero, SW arXiv:1207.4699

Spectral diffusion by sweeping the Stark force



Parra-Murillo, Madroñero, SW arXiv:1207.4699

Parra-Murillo, PhD thesis, Heidelberg University (2013)

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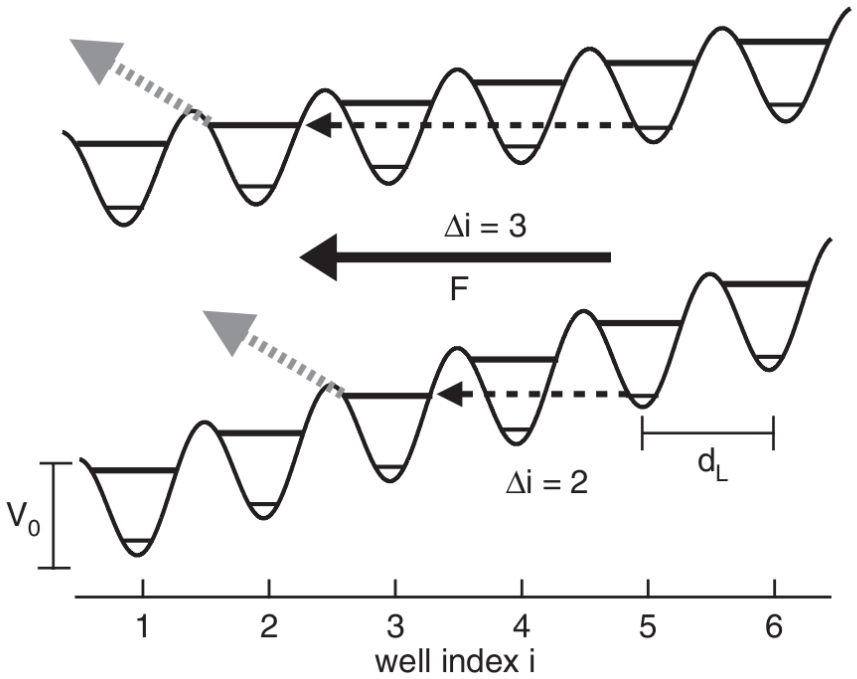
Current and future work

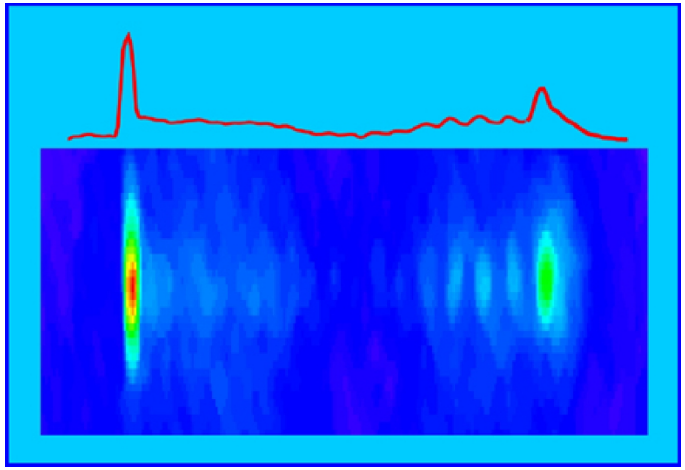
- Open many body systems:
  - controlling quantum dynamics
  - rich interplay of discreteness, interaction, and openness/loss.
- Still open – Open two-band Wannier-Stark problem:

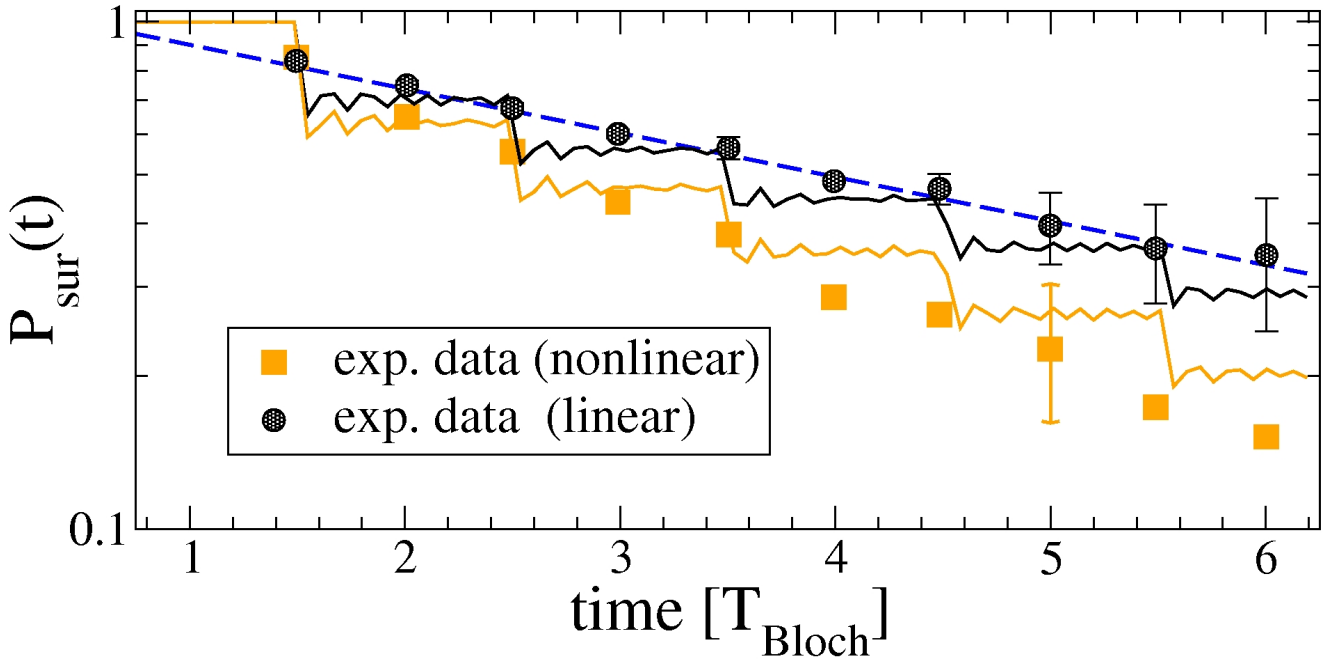


Your questions?

Thank you very much!

Survival probability



$$P_{\mathrm{sur}}(t) \equiv \int_{-p_{\mathrm{c}}}^{\infty} dp_x \left(\int dp_y dp_z |\psi(\vec{p},t)|^2 \right)$$

$$p_{\mathrm{c}} \;\geq\; 3\, p_{\mathrm{R}}$$