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# Selfconsistent diagrammatic transport for light including time reversal symmetric entropy production Regine Frank<sup>1\*</sup>, Bart A. Van Tiggelen<sup>2</sup>

<sup>1</sup> Serin Physics Laboratory, Department of Physics and Astronomy Rutgers University, 136 Frelinghuysen Road, Piscataway, NJ 08854-8019, USA, email: regine.frank@googlemail.com, http://www.ppnec.org <sup>2</sup> Université Grenoble Alpes, Centre National de la Recherche Scientifique, LPMMC, 38000 Grenoble, France 11th Workshop on Quantum Chaos and Localisation Phenomena 25 - 26 May 2023 - Warsaw, Poland

Transport of Light in Random Media including Absorption Gain – Mono- or Polydisperse Mie Resonators



- Always positive entropy production due to absorption and gain in complex also under time reversal
- Entropy production in lasers is always  $\geq 1$ , also for a monochromatic 4-let time reversal parity is +1

# Generalized Bethe-Salpeter Equation – Continuity Equation Current Density Equation – Fick's Law

The intensity correlation, the disorder averaged particle-hole Green's  $\Phi^{\omega}_{\vec{a}\vec{a}'}(\hat{Q},\Omega)$  is described by the Bethe-Salpeter equation

$$\Phi_{\vec{q}\vec{q'}} = G_{q_+}^R(\omega_+)G_{q_-}^A(\omega_-) \times \left[\delta(\vec{q}-\vec{q'}) + \left|\frac{\mathrm{d}^3q''}{(2\pi)^3}\gamma_{q\,q''}\Phi_{\vec{q}''\vec{q'}}\right]\right].$$

Continuity equation and current relaxation equation relate the correlator  $J_E$  to the gradient of the density correlator  $P_E$ .

$$P^{\omega}_{\scriptscriptstyle 
m E}(\,ec{Q},\Omega)\,=\,\left[\!rac{\omega}{c_{\scriptscriptstyle 
m p}}\!
ight]^{2}\Phi_{
ho
ho}\qquad\qquad \Phi_{j
ho}\,=\,\left[\!rac{c_{\scriptscriptstyle 
m p}}{\omega v_{\scriptscriptstyle 
m E}}\!
ight]J^{\omega}_{\scriptscriptstyle 
m E}(\!ec{Q},\Omega)\,.$$

**Conservation Laws: Ward-Takahashi Identity For Open Me Onsager Scenario** 

$$\Sigma_{\vec{q}_{+}}^{\omega_{+}} - \Sigma_{\vec{q}_{-}}^{\omega_{-}*} - \left| \frac{\mathrm{d}^{3}q'}{(2\pi)^{3}} \left[ G_{\vec{q}_{+}'}^{\omega_{+}} - G_{\vec{q}_{-}'}^{\omega_{-}*} \right] \gamma_{\vec{q}'\vec{q}}^{\omega}(\vec{Q},\Omega) \right. \\ = f_{\omega}(\Omega) \left[ \operatorname{Re}\Sigma_{\vec{q}}^{\omega} + \left| \frac{\mathrm{d}^{3}q'}{(2\pi)^{3}} \operatorname{Re}G_{\vec{q}'}^{\omega} \gamma_{\vec{q}'\vec{q}}^{\omega}(\vec{Q},\Omega) \right] \right].$$

In presence of loss or gain, effects are enhanced by the prefactor

$$f_{\omega}(\Omega) = \frac{(\omega \Omega \text{Re}\Delta\epsilon + i\omega^2 \text{Im}\Delta\epsilon)}{(\omega^2 \text{Re}\Delta\epsilon + i\omega \Omega \text{Im}\Delta\epsilon)},$$

(4)

which now does not vanish in the limit of  $\Omega \to 0$ .  $\Delta \epsilon = \epsilon_s - \epsilon_b$  is the dielectric contrast.

and	Disentangeling the Cooperon Contribution
	(a) $\frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{-}  \vec{k}'_{-}} = \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{\gamma}} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{C}} + \frac{\vec{k}_{-}  \vec{k}'_{-}}{\vec{k}_{-}  \vec{k}'_{-}} + \frac{\vec{k}_{-}  \vec{k}'_{-}} + \frac{\vec{k}'_{-}  \vec{k}'_{-}} + \frac$
	$+ \underbrace{\vec{k}_{+}}_{\vec{k}_{-}} \underbrace{\vec{k}'_{+}}_{\vec{k}_{-}} \underbrace{\vec{k}_{+}}_{\vec{k}_{-}} \underbrace{\vec{k}_{+}}_{\vec{k}_{-}} \underbrace{\vec{k}_{-}}_{\vec{k}_{-}} \underbrace{\vec{k}_{-}} \underbrace{\vec{k}$
x matter,	(b) $\frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{-}  \vec{k}'} = \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{-}  \vec{k}'} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{-}  \vec{k}'} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{-}  \vec{k}'} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{-}  \vec{k}'_{+}} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}_{+}  \vec{k}'_{+}} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}'_{+}  \vec{k}'_{+}} + \frac{\vec{k}_{+}  \vec{k}'_{+}}{\vec{k}'_{+}} + \frac{\vec{k}_{+}  \vec{k}'_{+}} + \frac{\vec{k}_{+}  \vec{k}'_{+}} + \frac{\vec{k}_{+}  \vec{k}'_{+}} + \frac{\vec{k}'_{+}  k$
evel laser; n and	$   \begin{array}{ccccccccccccccccccccccccccccccccccc$
function,	k̄_k̄_k̄_k̄_Weighted Essentially Non-Oszillatory Solvers Self-Consistency for Diffusion E
(1) current	
(2) edia -	• adaptive stencils in the sense of adaptive meshes, clust
(3)	• capable for non-linearities and roughes, steep gradi Galerkin methods)

- Lax-equivalence theorem: convex combination of all candidates of lower- order difference quotients of the stencil with an attributive weight  $g_i = \frac{1}{h+D^2}$ ;  $D_i$  are the smoothness indicators of the stencil. The variable h > 0 is the machine accuracy prohibiting a division by 0.
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for Complex Media



(WENO) – Numerical Equations



ter methods ients (compare discontinuous)



peaked at  $2r_{scat} = 245.0 nm$ .



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# Selfconsistent Diffusion and Anderson Localization

Single Particle Characteristics: Scattering mean free path  $l_s = \frac{1}{2 \text{Im}(\sqrt{q^2 + i \text{Im}\Sigma(\omega)})}$  for disordered samples of TiO<sub>2</sub> Mie spheres, n = 2.7, with a Gaussian distribution of scatterers

The mean square width  $\sigma^2(t) = \frac{\int r^2 P_E(r,t) d^2 r}{\int P_E(r,t) d^2 r}$  is a measure of Anderson localized photons.