Eigenfunction and eigenmode-spacing statistics in chaotic photonic crystal graphs

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Overview

Motivation:
- Chaotic 1D graphs have been studied extensively with microwave cable networks. However, the wavefunctions are only measured at graph nodes.
- Recent studies show that the chaotic graphs face challenges from trapped modes on bonds that give rise to deviations from RMT.
- Through direct eigenfunction measurement, the trapped mode issues can be identified and quantified.

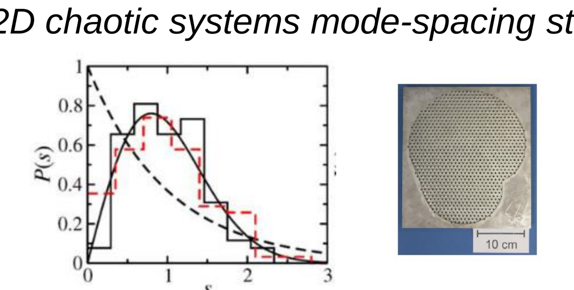
Achievements:
- Designed a chaotic graph structure using defect photonic crystals at microwave frequencies.
- Through numerical simulations, we conducted the mode-spacing and the first eigenfunction study for closed microwave graphs, and find good agreement with the GOE predictions.

Outlooks:
- Our design is ready for experimental realization.

Previous studies

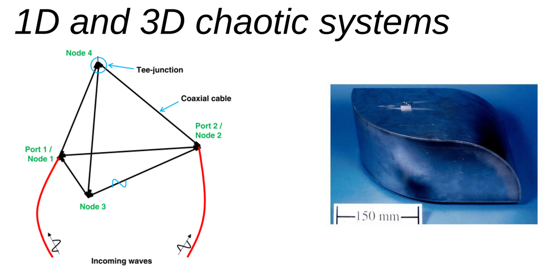
- **The mode-spacing statistics have been studied in chaotic system with different dimensions,** including GOE, GUE and GSE systems.

2D chaotic systems mode-spacing stats



Dietz, B, and A Richter. 2019. *Physica Scripta* 94 (1): 014002.

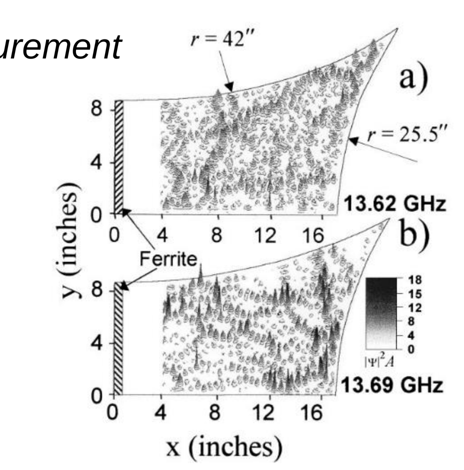
1D and 3D chaotic systems



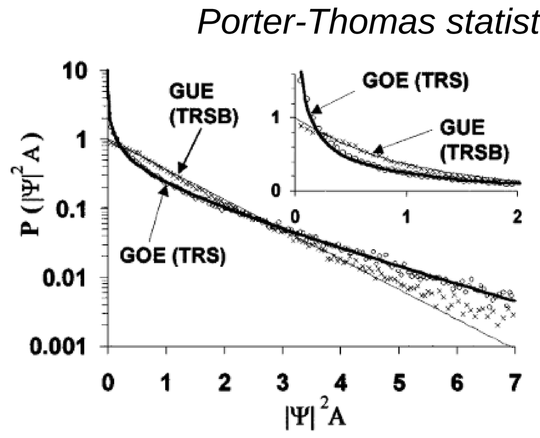
Dietz, B, and A Richter. 2015. *Chaos* 25 (9): 097601.
Chen, Lei, Tsampikos Kottos, and Steven M. Anlage. 2020. *Nat Comm* 11 (1): 5826.

- **The eigenfunction statistics have been tested in 2D systems, for both GUE and GOE cases.**

Measurement



Porter-Thomas statistics

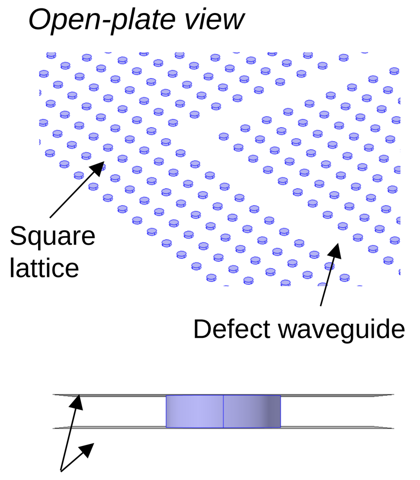


Wu, Dong Ho, Jesse S. A. Bridgewater, Ali Gokirmak, and Steven M Anlage. 1998. *Physical Review Letters* 81 (14): 2890–93.

Dielectric photonic crystal (PC) simulation

Square lattice of dielectric posts between conducting plates

Open-plate view



Structure dimension
lattice constant $a_0 = 36.8mm$
height $h_0 = 0.1a_0$
rod diameter $d_0 = 13.2mm$

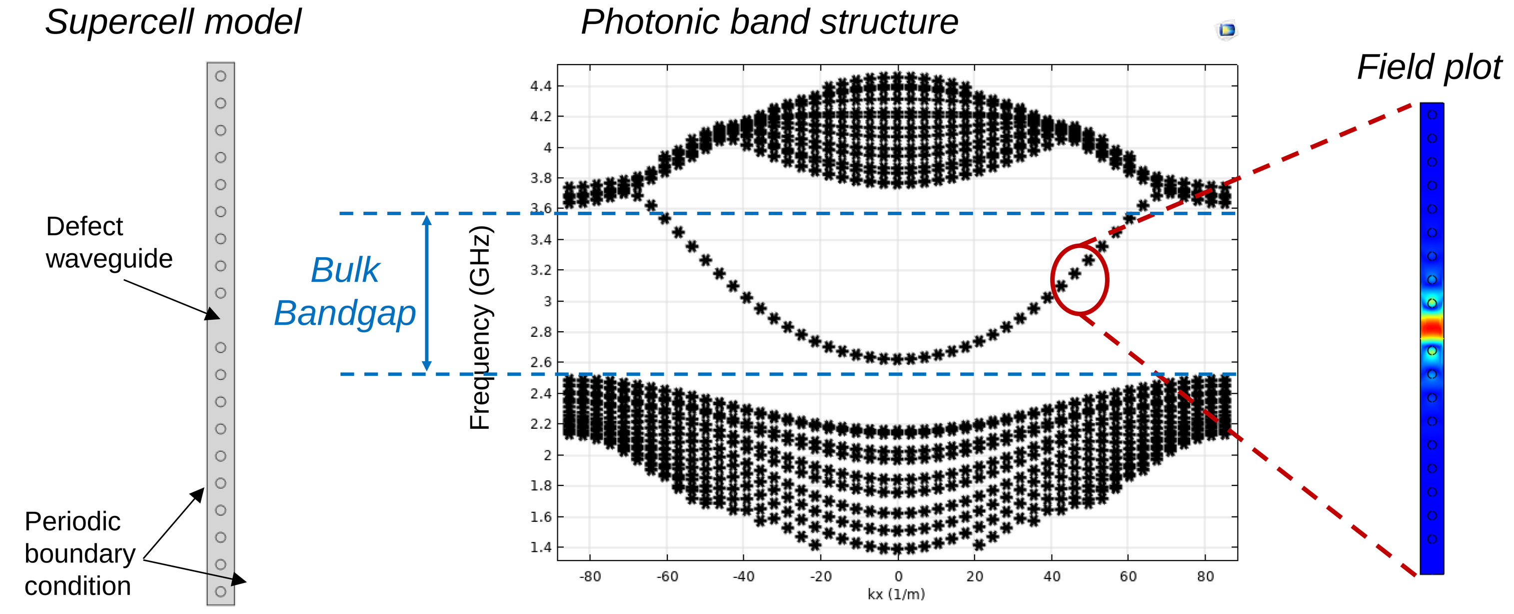
Dielectric rod material parameter
$\mu_r = 1, \quad \epsilon_r = 11.56$

Boundary condition
metallic top and bottom surfaces

Simulation environment
Use the eigenfrequency solver in COMSOL Multiphysics with RF Physics.

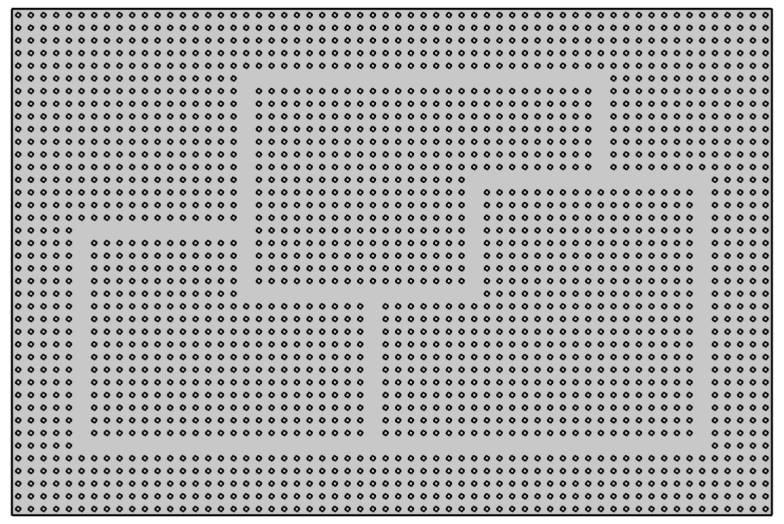
The photonic band structure simulations are implemented with periodic boundary conditions and wavenumber sweep.

Photonic crystal defect waveguide



Only the waveguide modes exist in the bulk bandgap region (2.6~3.5 GHz).

Chaotic PC graph

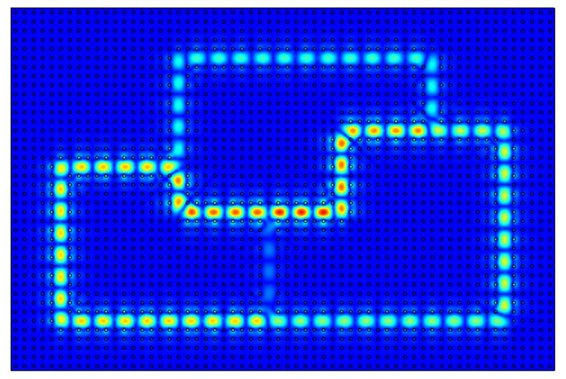


The chaotic graph structure is constructed by defect waveguide bonds with various lengths.

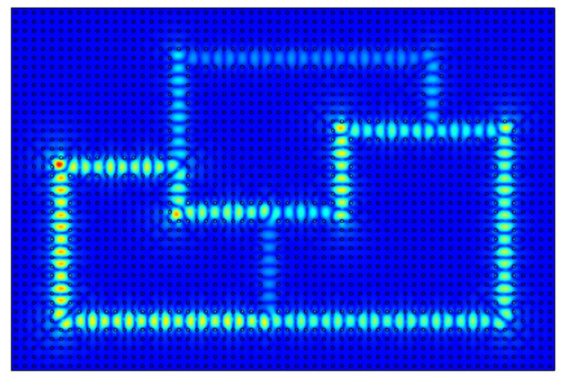
The shape of the graph is similar to a 3D tetrahedral graph.

New chaotic graphs can be constructed by varying the bond lengths.

Graph mode $|E_z|$, $f = 2.8GHz$



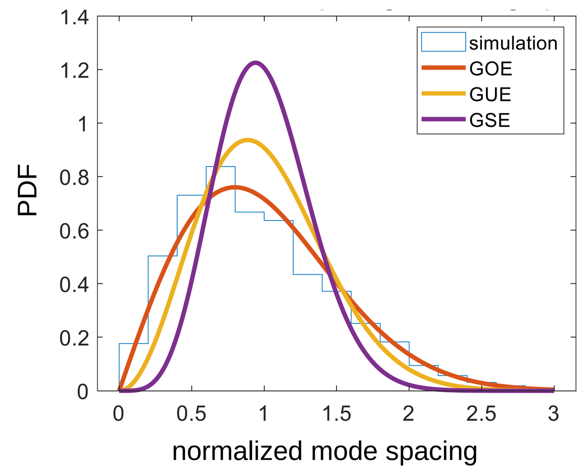
Graph mode $|E_z|$, $f = 3.6GHz$



The PC-graph modes are TM polarized.

The wavelength is usually 3 times a_0.

Mode-spacing statistics



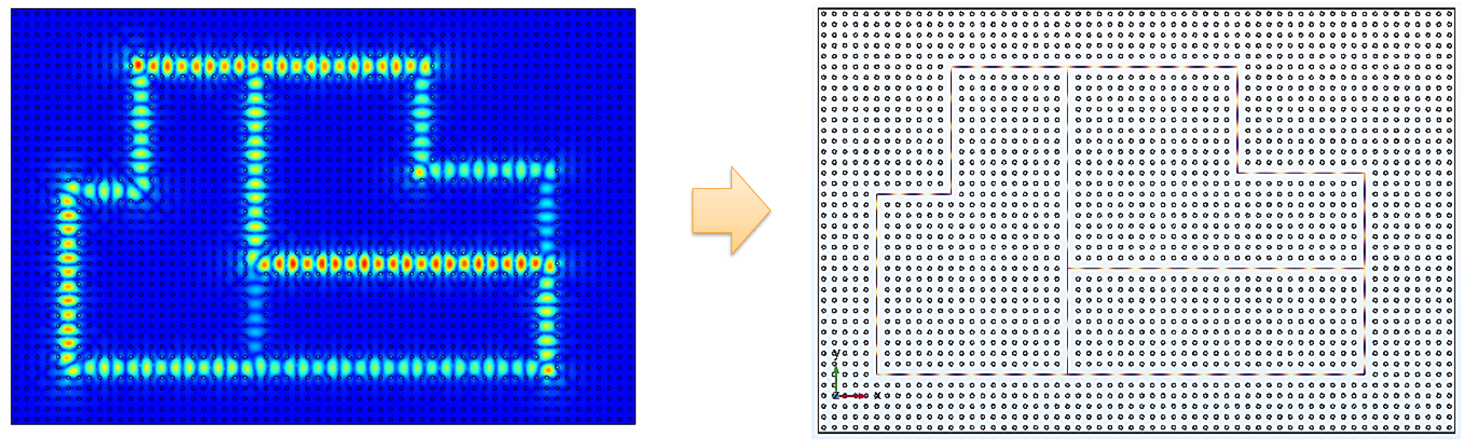
The graph eigenmodes are obtained from eigenvalue simulations with COMSOL Multiphysics.

For each PC-graph realization, we usually obtain 80 eigenmodes solutions.

The mode-spacing studies are conducted with ~800 modes spacing values from 10 different graph structures, and find good agreement with GOE predictions.

Eigenfunction statistics

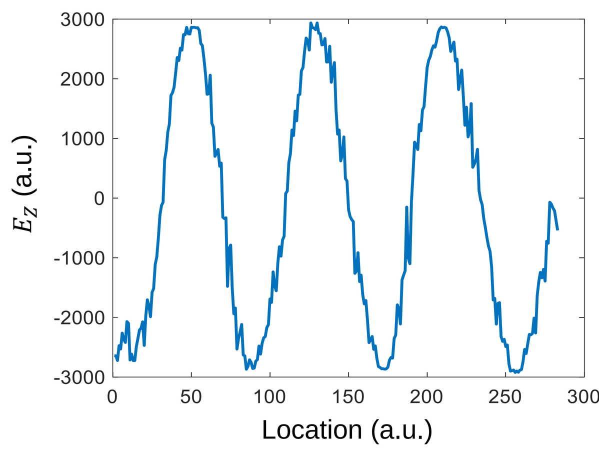
Graph mode $|E_z|$ in the middle plane / Tracing $|E_z|$ at the center of the bond



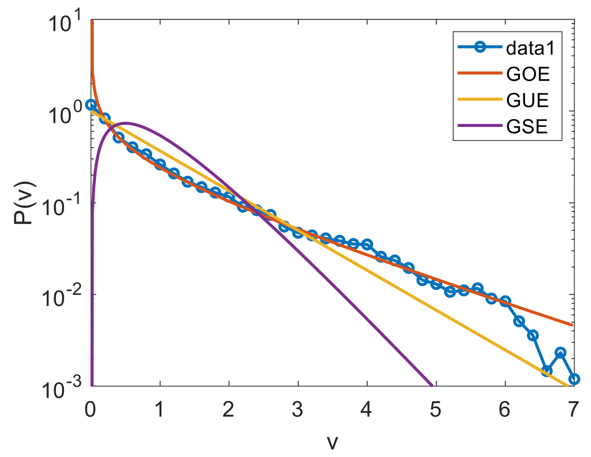
For 1D graph system, the telegrapher equation is formally equivalent to the 1D Schrodinger equation, where the wavefunction is represented by the wave voltage.

For thin parallel plate waveguides, the voltage between the top-bottom metallic plate is represented by the E_z value at the middle cutting plane.

Method I: grid-wise representation

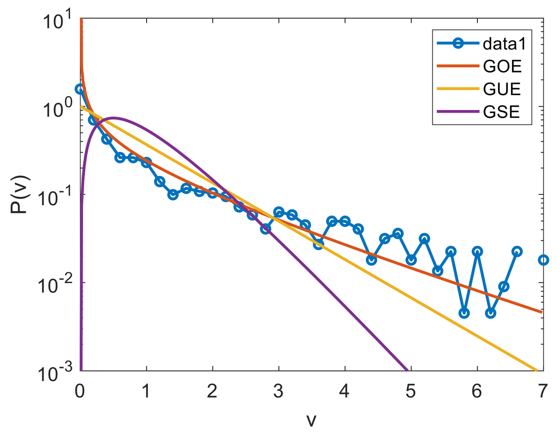


The eigenfunction amplitude on a PC-graph bond shows simple sinusoidal pattern.



The eigenfunction statistics agree well with the GOE Porter-Thomas curve, with data from 48 graph modes.

Method II: mean-bond-value representation



Each eigenfunction datapoint will be represented by the mean bond intensity, normalized by the bond lengths [L. Kaplan, Phys. Rev. E **64**, 036225 (2001)].

We have 14×78 data points: 14 is the number of the bonds, and 78 is the number of modes per graph.

The normalization is conducted for all bonds that belong to the same eigenmode.

Summary and future steps

- We have designed and simulated PC defect waveguide chaotic graphs whose eigenvalue and eigenfunction statistics show nice agreement with theoretical predictions.
- Future steps: further analysis on trapped graph modes statistics; experimental realizations of the PC graph; expand to topologically non-trivial photonic crystal systems*.

*Ma, Shukai, Bo Xiao, Yang Yu, Kueifu Lai, Gennady Shvets, and Steven M Anlage. 2019. *Physical Review B* 100 (8): 085118. Ma, Shukai, and Steven M. Anlage. 2020. *Applied Physics Letters* 116 (25): 250502.

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