Experimental Demonstration of Complex Wigner Time Delay in Sub-Unitary Scattering Systems

Lei Chen[1], Yan V. Fyodorov[2], Steven M. Anlage[1]

[1]Quantum Materials Center, Department of Physics, University of Maryland, College Park, Maryland 20742, USA

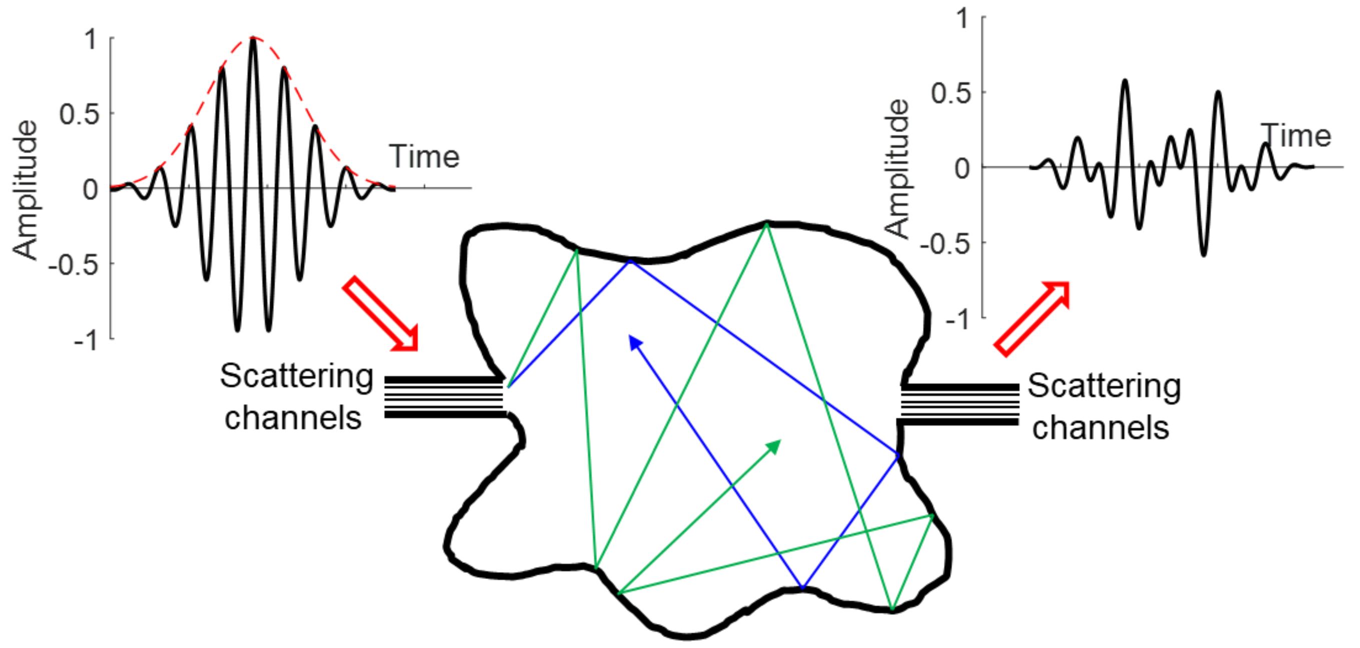
[2]Department of Mathematics, King's College London, London WC26 2LS, United Kingdom

Overview

Motivation: Wigner time delay describes the scattering properties (phase) of a complex scattering region.

Development: A complex generation of Wigner time delay in the sub-unitary scattering systems can fully characterize both the phase delay and the waveform distortion of waves in a complex scattering systems.

Significance: Complex Wigner time delay provides easy access to the zeros and poles of the S-matrix, and directly connects to Coherent Perfect Absorption (CPA).



Definition

Flux-Conserving (Unitary) Systems:

$$|\psi_{\text{out}}\rangle = S|\psi_{\text{in}}\rangle$$

Wigner-Smith Delay Time:

$$\tau_W(E) \equiv \frac{1}{M}\mathrm{Tr}\left(-i\hbar\, S^\dagger(E)\frac{dS}{dE}\right) = \frac{-i\hbar}{M}\frac{d}{dE}\ln\det S(E)$$

Wigner, *Phys. Rev.* **98**, 145 (1955).
Smith, *Phys. Rev.* **118**, 349 (1960).

M: Number of scattering channels

"Complex" Wigner Time Delay

Lossless system:

Assume $\det(S) = e^{i\phi(f)}$

M is the number of channels

$$\tau_W(f) = \frac{1}{M}\frac{d}{df}\phi(f) = -\frac{i}{M}\frac{d}{df}\ln[\det(S)]$$

System with loss:

Assume $\det(S) = A(f)e^{i\delta(f)}$

$$\tau(f) = -\frac{i}{M}\frac{d}{df}\ln[\det(S)]$$

"Complex"! $$= \underbrace{-\frac{i}{M}\frac{d}{df}\ln[A(f)]}_{\mathrm{Im}\,\tau} + \underbrace{\frac{1}{M}\frac{d}{df}\delta(f)}_{\mathrm{Re}\,\tau}$$

S-matrix: Zeros & Poles

Sub−Unitary Scattering Systems:

$$\det S_\gamma(E) \coloneqq \det S(E+i\gamma) = \frac{\det[E-H+i(\gamma+\Gamma_A-\Gamma_W)]}{\det[E-H+i(\gamma+\Gamma_A+\Gamma_W)]} = \prod_{n=1}^{N}\frac{E+i\gamma-z_n}{E+i\gamma-\mathcal{E}_n}$$

γ: uniform absorption strength
H: Hamiltonian of the closed system
$\Gamma_W = \pi WW^\dagger, W$ coupling matrix
$\Gamma_A = \pi AA^\dagger, A$ absorption matrix

Chen, Anlage, Fyodorov, *Phys. Rev. E* **103**, L050203 (2021).

$z_n = \mathbf{Re}z_n + i\mathbf{Im}z_n$: zeros of S−matrix, eigenvalues of $H + i(\Gamma_W - \Gamma_A)$

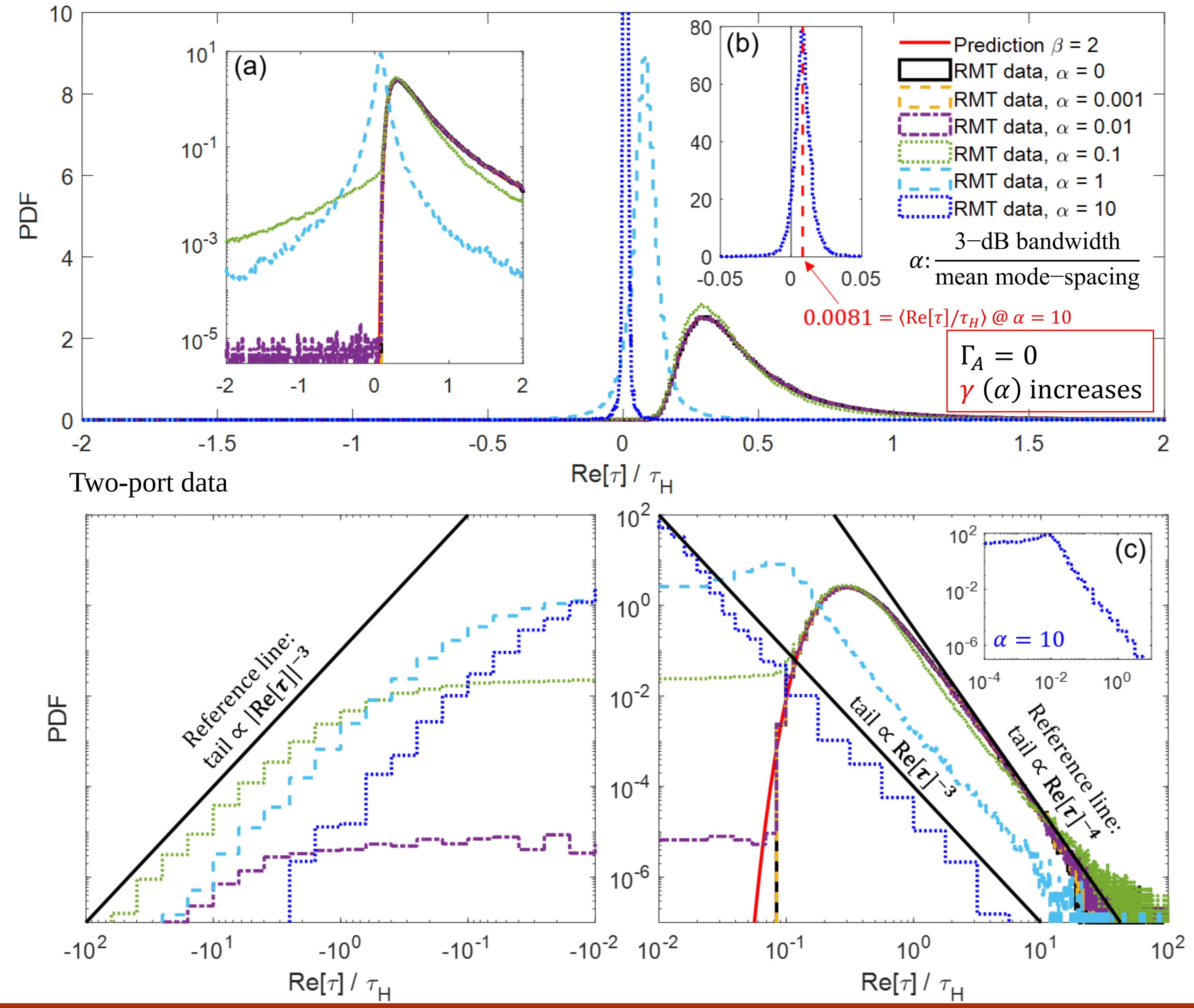
$\mathcal{E}_n = E_n - i\Gamma_n$: poles of S−matrix, eigenvalues of $H - i(\Gamma_W + \Gamma_A)$

Complex Wigner Time Delay:

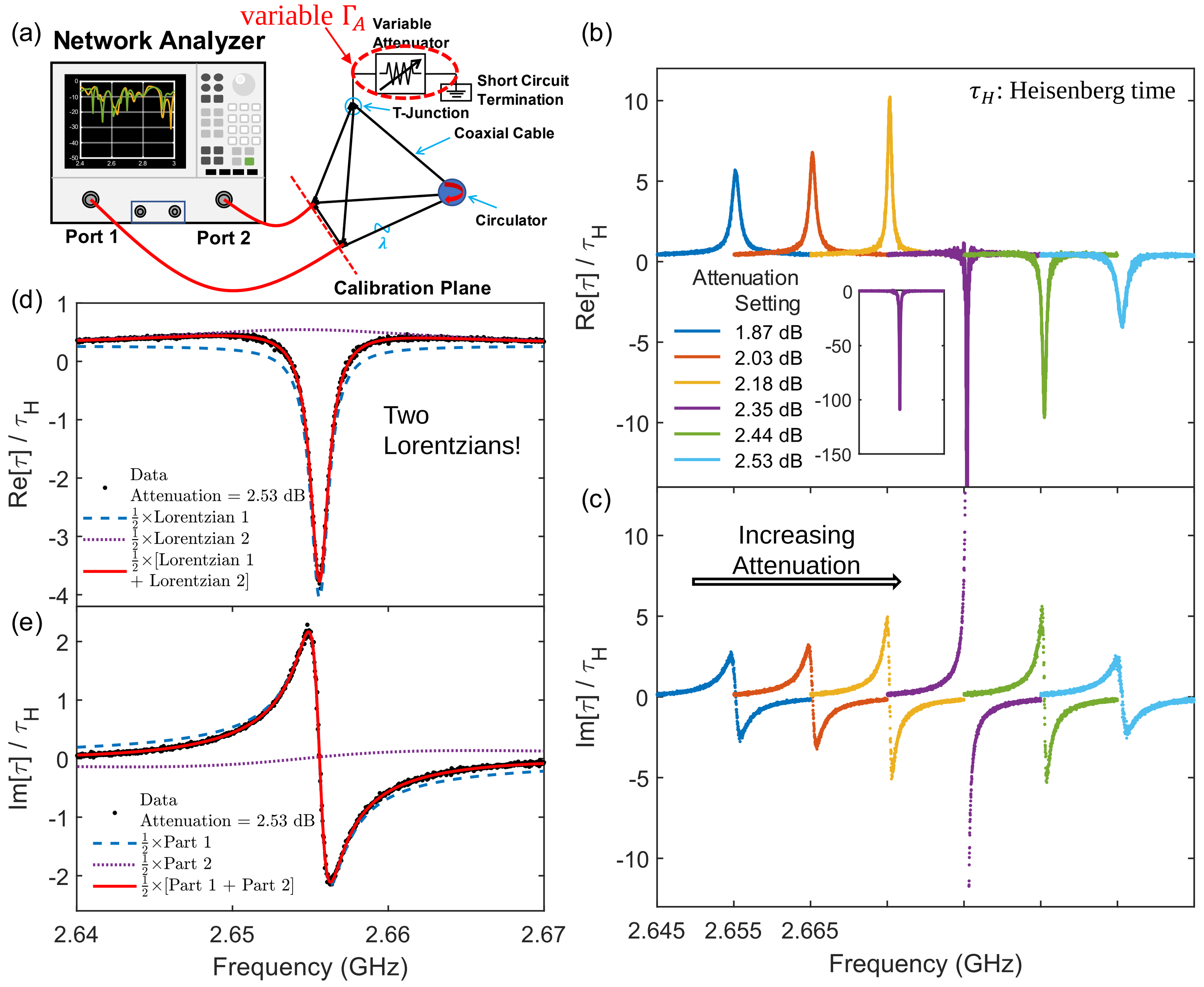
$$\mathrm{Re}\,\tau(E;A,\gamma) = \frac{1}{M}\sum_{n=1}^{N}\left[\frac{\mathrm{Im}z_n - \gamma}{(E-\mathrm{Re}z_n)^2 + (\mathrm{Im}z_n-\gamma)^2} + \frac{\Gamma_n + \gamma}{(E-E_n)^2 + (\Gamma_n+\gamma)^2}\right]$$

$$\mathrm{Im}\,\tau(E;A,\gamma) = -\frac{1}{M}\sum_{n=1}^{N}\left[\frac{E-\mathrm{Re}z_n}{(E-\mathrm{Re}z_n)^2 + (\mathrm{Im}z_n-\gamma)^2} - \frac{E-E_n}{(E-E_n)^2 + (\Gamma_n+\gamma)^2}\right]$$

Evolution of Re[τ] Distribution with Increased Loss (RMT Results)



Experimental Tests (single isolated mode) with Microwave Graphs

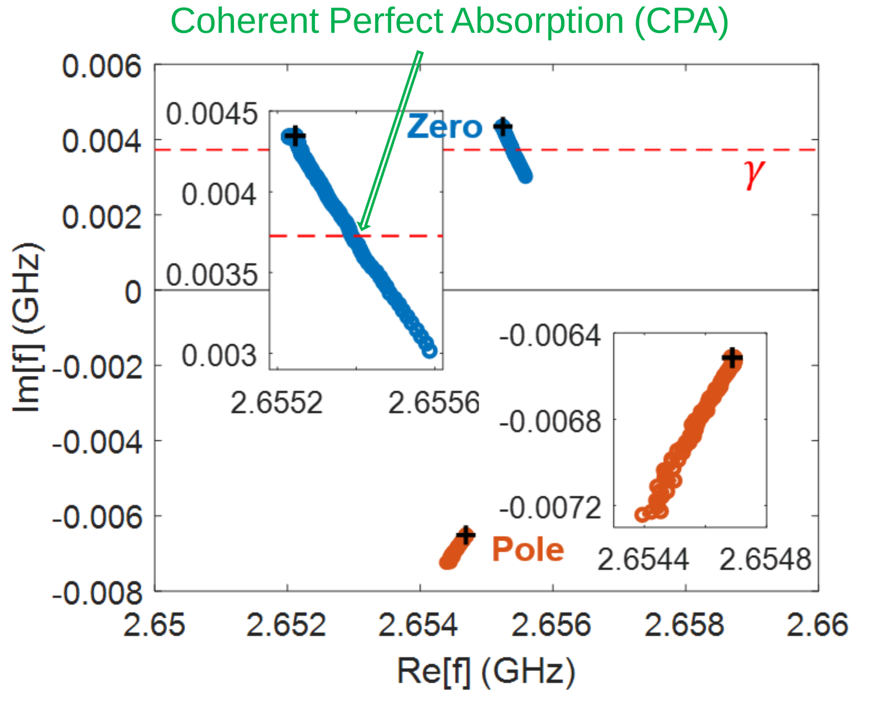


Graph Advantages:
Access to isolated modes,
compile good statistics,
easy to vary M,
GOE/GUE,
variable Γ_A,
variable γ
…

Disadvantages:
Statistics can show deviations from RMT.
[Dietz, *et al.*, *PRE* **95**, 052202 (2017)]

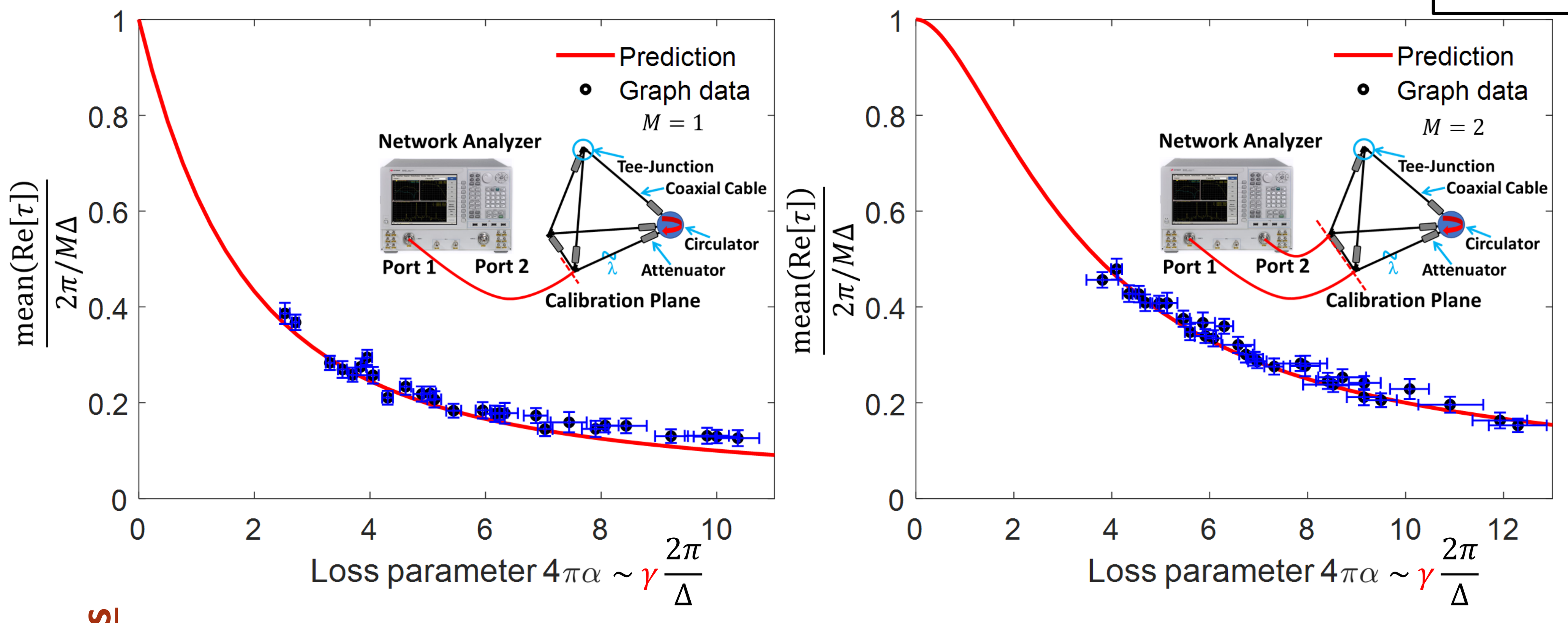
Chong, et al, *PRL* **105**, 053901 (2010).
Chen, Kottos, Anlage, *Nat. Comm.* **11**, 5826 (2020).

Coherent Perfect Absorption (CPA)



Counting Resonance Widths via Complex Time Delays

$$\langle \mathrm{Re}[\tau(E;\gamma)]\rangle_E = \frac{2\pi}{M\Delta} * \mathrm{Prob}(\Gamma_n > \gamma)$$



- One/Two-port GUE Tetrahedral Graph
- Increase uniform loss (γ) of the system by changing the attenuators.
- Create ensemble data by changing cables with different length.

Conclusions

- ✓ The complex generalization of the Wigner time delay holds both for arbitrary uniform (global) and spatially-localized loss, and directly relates to poles and zeros of the scattering matrix.
- ✓ CPA coincides with the divergence of Re[τ] and Im[τ].
- ✓ New interesting results in statistics of complex time delay in the sub-unitary scattering systems.
- ✓ Complex time delays can be used as a counter for the resonance widths of the system.

We acknowledge the funding supports from AFOSR COE Grant No. FA9550-15-1- 0171, and ONR Grant No. N000141912481.