Investigations of the enhancement factor in an open wave chaotic system with time-reversal-invariance violation

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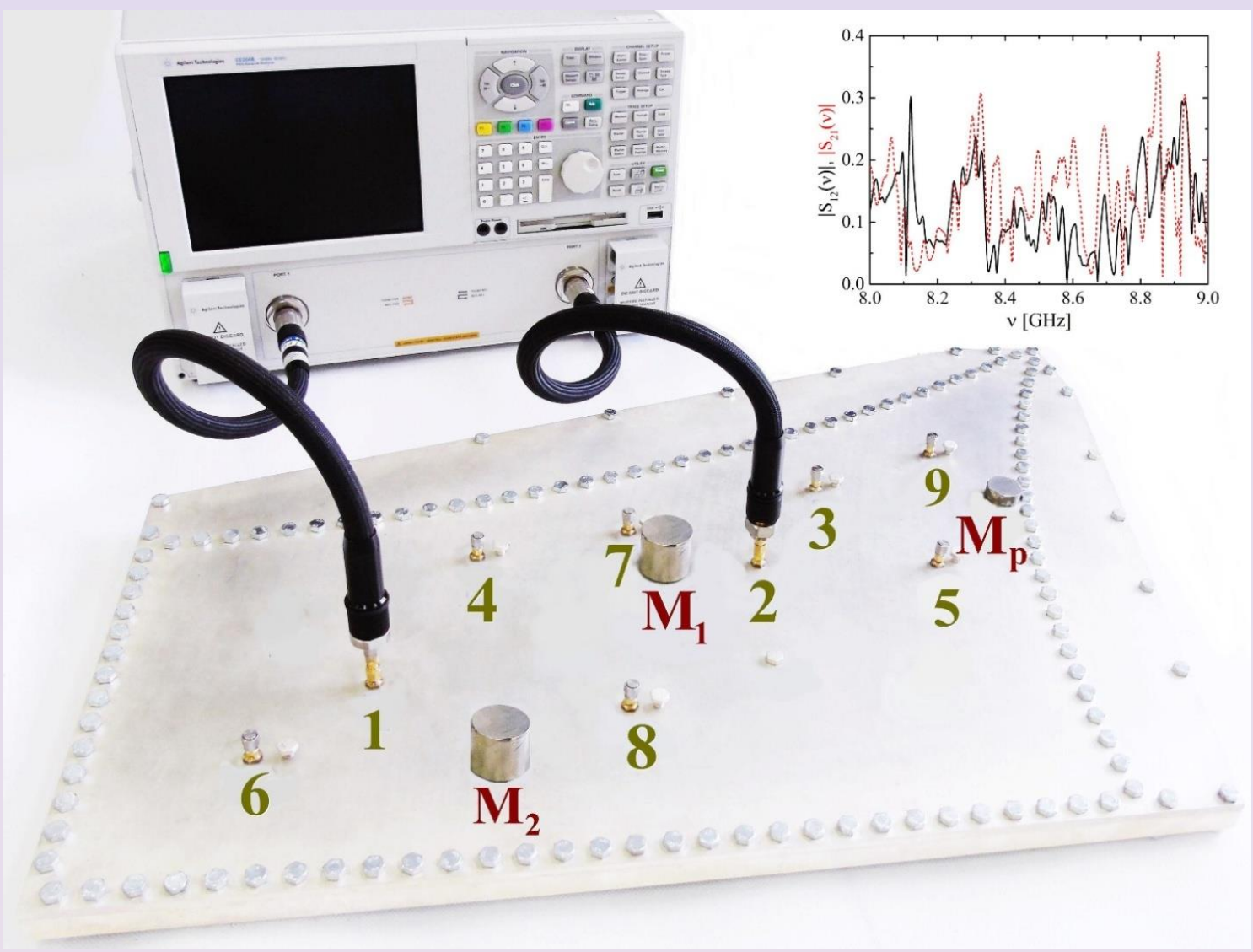
INTRODUCTION

We show experimentally and confirm theoretically that above a certain size of $\mathcal{T}$-invariance violation (TIV) the increase of the openness of a wave chaotic system can lead to an increase of the elastic enhancement factor (EEF). In the experiment a quantum billiard with partially violated time-reversal invariance, characterized by the $\mathcal{T}$-invariance violation parameter $\xi \in [0,1]$, is simulated with a flat quarter-bow-tie microwave cavity. TIV was induced by two cylindrical ferrites magnetized by an external magnetic field. The elastic enhancement factor $F_M(\eta, \gamma, \xi)$ is investigated as a function of internal absorption γ and openness η. In these investigations we focus on the frequency range of strongest TIV where the increase of the number of open channels M causes a boost of the elastic enhancement factor, instead of the expected lowering [1-3].

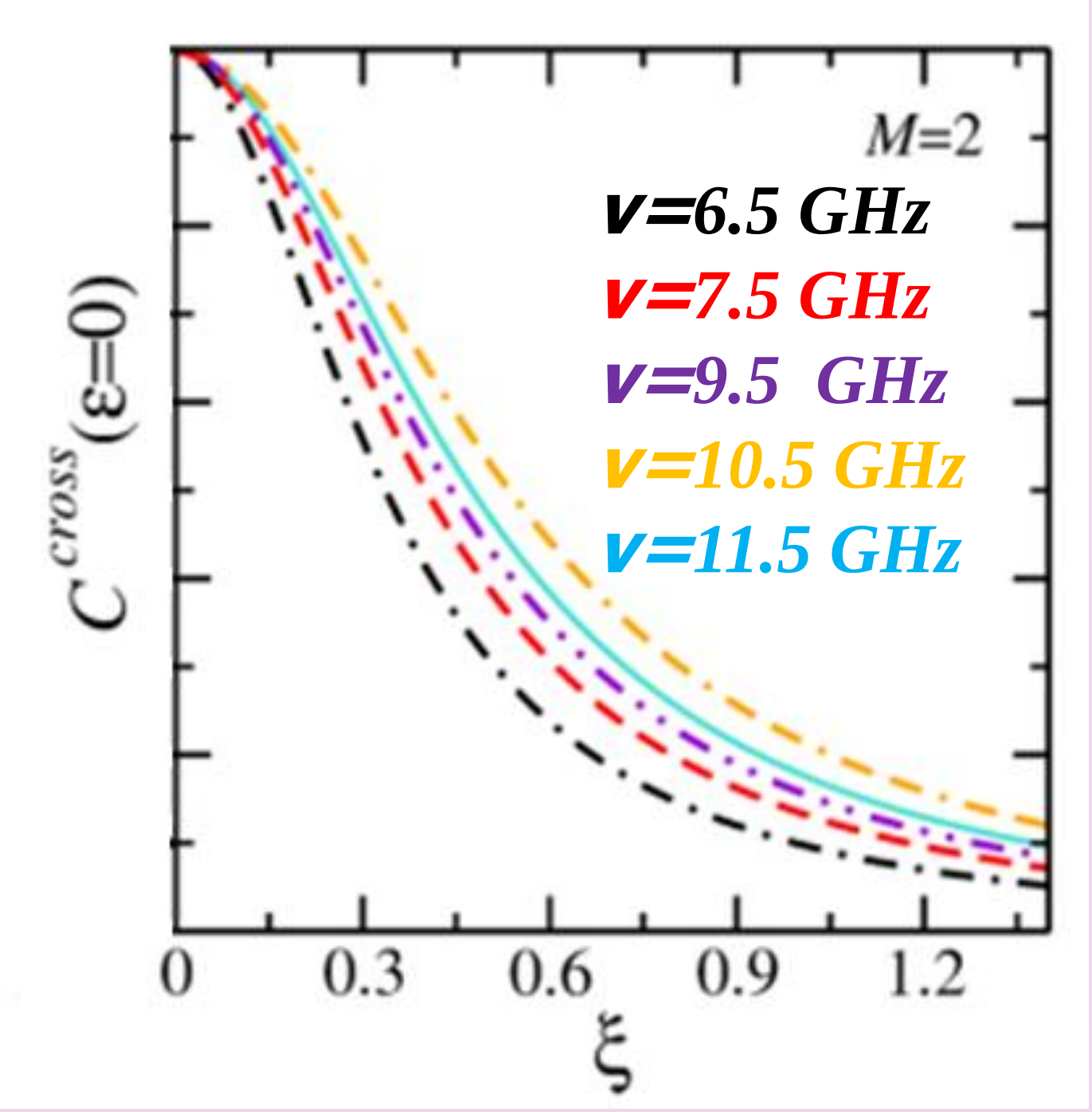
EXPERIMENT



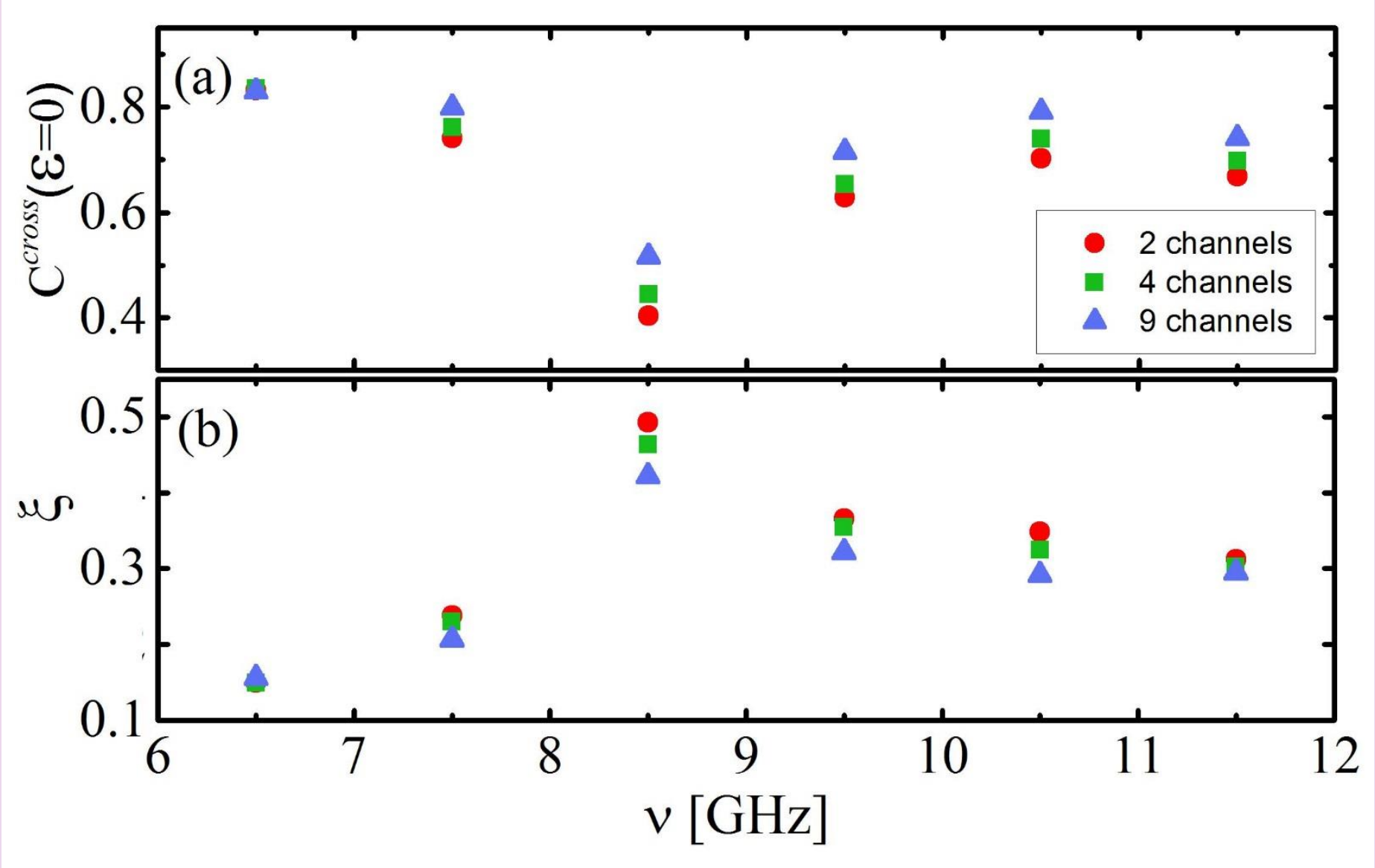
In the experiment the quantum billiard is simulated by a aluminum flat microwave cavity (area A=1828.5 cm^2, height h=1.2 cm) covered by 20 µm layer of silver. Billiards of that shape generate a chaotic dynamics. The two-dimensional Schrödinger equation for the quantum billiard is mathematically equivalent to the Helmholtz equation describing the electromagnetic field inside the cavity. The cut-off frequency of $\nu_{max}= c/2d \approx 12.49$ GHz. The homogeneous magnetic field of strength B ≈ 495 mT leads to $S_{12}(\nu) \neq S_{21}(\nu)$ of the measured two-port scattering matrix $\hat{S}(\nu)$. The microwave antennas 1 and 2 with the length 5.8 mm were connected to the Agilent E8364B microwave vector network analyzer. Randomly distributed open channels $2 \le M \le 9$ were realized by 7 antennas shunted with 50 Ω loads. In order to create 100 realizations for the cavity a metallic perturber M_p was moved along the walls of the cavity.

THE STRENGTH OF TIV

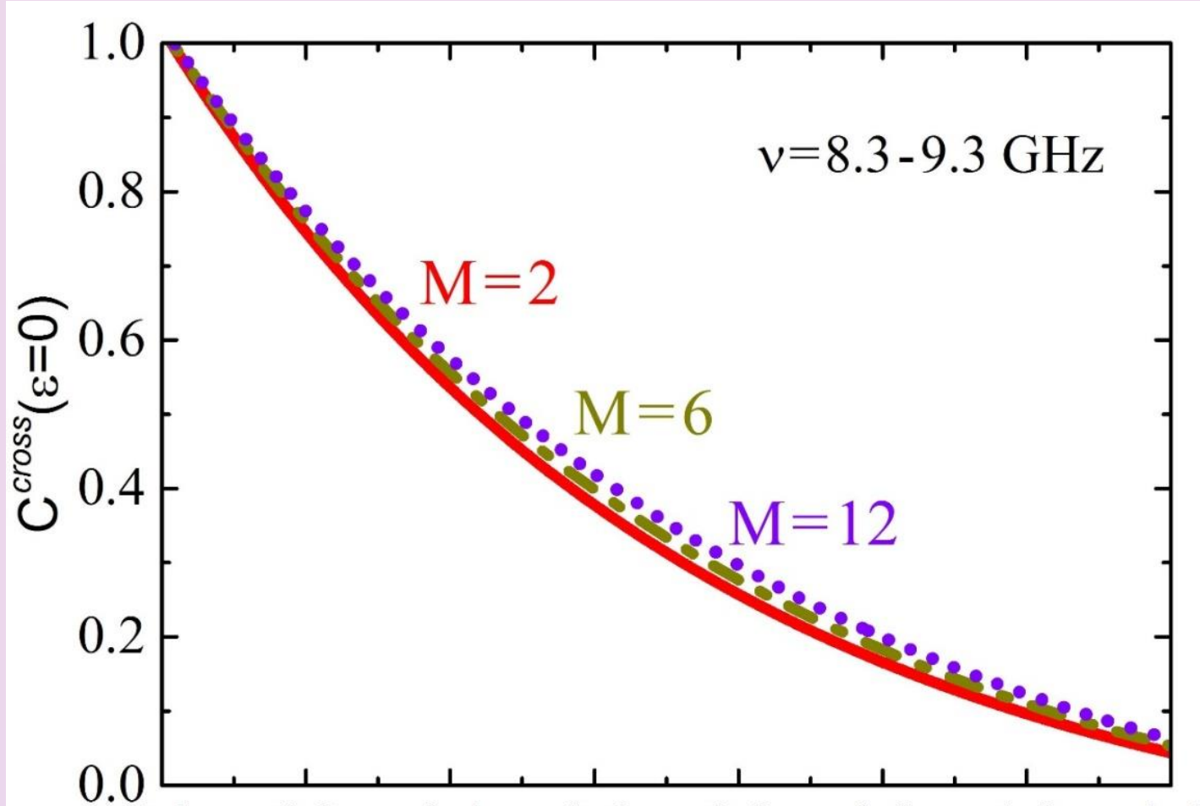
$$C_{ab}^{cross}(0) = C_{ab}^{cross}(\varepsilon = 0; \eta, \gamma, \xi) = \frac{\mathrm{Re}[\langle S_{ab}^{fl}(\nu) S_{ba}^{fl*}(\nu)\rangle]}{\sqrt{\langle |S_{ab}^{fl}(\nu)|^2 \rangle \langle |S_{ba}^{fl}(\nu)|^2 \rangle}}$$



The size of TIV can be quantified by using the cross- correlation coefficient $C_{ab}^{cross}(0)$. It decreases with the openness of the cavity and is largest in the frequency interval $\nu \in [8,9]$ GHz. There, $\xi \approx 0.49$.



(a) *Experimentally determined* $C_{ab}^{cross}(0)$ *over 100 cavity realizations.*
(b) *The strength* ξ *of TIV.*

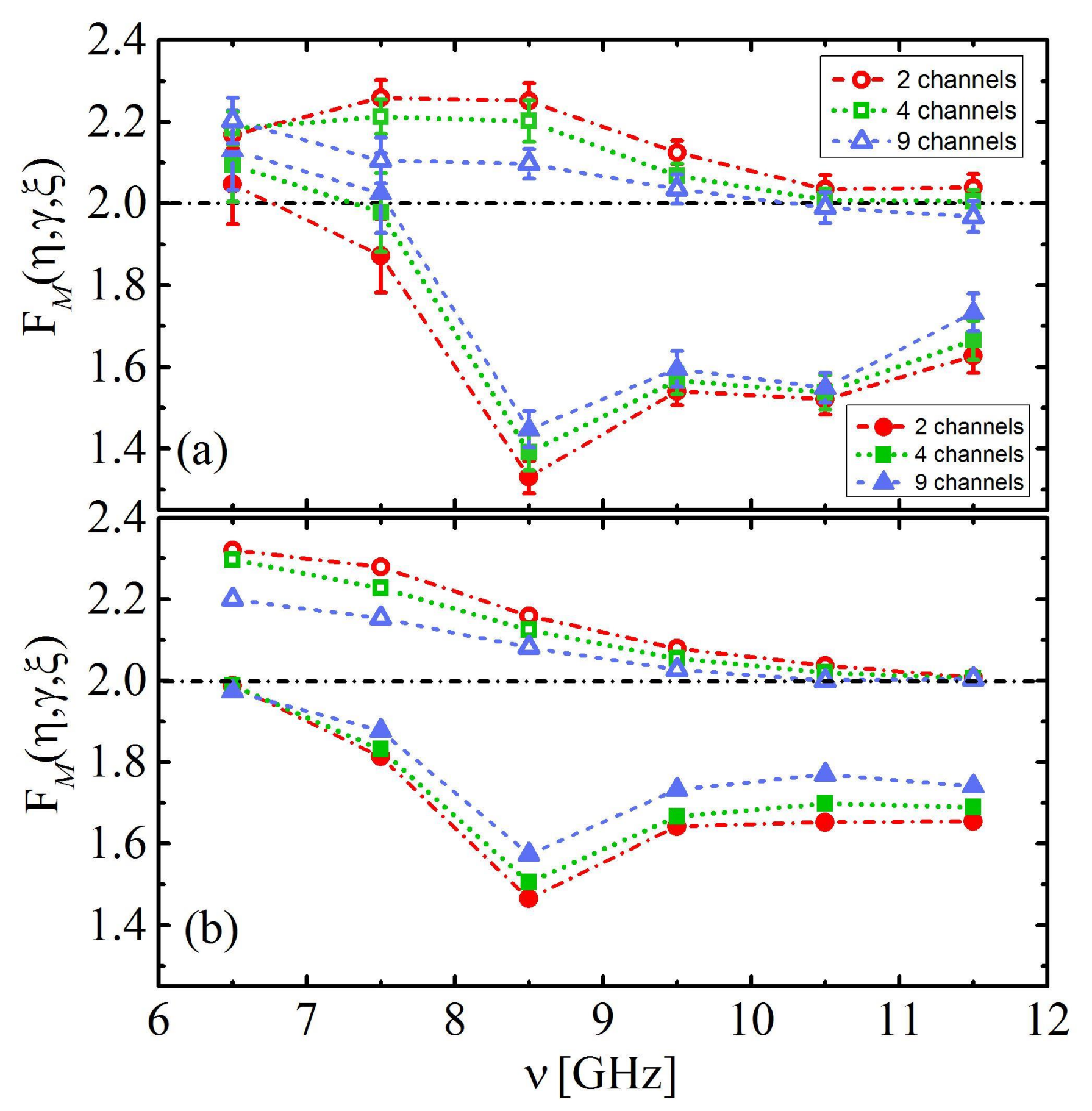


The experimental cross-correlation coefficient.

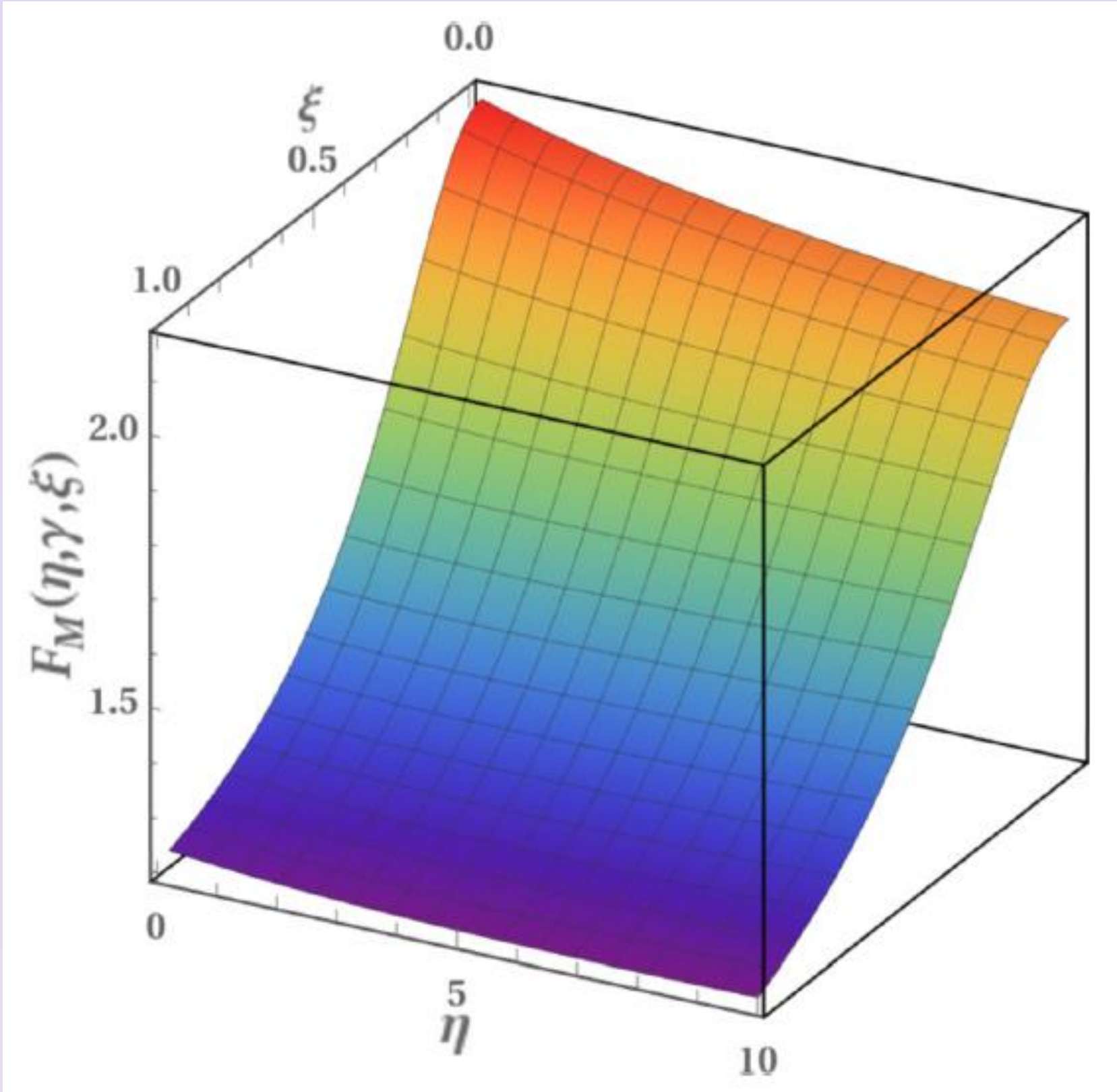
ELASTIC ENHANCEMENT FACTOR

The elastic enhancement factor $F_M(\eta, \gamma, \xi)$ as function of ξ and M can be expressed in terms of the scattering matrix elements $|S_{ab}{}^{fl}|^2 \equiv C_{ab}(0; \eta, \gamma, \xi)$

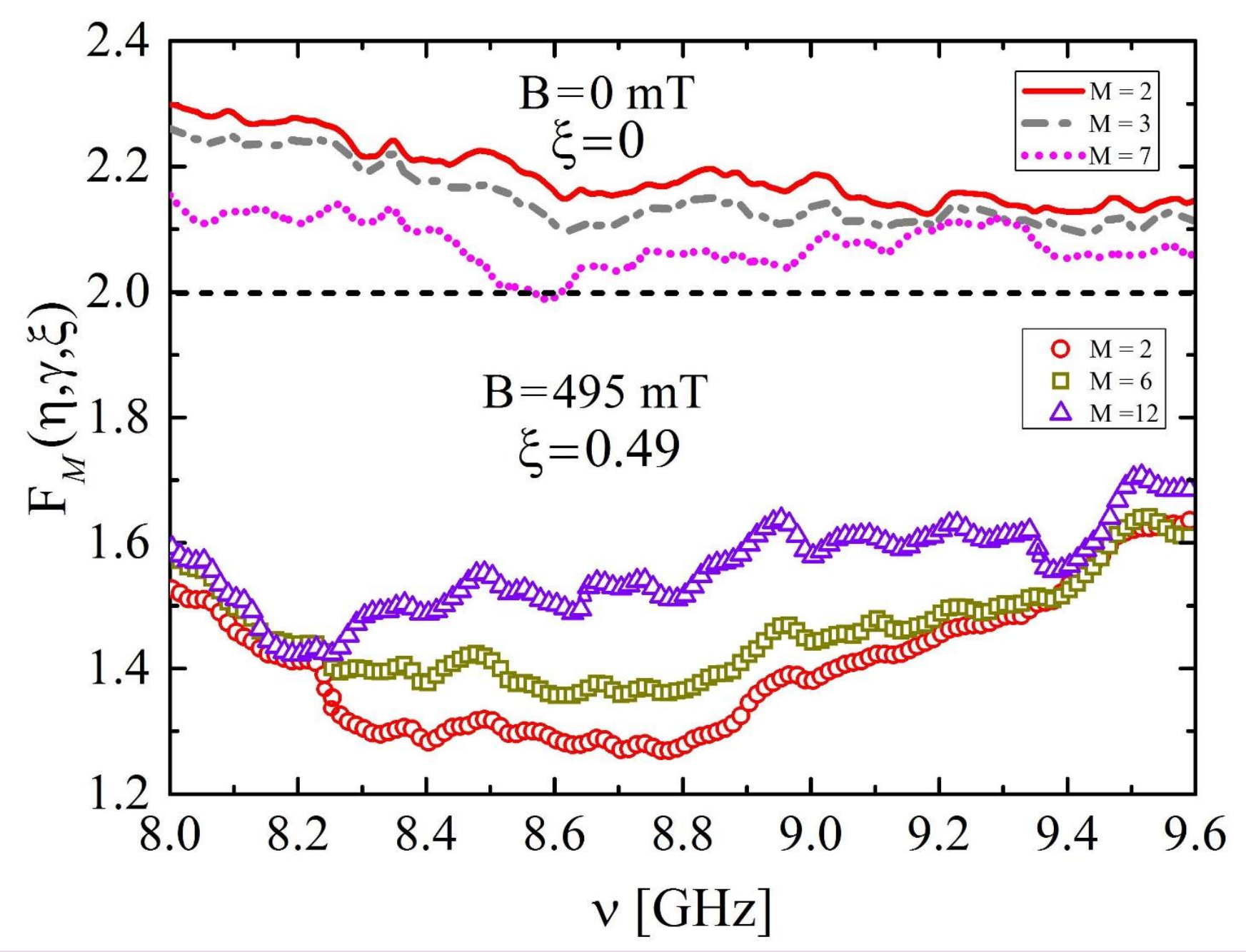
$$F_M(\eta,\gamma,\xi) = \frac{\sqrt{C_{aa}(0;\eta,\gamma,\xi) C_{bb}(0;\eta,\gamma,\xi)}}{\sqrt{C_{ab}(0;\eta,\gamma,\xi) C_{ba}(0;\eta,\gamma,\xi)}}$$



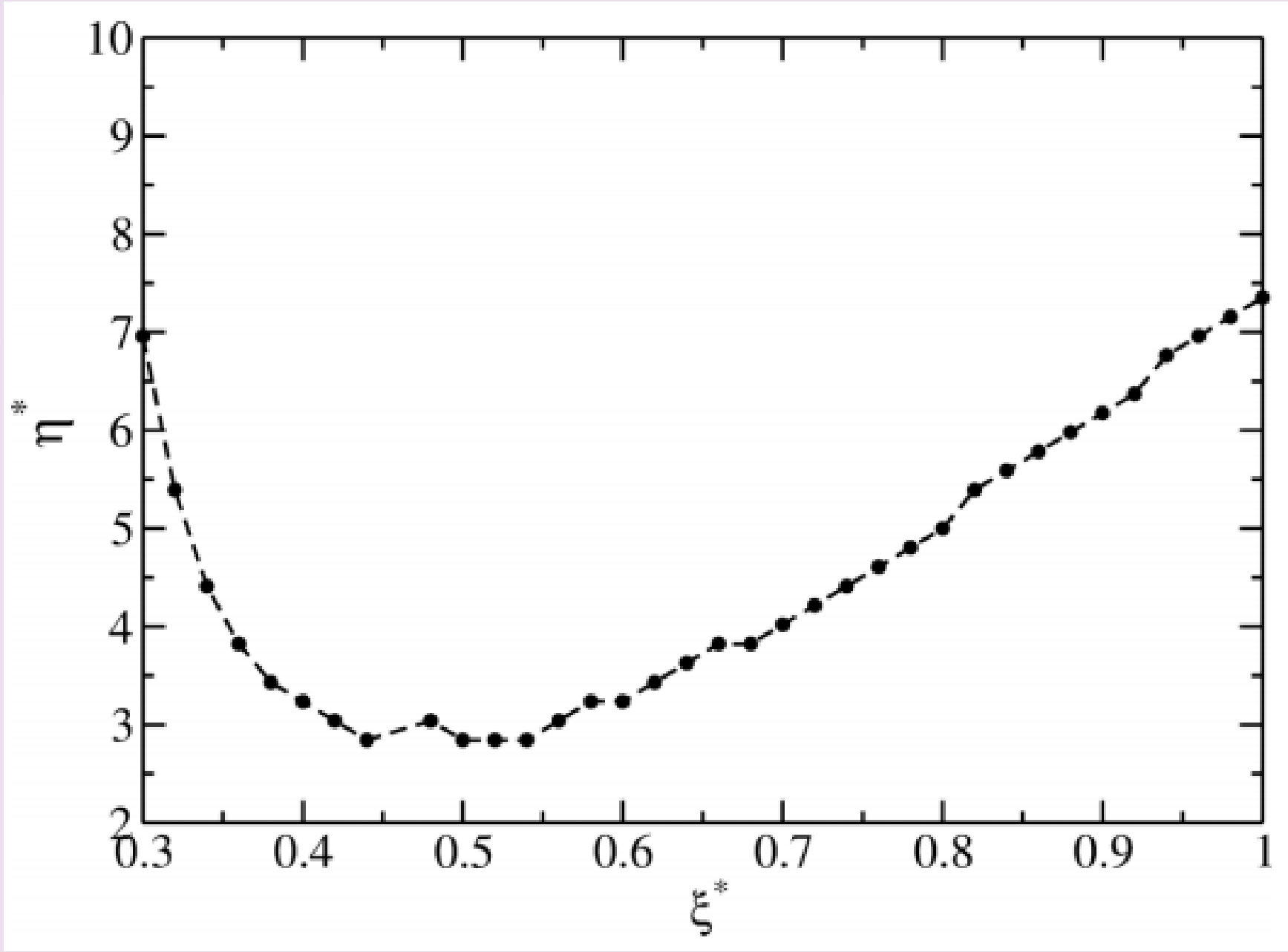
(a) *Experimental EEF with standard deviations was obtained without and with magnetized ferrite inside the cavity by averaging over 100 microwave billiard realizations (respectively empty and filled symbols). The black dash-dotted line separate the cases of preserved and violated* $\mathcal{T}$*-invariance.*
(b) *Same as (a) for RMT results.*



Three- dimensional plot of the computed EEF versus η *for fixed M =10 open channels and* γ *=10 (T and* ξ *were varied)- results for random matrix theory.*



Experimental EEF averaged over 100 realizations of the cavity in the frequency range $\nu \in [8; 9.6]$ *GHz.*



Changes of EEF - extracted from the 3D plot. The results are in accordance with experimental findings. The experimental values of openness are larger than η^* *for* $\xi > 0.2$ *hence the effect of* $\mathcal{T}$*-invariance violation on the elastic enhancement factor dominates over that of the openness.*

For a fixed number M of open channels and partial TIV (when $\beta=2$), the elastic enhancement factor decreases with increasing size of TIV induced by the magnetized ferrite. The increase of the number *M* leads to a decrease of the electric-field intensity and causes a boost of the EEF. The opposite behavior of the enhancement was observed for $\xi=0$. The RMT results reproduce the course of the experimental ones. The strong dip in the range $\nu \in [8,9]$ GHz coincides with that of the largest TIV, for $\xi \approx 0.49$. The experimental and numerical results corroborate the crossover from GOE (for $\xi=0$) to GUE (for $\xi=1$). The measured frequency range $\nu \in [6,12]$ GHz corresponds to the Ohmic absorption strength $6 \le \gamma \le 15$ due to the presence of the lossy ferrites.

CONCLUSIONS Elastic enhancement factor depends on the size of $\mathcal{T}$-invariance violation. The increase of the number of open channels *M* causes a boost of the elastic enhancement factor. The experimental results are in good agreement with the theoretical predictions.

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