ELASTIC ENHANCEMENT FACTOR AS A MEASURE OF DEGREE OF INTERNAL CHAOS.

Yaroslav A. Kharkov, Valentin V. Sokolov

Novosibirsk State University; Budker Institute of Nuclear Physics, Novosibirsk State Technical University.

21st May 2013

What is the Enhancement Factor?

Resonance compound nuclear reactions.

Enhancement factor is the typical ratio F of the cross sections of elastic and inelastic resonance processes.

Which kind of information the enhancement factor carries? What it can tell us about properties of the intermediate compound nucleus?

1. *P.A. Moldauer, Phys. Rev.* **123**, *968 (1961).*

2. *A. Müller and H.L. Harney, Phys. Rev. C* **35**, *1228 (1986).*

3. *J.J.M. Verbaarschot, Ann. Phys. (NY)* **168**, *368 (1986).*

4. *C.H. Lewenkopf and H.A. Weidenmüller, Ann. Phys. (NY)* **212**, *53 (1991).*

5. *N. Lehmann, D. Savin, V.V. Sokolov, H.-J. Sommers, Pysica D* **86**, *572, (1995).*

6. *Y.V. Fyodorov, D.V. Savin and H.-J. Sommers, J. Phys. A: Math. Gen.* **38**, *10731, (2005).*

7. *G.L. Celardo, F.M. Izrailev, V.G. Zelevinsky and G.P. Berman, Phys. Rev. E* **76**, *031119 (2007).*

QUANTUM CHAOS

and

Ensembles of random Hamiltonians H.

i. CLOSED SYSTEMS; INTERNAL DYNAMICS.

View from the inside: spectral statistics ($N \gg 1$);

The global scale: mean level density and mean level spacing

$$\rho(E) = \langle \sum_n \delta(E - E_n) \rangle = -\frac{1}{\pi} \Im \Big\langle Tr \frac{1}{E + i0 - \mathbf{H}} \Big\rangle = -\frac{1}{\pi} \Im \langle Tr\, G(E + i0) \rangle = \frac{1}{d}\,.$$

Characteristic **Heisenberg time** $t_H = 2\pi\rho(E) = 2\pi/d\,.$

Local universal spectral fluctuations. Density-density correlation function; unfolding .

$$\mathbf{C_2}(\varepsilon) = \frac{1}{\rho^2}\,\langle \sum_n \delta(E + \tfrac{\varepsilon}{2} - E_n)\,\sum_{n'} \delta(E - \tfrac{\varepsilon}{2} - E_{n'})\rangle - 1$$

$$= \delta\,(\rho\,\varepsilon) - Y_2\,(\rho\,\varepsilon)\ .$$

Dyson's binary form factor

$$Y_2\,(\rho\,\varepsilon) = 2\int_0^\infty B_2(s)\,\cos\,(2\pi\rho\varepsilon s)\,ds\,.$$

The number N of involved excited levels is supposed to be very large, $N \gg 1$, while the functions Y_2 and B_2 are calculated. The non-perturbative super-symmetry method is ordinary utilized.

ENSEMBLES OF HAMILTONIANS and T-INVARIANCE:

T-invariant systems are described by the ensemble of real symmetric Hamiltonian matrices: GOE.

Systems without T-invariance (very strong magnetic field!). Ensemble of complex Hermitian Hamiltonian matrices becomes relevant: GUE.

When the magnetic field is not strong enough, some transient Ensemble is used.

The correlation functions Y_2 and B_2 carry information on the degree of time-reversal symmetry.

ii. OPEN SYSTEMS: Effective Hamiltonian $\mathcal{H} = \mathbf{H} - \frac{\mathbf{i}}{\mathbf{2}}\,\mathbf{A}\mathbf{A}^{\dagger}$

View from outside: universal fluctuations of scattering amplitudes and cross sections.

The slow-varying reference quantities are: the mean scattering matrix and transmission coefficients

$$\langle S^{ab}\rangle = \langle S^{aa}\rangle\,\delta^{ab}; \qquad 1 - \left|\langle S^{aa}\rangle\right|^2 = T^a\,.$$

Equivalent channels (number $M \gg 1$**):** $T^a = T^b = ... = T\,.$

Characteristic time scale:

1. *E. Wigner, Phys. Rev.* **98**, *145 (1955).*

2. *E.T. Smith, Phys. Rev.* **118**, *349;* **119**, *2098 (1960).*

Channel-weighted Wigner delay time

$$\mathbf{Q}(E) = -\frac{2}{M}\,\Im\, Tr\frac{1}{E-\mathcal{H}} = \frac{2}{M}\sum_n \frac{\frac{1}{2}\Gamma_n}{(E-E_n)^2+\frac{1}{4}\Gamma_n^2}\,.$$

$$M\,\langle\mathbf{Q}\rangle(E) \Leftrightarrow 2\pi\rho(E) = t_H\,.$$

Mean delay time $\langle \mathbf{Q} \rangle$ defines the

dwell time $t_W = \langle \mathbf{Q} \rangle / T = \frac{1}{MT} t_H$ **and Weisskopf width** $\Gamma_W = 1/t_W$ **.**

The ratio $t_H/t_W = MT = \eta$ **characterizes the openness of the intermediate system.**

If $\eta \gg 1, \;\; t_H \gg t_W$ **- no information can be extracted from the outside on the spectral properties of the internal system!**

Only when $\eta \ll 1, \;\; t_H \ll t_W$**, transition amplitudes do carry information on the internal dynamics. That is the case of our main interest.**

The quantity of fundamental importance: the two-point correlation function of S-matrix elements (local scale fluctuations).

J.J. Verbaarschot, H.A. Weidenmüller and M.R. Zirnbauer, Phys. Rep. **129**, *367 (1985).* **(GOE)**

Y.V. Fyodorov, D.V. Savin and H-J. Sommers, Journ. Phys. A, **38**, *10731 (2005).* **(GUE)**

$$\mathbf{C_2}^{abcd}(\varepsilon) = \langle S^{ab}(E+\tfrac{1}{2}\varepsilon)\, S^{*cd}(E-\tfrac{1}{2}\varepsilon)\rangle - \langle S^{ab}(E+\tfrac{1}{2}\varepsilon)\rangle\, \langle S^{*cd}(E-\tfrac{1}{2}\varepsilon)\rangle$$

$$= \Upsilon_1^{(\beta)}(\rho\varepsilon)\, \delta^{ab}\, \delta^{cd} + \Upsilon_2^{(\beta)}(\rho\varepsilon)\, \left(\delta^{ac}\, \delta^{bd} + \delta_{1\beta}\, \delta^{ad}\, \delta^{bc}\right) \;;$$

where $\beta = 1$ **(GOE) or** $\beta = 2$ **(GUE).**

Enhancement factor is the ratio

$$\mathbf{F}^{(\beta)} = \langle |S^{aa}|^2 \rangle / \langle |S^{ab}|^2 \rangle = \mathbf{C_2}^{aaaa}(0)/\mathbf{C_2}^{abab}(0)$$

$$= 1 + \delta_{1\beta} + \Upsilon_1^{(\beta)}(0)/\Upsilon_2^{(\beta)}(0) = \begin{cases} 2 + \Upsilon_1(0)/\Upsilon_2(0), & \beta = 1\,, \\ 1 + \Upsilon_1(0)/\Upsilon_2(0), & \beta = 2\,. \end{cases}$$

Strong openness η $(t_H \gg t_W)$. **Semiclassical limit:** $M \gg 1$ equivalent channels with $T \lesssim 1$ (**strongly overlapping** resonances).

N. Lehmann, D. Saher, V.V. Sokolov, H.-J. Sommers, Nucl. Phys, A **582**, *223, (1995).*

the contribution $\Upsilon_1/\Upsilon_2 \sim 1/M \to 0$, and $\mathbf{F}^{(\beta)} = 1 + \delta_{1\beta} = \begin{cases} 2, & \beta = 1\,, \\ 1, & \beta = 2\,. \end{cases}$

No sensitivity to the **spectral properties**, just **interference**.

<u>Weak</u> openness η $(t_H \ll t_W)$.

J.J.M. Verbaarschot, Ann. Phys. (NY) **168**, *368 (1986).* **(GOE)**

Y.V. Fyodorov, D.V. Savin and H.-J. Sommers, J. Phys. A: Math. Gen. **38**, *10731, (2005).* **(GUE)**

In both this cases the ratio

$$\Upsilon_1(0)/\Upsilon_2(0) = \eta \int_0^\infty ds\, e^{-\eta s}\, [1 - B_2(s)] = 1 - \eta \int_0^\infty ds\, e^{-\eta s}\, B_2(s)$$

is directly related to the corresponding spectral form factor!

On the other hand, time delay correlation function looks as

$$\mathbf{K}(\varepsilon;\eta) = \frac{1}{\langle \mathbf{Q} \rangle^2} \langle \mathbf{Q}(E+\frac{1}{2}\varepsilon)\mathbf{Q}(E-\frac{1}{2}\varepsilon)\rangle - 1 = \int_0^\infty ds\, e^{-\eta s}\left[1 - B_2(s)\right]\cos\left(2\pi s \rho \varepsilon\right) .$$

N. Lehmann, D. Savin, V.V. Sokolov, H.-J. Sommers, Physica D **86**, *572, (1995).* **(GOE)**
Y.V. Fyodorov and H.-J. Sommers, Phys. Rev. Lett. **76**, *4709, (1996).* **(GUE)**

In particular, the variance is

$$var\, Q(\eta) = \mathbf{K}(0;\eta) = \int_0^\infty ds\, e^{-\eta s}\left[1 - B_2(s)\right] .$$

Thereby the following remarkable connection is established between the enhancement factor and the time delay variance:

$$\mathbf{F}^{(\beta)}(\eta) = 1 + \delta_{1\beta} + \eta\, var\, Q(\eta) = 2 + \delta_{1\beta} - \eta \int_0^\infty ds e^{-\eta s}\, B_2{}^{(\beta)}(s)\,.$$

The last term vanishes together with η and approaches to unity when η grows. The maximal value is 3 or 2 depending on the symmetry class.

Enhancement Factor

from regular to chaotic regime

G.L. Celardo, F.M. Izrailev, V.G. Zelevinsky and G.P. Berman, Phys. Rev. E **76**, *031119 (2007)*.

S. Sorathia, PHD thesis, Puebla, Mexico (2010).

Transient matrix Ensembles: $\mathbf{F}^{(\beta)}(\eta|\kappa)$.

THE MODEL.

1. The decay amplitudes are supposed to be Gaussian random variables (parameter γ controls the strength of the coupling to continuum):

$$\langle A^a_m \rangle = 0, \quad \langle A^a_m{}^* \, A^b_n \rangle = \frac{\gamma}{N} \, \delta^{ab} \, \delta_{mn} \, .$$

Why?

Angular momentum conservation: transitions go via an internal degenerate multiplet $|JM\rangle$ with a fixed value J of angular momentum $\mathbf{J} = \sum_{\mathbf{a}} \mathbf{j_a}$.

Vladimir Zelevinsky, Alexander Volya, Pysics Report **391**, *311 (2004).*

Random walk in the angular momentum space and "geometrical chaos": Gaussian probability distribution $w(M)$.

2. The joint probability distribution $P(E_1, E_2, ..., E_N|\kappa)$ of the eigenenergies of the internal Hamiltonian $\mathbf{H}$ changes, depending on the parameter of chaoticity κ, from Poissonion distribution of fully independent levels ($\kappa = 0$) to that of highly correlated levels what is typical of the GO or GU Ensembles ($\kappa \to \infty$).

3. The mean level density ρ is supposed to be independent of the parameter κ.

1. A-averaging .

Keeping the internal Hamiltonian H diagonal,

$$\left\langle S^{ab}(E)\right\rangle = \delta^{ab}\left\langle \frac{1-i\frac{\gamma}{2}g(E)}{1+i\frac{\gamma}{2}g(E)}\right\rangle_{P(\{E_n\}|\kappa)}$$

where the mean trace $g(E) = \frac{1}{N}\langle Tr\frac{1}{E-\mathcal{H}}\rangle_A$ **satisfies**

$$g(E) = \frac{1}{N}\sum_n \left[E - E_n + \frac{\frac{i}{2}m\gamma}{1+\frac{i}{2}m\gamma\, g(E)}\right]^{-1}$$

$(m = M/N)$.

In the weak coupling approximation it reduces to

$$g(E) \approx \frac{1}{N} \sum_n \frac{1}{E - E_n + \frac{i}{2} m\gamma} = \frac{1}{N} Tr\, G(E + \frac{i}{2} m\gamma)\,.$$

This yields the mean scattering matrix (supposing that $m \ll 1$)

$$\left\langle S^{ab}(E=0) \right\rangle = \delta^{ab} \frac{1 - i\frac{\gamma}{2}\langle g \rangle_{P(\{E_n\}|\kappa)}}{1 + i\frac{\gamma}{2}\langle g \rangle_{P(\{E_n\}|\kappa)}} = \frac{1-\zeta}{1+\zeta}$$

The parameter $\zeta = \frac{\pi\gamma}{2Nd}$ measures the degree of resonance overlap. Weak overlap: $\zeta \ll 1$, $T \approx 4\zeta$ so that the openness equals $\eta = 4M\zeta = \underline{2\pi m\gamma/d}$.

In the same approximation

$$\Upsilon_1(0) = \left(\frac{\gamma}{N}\right)^2 \left\langle Tr G(\frac{i}{2}m\gamma)\, Tr G^\dagger(\frac{i}{2}m\gamma) \right\rangle_{P(\{E_n\}|\kappa)},$$

$$\Upsilon_2(0) = \left(\frac{\gamma}{N}\right)^2 \left\langle \left[Tr G(\frac{i}{2}m\gamma)\, G^\dagger(\frac{i}{2}m\gamma) \right] \right\rangle_{P(\{E_n\}|\kappa)}$$

resulting finally in

$$\mathbf{F}^{(\beta)}(\eta|\kappa) = 1+\delta_{1\beta}+\eta\, \mathbf{K}^{(\beta)}(0;\eta|\kappa) = 2+\delta_{1\beta}-\eta \int_0^\infty ds e^{-\eta s}\, B_2{}^{(\beta)}(s|\kappa)\,.$$

Limiting cases: $\mathbf{F}^{(\beta)}(\eta|0) \equiv 2 + \delta_{1\beta}$; $\mathbf{F}^{(\beta)}(\eta|\infty) \Rightarrow GE^{(\beta)}$.

In the transition region $0 < \kappa < \infty$**, the binary form factor** ${B_2}^{(\beta)}(s|\kappa)$ **is known currently only in the case** $\beta = 2$.

H. Kunz and B. Shapiro,Phys. Rev. E, **58**, *400, (1998).*

Main properties of the form factor $B_2(s|\kappa)$:

$B_2(s|0) \equiv 0 \Rightarrow$ **No level correlations;**

$B_2(s|\infty) = (1-s)\theta(1-s) \Rightarrow$ **maximal correlations, GUE.**

Besides, $\int_0^\infty ds B_2(s|\kappa \neq 0) \equiv \frac{1}{2}$ **.**

Two perfectly equivalent representations:

$$\mathbf{F}(\eta|\kappa) = 1 + \Psi(\eta|\kappa) + \eta \frac{d^2}{d\eta^2}\left[\frac{1}{\eta}\int_0^\kappa d\kappa' \Psi(\eta|\kappa')\right], \qquad (1)$$

$$\mathbf{F}(\eta|\kappa) = 1 + \frac{1-e^{-\eta}}{\eta} + \Psi(\eta|\kappa) - \eta \frac{d^2}{d\eta^2}\left[\frac{1}{\eta}\int_\kappa^\infty d\kappa' \Psi(\eta|\kappa')\right] \qquad (2)$$

where the only function to be calculated is

$$\Psi(\eta|\kappa) = \eta \int_0^\infty ds\, e^{-\kappa s(s+1)-s\eta}\, \frac{I_1(2\kappa\, s^{3/2})}{\kappa\, s^{3/2}} \geqslant 0\,,$$

in particular

$$\Psi(\eta|0) \equiv 1, \quad \Psi(\eta|\infty) \equiv 0\,.$$

The form (1) is more convenient when $\kappa \ll 1$ (almost regular internal dynamics), whereas the form (2) is most useful if $\kappa \gg 1$ (the internal dynamics is almost perfectly chaotic.)

Limiting cases: $\mathbf{F}(\eta|0) \equiv 2; \quad \mathbf{F}(\eta|\infty) = 1 + \frac{1-e^{-\eta}}{\eta}$ **(GUE).**

The universal slope

$$\frac{d\mathbf{F}(\eta|\kappa \neq 0)}{d\eta}\Big|_{\eta=0} \equiv -\frac{1}{2}\,.$$

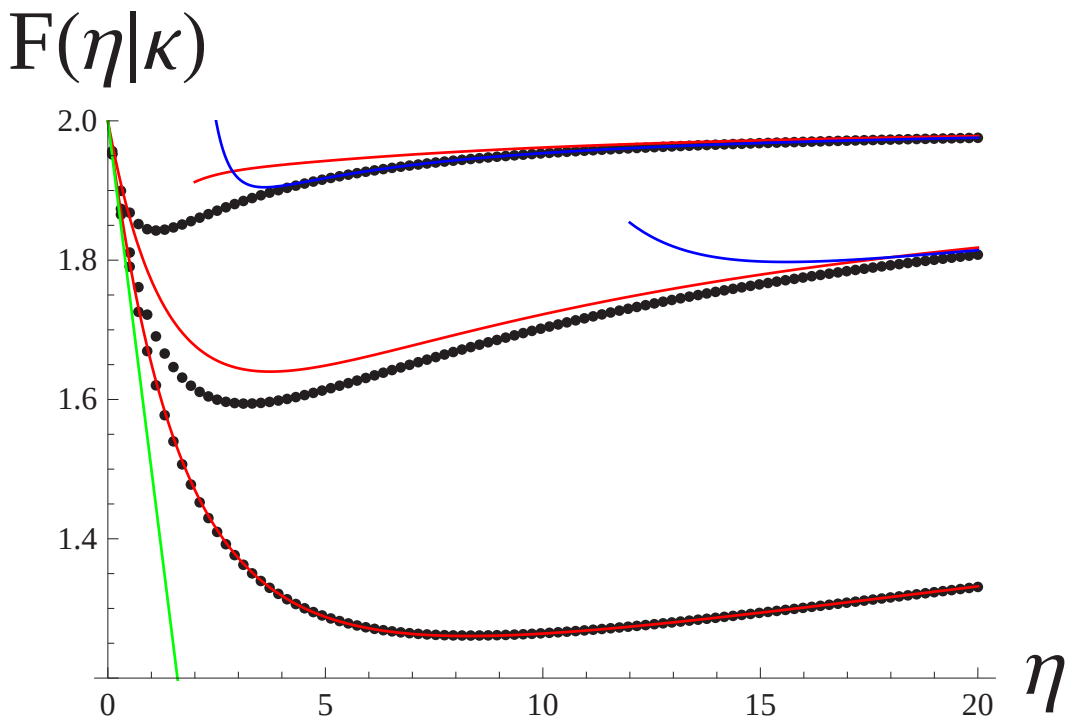

Figure 1: F versus η for (from top to bottom) $\kappa = 0.5,\ 5,\ 50$. Black points - exact numerics; blue lines - small κ approximation derived from eq.(1); red lines - large κ approximation derived from eq.(2); green line indicates the slope at the point $\eta = 0$.

As the openness η grows, the enhancement factor $F(\eta|\kappa)$ goes down until some minimal value F_{min}. This happens at some critical openness $\eta_c(\kappa)$ fixed by the condition $\frac{d\mathbf{F}(\eta|\kappa)}{d\eta} = 0$. Corresponding value of the chaoticity parameter $\underline{\kappa(F_{min})}$ can be, finally, extracted from the equation

$$F(\eta_c(\kappa)|\kappa) = F_{min}$$

The enhancement factor measures the degree of internal chaos.

Thank you for your attention!