Universal conductance fluctuations in spin glasses

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Results of numerical studies of conductance of a two-dimensional tight-binding system with random potentials and coupled to a three-dimensional Ising spin glass are presented. Numerical values of the conductance dispersion, in an ensemble of the potentials, support results of diagrammatic calculations for universality classes relevant for this system. Conductance of a sample is studied as a function of "time" for various Monte Carlo trajectories of the spin glass. The dispersion of the conductance noise decreases on cooling of the spin system without a noticeable anomaly at T_c. Power spectra of the noise are power laws in frequency with the exponent of -0.5 (at least in the vicinity of T_c) and the amplitude of a spectrum decreases monotonically with T. The fluctuations in the noise are only statistically related to the number of spin flips involved. The dispersion of the fluctuations at a given temperature depends on initial spin configurations. Locations of spins that effect conductance must also depend on the spin configuration. History-dependent effects are present both above and below T_c. Conductance noise in ferromagnets is compared to that occurring in spin glasses.

I. INTRODUCTION

Advances in materials technology have recently enabled fascinating studies of electronic mesoscopic systems.[1,2] In these systems the length over which the electrons can scatter inelastically may exceed the system size and allow for observation of quantum-interference effects of the electronic de Broglie waves.[3,4] This happens typically in micrometer-sized systems which are studied in the millikelvin temperature range. One manifestation of the quantum-interference laws is the emergence of the Aharonov-Bohm effect in mesoscopic metal rings. Another is the reproducible noiselike sensitivity of conductivity changes in values of parameters that define the system such as magnetic field, Fermi energy, and configuration of impurities. This sensitivity, as measured by the variance of the response, appears to be universal and is known under the name of universal conductance fluctuations.

Altshuler and Spivak[5] as well as Feng *et al.* have suggested that the quantum-interference effects would be put to use as a probe of spin dynamics in those mesoscopic systems which contain localized lattice spin degrees of freedom. Studying spin glasses would be of a particular interest since the dynamics of these systems is complex and poorly understood. Feng *et al.*[6] have proposed that since the conductance of the system is sensitive to the configurations of spins in the system, the chaotic nature of spin reorganizations would result in measurable changes of conductance on varying the temperature and as a function of time.

Effects of quantum interference in spin glasses have recently been studied experimentally. Israeloff *et al.*[7] have detected low-frequency resistance noise in thin films of the metallic $Cu_{1-x}Mn_x$ $(0.045 \le x \le 0.195)$ and compared it to the $1/f$ noise in the magnetization, which was measured by the imaginary part of the dynamic susceptibility. The power spectrum of the resistance noise was found to increase on cooling down through the spin-glass freezing temperature and then to exhibit a tendency toward saturation.

In a recent paper, Vegvar, Levy, and Fulton[8] have reported observation of universal conductance fluctuations induced by varying the magnetic field in small (mesoscopic) samples of $Cu_{1-x}Mn_x$. The pattern in the fluctuations has been found to be sensitive to the thermal history of the system. A sensitivity of the resistance to the thermal history as well as temperature and frequency dependences of the noise in small samples of $Cu_{0.91}Mn_{0.09}$ have also been studied by Israeloff, Alers, and Weissman.[9] The experimental results have been analyzed in view of predictions of various models of glassy dynamics.

Similar studies have been recently undertaken by Grabecki *et al.*[10] The system considered is the semiconducting p-type $Hg_{0.80}Cd_{0.19}Mn_{0.01}Te$ bicrystal. Its low-temperature conductance is due to a two-dimensional electron gas adjacent to the grain-boundary plane.[11] Conduction fluctuations were measured as function of magnetic field (up to 9 T) and down to $T=30$ mK. An ac susceptibility cusp (at 20 Hz) is found to develop around $T=100$ mK. At such low concentration and temperatures, the spin-glass-like ordering could be due to the dipole-dipole interactions.[12] There is, however, a possibility that as in the dilute dipolar-coupled $LiHo_{0.045}Y_{0.055}F_4$, studied by Reich, Rosenbaum, and Aepli,[13] the cusp is merely a nonequilibrium effect. The studies of Israeloff *et al.*[7,9] of $Cu_{1-x}Mn_x$ have focused on the power spectra of the noise, whereas investigations of $Hg_{0.80}Cd_{0.19}Mn_{0.01}Te$ were primarily concerned with properties of the resistance fluctuations generated by varying the magnetic field.

In this paper we report results of numerical studies of conductivity in a two-dimensional (2D) small tight-binding system with random potentials, which is coupled to a three-dimensional Edwards-Anderson spin Ising spin glass. The electrons are constrained to move in a central plane that cuts through the spin-glass system. This geometry is motivated by the structure of the bicrystal system considered by Grabecki *et al.*[10] The lattice spins were chosen to be of the Ising type for two reasons. First, theoretical studies should start with a simplest model possible and then become dressed with complications. Second, various sources of anisotropy induce crossover to Ising spin-glass behavior with a phase transition in three-dimensional systems.[14,15] It should be noted, however, that the dynamics of a continuous-symmetry spin system may differ from the purely relaxational dynamics of Ising systems. Our model may enhance the effects of spin freezing if what is observed in the experiments is merely an apparent freezing.

In Sec. II we describe our model in detail and define its parameters. In Sec. III we derive a Kubo-formalism-based expression for the conductivity that we used. Conductance and conductivity coincide for two-dimensional systems, and so we use these terms interchangeably. In Sec. IV we discuss the influence of electron scattering by a system of frozen spins upon the universal conductance fluctuations. In particular, we demonstrate good agreement between results of our numerical simulations and predictions of diagrammatic calculations. In Sec. V we describe our Monte Carlo algorithm and give an estimate of the numerically accessible time domain. We then present results on the conductance noise associated with the spin-disorder scattering. We find that the amplitude of noise decreases on cooling and that the power spectra are essentially monotonic in temperature. They are described by the $f^{-0.5}$ law, which is a 2D analog of the $1/f$ noise.

In Sec. VI we attempt to understand the conductance fluctuations in terms of the lattice spin flips and point to difficulties in making simple interpretations. We conclude that uses of conductance fluctuations as a reliable probe of the spin dynamics may prove not to be easy. It is possible that only laws based on large statistics contain precise information about the spin-glass dynamics.

II. DESCRIPTION OF THE SYSTEM

The Hamiltonian of the system is given by

$$H = H_e + H_{e\text{-}s} + H_s \;, \tag{1}$$

where H_e describes electrons hopping on a two-dimensional square lattice, H_s describes the three-dimensional spin-glass system, and $H_{e\text{-}e}$ corresponds to the interaction between the two subsystems. One can express H_e as

$$H_e = \sum_{l,\mathbf{a}} \Psi_l^\dagger U(l,\mathbf{a}) \Psi_{l+\mathbf{a}} + \sum_l \Psi_l^\dagger V(l) \Psi_l - g_e \mu_B B \sum_l \Psi_l^\dagger \sigma^z \Psi_l \;, \tag{2}$$

where

$$\Psi_l^\dagger = (c_{l+}^\dagger, c_{l-}^\dagger) \;, \quad \Psi_l = \begin{bmatrix} c_{l+} \\ c_{l-} \end{bmatrix} \;, \tag{3}$$

represent creation and annihilation operators for electrons at site l and spin states $+$ or $-$. The summation is over all sites of the square lattice.

The matrix U is given by

$$U(l,\mathbf{a}) = \begin{bmatrix} u & 0 \\ 0 & u \end{bmatrix} \;, \tag{4}$$

where u is the hopping matrix element. The hopping is confined to the nearest-neighbor sites, and the vector $\mathbf{a} = (a_x, a_y)$ connects two such sites. For the square lattice, $a_x, a_y = \pm 1$ in units of the lattice constant. The matrix V is defined by

$$V(l) = \begin{bmatrix} \varepsilon(l) & 0 \\ 0 & \varepsilon(l) \end{bmatrix} \;, \tag{5}$$

where $\varepsilon(l)$ is the potential at site l. The last term in Eq. (2) corresponds to the Zeeman spin splitting in a magnetic field B; g_e is the electronic g factor, and μ_B is the Bohr magneton. The effects of magnetic field on the electronic states will not be considered in this paper.

We assume the following geometry for the electronic subsystem. Atomic sites lie on the square lattice. The central $L_x \times L_y$ section is disordered. In the section potentials $\varepsilon(l)$ are random. We chose them to be uniformly distributed between $-w/2$ and $w/2$, with $w = 2u$. Infinite perfect leads are attached in the x direction, to the left- and right-hand sides of the section. In the leads, $\varepsilon(l) = 0$. The form of boundary conditions on the other sides is not essential, and we chose the periodic boundary conditions. We typically study systems with $L_x = L_y = L = 20$. According to Stone,[16] the universal conductance fluctuations actually show an L dependence, which saturates for L close to and exceeding 20, hence the choice of our system size.

Consider now the lattice spin system. The magnetic sites typically occupy only a fraction of the atomic sites. As a simple model of this situation, we take the Ising spins to occupy sites of the $L_s \times L_s \times L_s$ cubic lattice. A central plane in this lattice coincides with the plane on which the tight-binding system is defined, but the spin lattice forms a superlattice on the disordered $L_x \times L_y$ section. We took L_s to be 9 so that when looking along each principal direction every other site of the disordered section houses a lattice spin.

The spin Hamiltonian is given by

$$H_s = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j - g_s \mu_B B \sum_i S_i \ , \tag{6}$$

where $S_i = \pm 1$ and J_{ij} takes values distributed with the Gaussian probabilities with 0 mean and dispersion equal to J_0. The spin-glass freezing temperature of this system is known to be of order J_0.[15] In the Zeeman term, g_s is the lattice spin g factor.

The coupling between the two subsystems is given by

$$H_{e\text{-}s} = -J_{e\text{-}s} \sum_{\mathbf{m}} \Psi_{\mathbf{m}}^\dagger \sigma \cdot \mathbf{S}_{\mathbf{m}} \Psi_{\mathbf{m}} \ , \tag{7}$$

where $\mathbf{m}$ runs through those sites of the disordered segment which contain a lattice spin. $\sigma = (\sigma^x, \sigma^y, \sigma^z)$ and σ^α are the Pauli matrices. In the present work we consider only Ising spins, and so only the σ^z term enters Eq. (7). We took $J_{e\text{-}s} = w/10$ ($J_{e\text{-}s}$ is typically of order 0.5 eV). Note that $H_{e\text{-}s}$ modifies the effective site potentials randomly and in a way which differentiates between the up and down electronic states. The electronic part $H_{\rm el} = H_e + H_{e\text{-}s}$ can be separated into two parts: one corresponding to electrons with spin up and one to spin down. Each flip of a lattice spin makes an imprint on the landscape of the potentials by changing $H_{\rm el}$ and thus affects the electronic wave functions. Time evolution in the spin system produces a noise in the conductivity which will be an object of our studies. We neglect any influences that electrons might have on the spin system.

III. DETERMINATION OF CONDUCTIVITY

In order to calculate the conductivity, we follow Elliot, Krumhansl, and Leath[17] and make use of the Kubo formalism. The formula is based on the linear-response theory, which relates conductivity to a current-current correlation function, where the current is defined by

$$\mathbf{j}_\mu = \sum_{l} \mathbf{j}_\mu(\mathbf{l}) = (ie\mathbf{a}_\mu/\hbar) \sum_{l} [\Psi_l^\dagger U(\mathbf{l}, \mathbf{a}_\mu) \Psi_{l+\mathbf{a}_\mu} - \text{H.c.}] \ , \tag{8}$$

and $\mathbf{a}_\mu$ is a vector toward a neighbor in the μth direction.

We define the Green's function as

$$G(E) = (E - H_e - H_{e\text{-}s})^{-1} \ , \tag{9}$$

the matrix elements of which will be noted by $G(\mathbf{l}, \mathbf{n}, E)$. Note that $G(\mathbf{l}, \mathbf{n}, E) = \langle\langle \Psi_l | \Psi_{\mathbf{n}}^\dagger \rangle\rangle_E$ is a 2×2 matrix. The static conductivity is then given by[16]

$$\begin{aligned}\sigma_{\mu,\nu}(E_F) = \frac{2\pi e^2}{\hbar\Omega} \mathbf{a}_\mu \mathbf{a}_\nu u^2 \int_{-\infty}^{\infty} dE \frac{\exp[(E-E_F)/k_B T]}{\{\exp[(E-E_F)/k_B T]+1\}^2} \sum_{\mathbf{l},\mathbf{n}} [&\mathrm{Im}G(\mathbf{l},\mathbf{n},E)\mathrm{Im}G(\mathbf{n}+\mathbf{a}_\mu, \mathbf{l}+\mathbf{a}_\nu, E) \\ &+ \mathrm{Im}G(\mathbf{l}+\mathbf{a}_\nu, \mathbf{n}+\mathbf{a}_\mu, E)\mathrm{Im}G(\mathbf{n},\mathbf{l},E) \\ &- \mathrm{Im}G(\mathbf{l},\mathbf{n}+\mathbf{a}_\mu,E)\mathrm{Im}G(\mathbf{n},\mathbf{l}+\mathbf{a}_\nu,E) \\ &- \mathrm{Im}G(\mathbf{l}+\mathbf{a}_\nu,\mathbf{n},E)\mathrm{Im}G(\mathbf{n}+\mathbf{a}_\mu,\mathbf{l},E)] \ ,\end{aligned} \tag{10}$$

where $\Omega = L_x \times L_y$ is a volume of the electronic sublattice. This equation can be simplified by considering the limit of $T \to 0$ and by making use of the current conservation in each plane perpendicular to the leads.[18] Specifically, for the 2D geometry, we get

$$\begin{aligned}\sigma_{xx} = \frac{e^2 a_x L_x u^2}{h L_y a_y} \sum_{l_y, n_y} [&\mathrm{Im}G(l_x, l_y; n_x, n_y, E_F)\mathrm{Im}G(n_x + a_x, n_y, l_x + a_x, l_y, E_F) \\ &+ \mathrm{Im}G(l_x + a_x, l_y; n_x + a_x, n_y, E_F)\mathrm{Im}G(n_x, n_y, l_x, l_y, E_F) \\ &- \mathrm{Im}G(l_x, l_y; n_x + a_x, n_y, E_F)\mathrm{Im}G(n_x, n_y, l_x + a_x, l_y, E_F) \\ &- \mathrm{Im}G(l_x + a_x, l_y; n_x, n_y, E_F)\mathrm{Im}G(n_x + a_x, n_y, l_x, l_y, E_F)] \ ,\end{aligned} \tag{11}$$

which is related to the continuum version of the equations derived by Lee, Stone, and Fukuyama.[4]

The disordered segment has the coordinates l_x and l_y between 1 and L_x and 1 and L_y, respectively. Here $L_x = L_y = L$. The coordinates n_x and n_y have similar properties. Equation (7) is valid for any $-\infty < l_x, n_x < \infty$, i.e., also where the sites belong to the perfect leads. It is convenient to pick $l_x = n_x = L_x$. The system can be treated similar to a one-dimensional system[19,20] where the Green's functions are expressed as continuous fractions. In our case it is convenient to use the matrix notation in which the elements of matrices are labeled by l_y and n_y. The Green's functions needed in formula (11) are determined in a recursive way from

$$\begin{aligned}G_{L_x,L_x}^{-1}(E) = E\hat{1} - \hat{H}_{L_x L_x} &- u G_{L_x-1,L_x-1}^{L} u \\ &- u G_{L_x+1,L_x+1}^{R} u\end{aligned} \tag{12}$$

and

$$G_{L_x,L_x+1} = G_{L_x,L_x} u G_{L_x+1,L_x+1}^{R} \ . \tag{13}$$

In these equations $G_{L_x L_x}(l_y, n_y, E) = G(L_x, l_y, L_x, n_y, E)$, $\hat{1}$ is an $L_y \times L_y$ unit matrix and $\hat{H}_{L_x,L_x}$ is the Hamiltonian for the L_xth row without including any couplings to the left- and right-hand side neighbors. The Green's functions G_{L_x-1,L_x-1}^{L} and G_{L_x+1,L_x+1}^{R} describe electron propagation in the semi-infinite strips to the left and right from the L_xth row respectively.

Note that $G^R_{L_x+1,L_x+1}$ corresponds to a uniform semi-infinite system described by the Hamiltonian

$$H_0=\sum_{l,\mathbf{a}}{}' \Psi_l^\dagger U(l,\mathbf{a})\Psi_{l+\mathbf{a}}-g_e\mu_B B\sum_l \Psi_l^\dagger\sigma^z\Psi_l \,, \tag{14}$$

where the prime denotes summation over $l_x \geq L_x+1$. Performing the Fourier transformation in the u direction yields

$$H_0=\sum_{k_y}\left[\sum_{l_x a_x}{}' \Psi^\dagger_{l_x k_y}U(l_x,a_x)\Psi_{l_x+a_x,k_y} +2u\cos(k_y a_y)\sum_{l_x}{}'\Psi^\dagger_{l_x k_y}\Psi_{l_x k_y} -\mu_B B\sum_{l_x}{}'\Psi^\dagger_{l_x k_y}\sigma^z\Psi_{l_x k_y}\right] . \tag{15}$$

For the periodic boundary conditions in the y direction, the summation over k_y takes place for

$$k_y=\frac{2\pi m_y}{L_y a_y},\quad 0\leq m_y<L_y \,. \tag{16}$$

In the case of the free-boundary conditions in the y direction, we would have

$$k_y=\frac{\pi m_y}{(L_y+1)a_y},\quad 0<m_y\leq L_y \,. \tag{17}$$

The Hamiltonian H_0 is a sum of the one-dimensional Hamiltonians corresponding to different wave numbers k_y. Therefore, the matrix elements of $G^R_{L_x,L_x+1}$ are given by

$$G^R_{\sigma\sigma',L_x,L_x+1}(l_y,n_y) =\frac{\delta_{\sigma\sigma'}}{L_y}\sum_{k_y} e^{ik_y(l_y-n_y)a_y}G_{1D} \times[E-\mu_B B\sigma+2u\cos(k_y a_y)] \,, \tag{18}$$

where the Green's function for a one-dimensional semi-infinite uniform strip G_{1D} is given by

$$G_{1D}^{-1}(E)=\tfrac{1}{2}[E+i(4u^2-E^2)^{1/2}] \,, \tag{19}$$

for $|E|<2u$, and

$$G_{1D}^{-1}(E)=\tfrac{1}{2}[E+\mathrm{sgn}(E)(E^2-4u^2)^{1/2}] \,, \tag{20}$$

for $|E|>2u$. The left-hand side Green's function $G^L_{L_x-1,L_x-1}$ can be calculated in a recursive way by noting that

$$(G^L_{n_x,n_x})^{-1}=E\hat{1}-\hat{H}_{n_x,n_x}-uG^L_{n_x-1,n_x-1}u \,, \tag{21}$$

and by starting from $n_x=0$ when $G^L_{0,0}=G^R_{L_x+1,L_x+1}$ since the system is uniform in that row.

To conclude, the calculation of conductivity of a finite disordered system reduces to a sequence of iterations and matrix inversions corresponding to adding subsequent layers of the disordered segment one by one starting from the semi-infinite uniform strip. The conductivity $\sigma_{xx}(E_F)$ is the proportional to a transmission.[18] One can say also that $\sigma_{xx}(E_F)$ is proportional to a number of active channels. In the case of a uniform system, the number of channels is equal to a number of k_y's for which $\mathrm{Im}G_{1D}[E_F-\mu_B B\sigma+2u\cos(k_y a_y)]\neq 0$, i.e., to the number of plane waves with different k_y which are transmitted through the system.[21] Disorder is expected to reduce the number of active channels and thus to reduce σ. The final result for σ can be expressed in the following way: $\sigma=(e^2/h)$ (number of active channels).

IV. EFFECTS OF FROZEN SPINS ON UNIVERSAL CONDUCTANCE FLUCTUATIONS

Throughout this paper we study one randomly selected sample. It is defined by a particular configuration of the site potentials and of the exchange couplings. Conductivity depends on the value of the Fermi level E. Figure 1 shows the conductivity $\sigma=\sigma_{xx}$ as a function of E in a range between 0 and $0.6u$. The solid line is for a system in which electrons are decoupled from the lattice spins. The line is composed of many resonant peaks.

The dotted lines are for the same system, but with the spins "reattached." These lines correspond to two randomly selected and then frozen spin configurations. The influence of the spins is substantial, but the dotted lines follow the no-spin curve in an essentially "parallel shifted" fashion.

The size of the variance in σ as a function of E and for a fixed spin configuration is difficult to be made precise because of the small statistics in the number of reso-

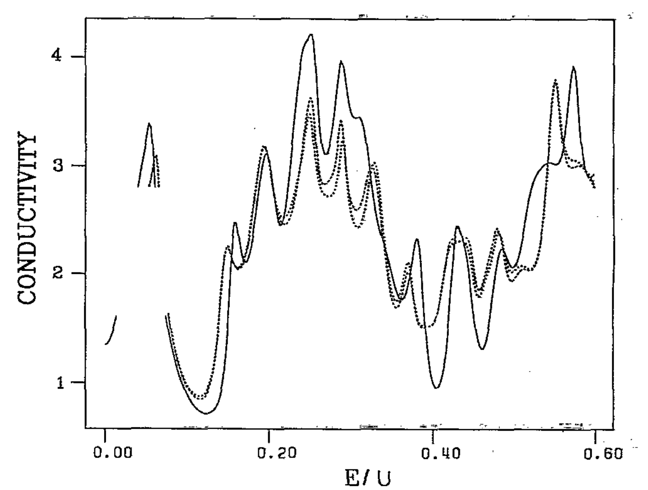

FIG. 1. Conductivity as a function of the Fermi energy in the random system under study. Conductivity is in units of e^2/h. The solid line is for the situation in which the interaction of electrons with the lattice spins is switched off. The dotted lines show the conductivity when the spins are coupled back to the tight-binding subsystem. The spin configurations correspond to two random spin configurations.

nances. It seems, however, that it is of the same order as fluctuations in σ for a given E in an ensemble of the electronic potentials. We took 2500 random sets of potentials and fixed E at 0. The average conductivity and the dispersion in this ensemble were found to be $2.39e^2/h$ and $0.64e^2/h$, respectively. For the electronic system size of 24×24, we get $2.32e^2/h$ and $0.60e^2/h$ for these quantities. If we disconnect the lattice spins from the electrons, by putting $J_{s\text{-}e}=0$, we get the same mean conductivities, but the dispersion increases to $0.76e^2/h$ for $L=20$ and to $0.75e^2/h$ for $L=24$.

Altshuler and Shklovskii[3] relate the dispersion to properties of density of eigenvalues of random matrices and derive for weak disorder, by diagrammatic techniques, the result

$$\langle \delta G\, \delta G \rangle^{1/2} = s\left[\frac{3K}{\beta}\right]^{1/2} \frac{e^2}{\hbar\pi^3}\sqrt{b_D}\ , \tag{22}$$

where δG is a fluctuation of conductance and b_D is a dimension-dependent factor: It is equal to 1.08, 1.51, and 2.0 for $D=1$, 2, and 3, respectively. The universality classes in the behavior of $\langle \delta G\, \delta G \rangle^{1/2}$ depend, in addition to D, on the asymmetry of the Hamiltonian via the factor β, on the s-fold (spin) degeneracy which is not removed by the random Hamiltonian, and on K, the number of the noninteracting series of energy levels. The factor β is equal to 1, 2, and 4 for the orthogonal, unitary, and symplectic ensembles of levels, respectively. If there are no spin-orbital and magnetic orbital effects present, $\beta=1$. When $J_{s\text{-}e}$ is set equal to zero, we should have $s=2$ and $K=1$, which yields, for $D=2$, $\langle \delta G\, \delta G \rangle^{1/2}=0.863e^2/h$, in agreement with the value proposed by Lee, Stone, and Fukuyama,[4] for this universality class of fluctuations. Our numerical result of $0.75-0.76e^2/h$, possibly affected by the small system size, is close to the above estimate. When $J_{s\text{-}e}$ is nonzero, we expect for the Ising spins that the splitting between the up and down states appears so that $s=1$, while $K=2$. Thus formula (22) predicts fluctuations of order $0.61e^2/h$. Our value for the fluctuations with spins agrees very closely with this estimate.

Finally, we note that the effect of spin-disorder scattering on universal conductance fluctuations in a paramagnetic phase was examined by Bobkov, Falkov, and Khmelnitskii.[23] For frozen Heisenberg spins in two or three dimensions, their result agrees within 12% with that of Eq. (22) for $s=1$, $K=2$, and $\beta=2$.

V. SPIN-INDUCED CONDUCTANCE NOISE

We now study what happens to our chosen sample on thermal processing as defined by a standard Monte Carlo simulation. In each Monte Carlo step per spin, we pick $L_s\times L_s\times L_s$ sites, one at a time, and attempt to flip the spin there. If flipping the spin lowers the system's energy, the flip is actually performed. Otherwise, the flip is made with the Boltzmann probability related to the ratio of energy increase to k_BT. It is difficult to relate the time τ_0 corresponding to one Monte Carlo step per spin to an experimental time scale. The time τ_0 is determined by the rate of energy dissipation by the spin system, which usually depends on the strength of the coupling to phonons. It is thus customary to consider τ_0 to be of order of the inverse Debye frequency, i.e., picoseconds. One can envision mechanisms which could extend this time scale significantly though. The longest train of conductivities at a given T that we calculated for one energy was obtained for 8192 Monte Carlo steps per spin. This corresponds to rather short probing times scales, of the order of at least 10 ns. It is possible to extend the time scale here by up to two orders of magnitude by calculating the conductivity, say, every one-hundredth Monte Carlo step. It is the calculation of conductivity, and not the Monte Carlo simulation itself, that consumes most of the CPU time and limits the statistics.

In our simulations we cool the system through the freezing temperature of $1J_0/k_B$ by starting high up in the paramagnetic regime — at $T=3J_0/k_B$. At each T the Monte Carlo process evolves the system for 4500 steps per spin. The sequence of the temperatures is, in a typical run, 3 J_0/k_B, 2.5 J_0/k_B, 2 J_0/k_B, 1.5 J_0/k_B, 1.2 J_0/k_B, 1.1 J_0/k_B, 0.9 J_0/k_B, 0.8 J_0/k_B, 0.7 J_0/k_B, 0.6 J_0/k_B, 0.5 J_0/k_B, and 0.4 J_0/k_B. The conductivity is being calculated for several selected temperatures. The first 2000 Monte Carlo steps are excluded from the calculations to allow for thermal equilibration of the spin configurations. Since then, the conductivities are determined at the completion of each Monte Carlo step per spin (one stochastic passage through the system). At each T we have a series of between 2500 and 8192 values of σ. It seems, however, that this procedure results in conditions which are, at best, only partially stationary during the observation time. The experimental studies of noise[22] have pointed to the presence of aging effects lasting for waiting times exceeding observational time scales by three orders of magnitude.

We focus on two particular locations of the Fermi energy: one at $E=0$, which is close to a minimum in σ shown in Fig. 1, and another at $E=0.2u$, which is close to one of the resonances. When discussing Fig. 1 we noted that the dotted lines showing $\sigma(E)$ corresponding to frozen-spin configurations follow the no-spin curve in an essentially parallel shifted fashion. Thus, at $E=0$, the thermally induced changes in σ are generally observed to be smaller than those at $E=0.2u$ for each temperature. (In Fig. 1 the two dotted lines actually correspond to two consecutive Monte Carlo steps per spin at $T=0.8J_0/k_B$.)

Figures 2–7 show 500-step-long portions of the time evolution of the conductivity for 6 temperatures between 3 J_0/k_B and $0.4J_0/k_B$, all calculated for the same sample and for the two values of E. We observe that, for both energies, the magnitude of noise in σ systematically decreases with temperature. The smaller the temperature, the farther apart the "events," or pronounced features, become separated and the smaller their amplitude.

We observe that the noise in σ is very sensitive to the starting spin configuration. The sensitivity appears not only in the actual look of the time dependence, which is expected, but, more strikingly, in the dispersion of σ. This is illustrated in Fig. 8, which displays a cumulative dispersion as a function of time, as measured in Monte Carlo steps per spin. Each curve is for the same sample,

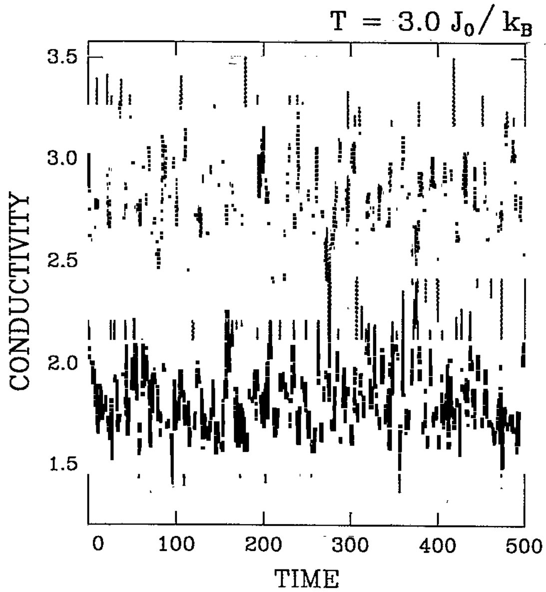

FIG. 2. Conductivity in units of e^2/h as a function of time for two values of the Fermi energy. The time is measured in number of Monte Carlo steps per spin. The lower, solid line is for $E=0$ and the upper, dotted line is for $E=0.2u$. The figure shows the first 500 values of the conductivity after an initial relaxation of 2000 steps at $T=3J_0/k_B$.

but the starting spin configurations differ. The figure shows dispersion as calculated by gathering the first 500–2500 data points. Each of the curves seems to reach some kind of an asymptote, the value of which depends on the initial configuration. The behavior of the cumulative dispersion as a function of time is shown here for three temperatures and for $E=0$. High in the paramagnetic regime ($T=2J_0/k_B$), the asymptotes are close to each other and give an average dispersion of $0.2e^2/h$. As the temperature decreases, the history dependence of the dispersion becomes more and more pronounced, but the asymptotes tend to be lower. E.g., for $T=1.1J_0/k_B$ and $0.8J_0/k_B$, they are on average $0.14e^2/h$ and $0.09e^2/h$, respectively, if 10 starting configurations and 2500 Monte Carlo steps are taken into account. When calculations are based on 8000 steps, we get $0.16e^2/h$ and $0.13e^2/h$ for dispersion at these two temperatures (the error bars are

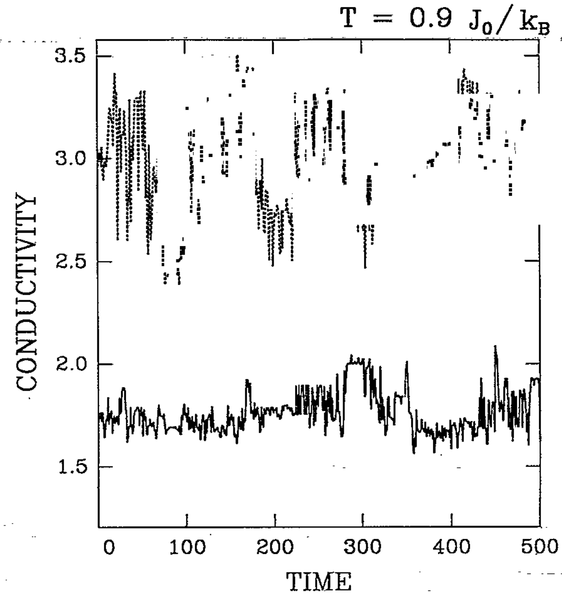

FIG. 4. Same as in Fig. 2, but for $T=0.9J_0/k_B$, i.e., just below the freezing temperature.

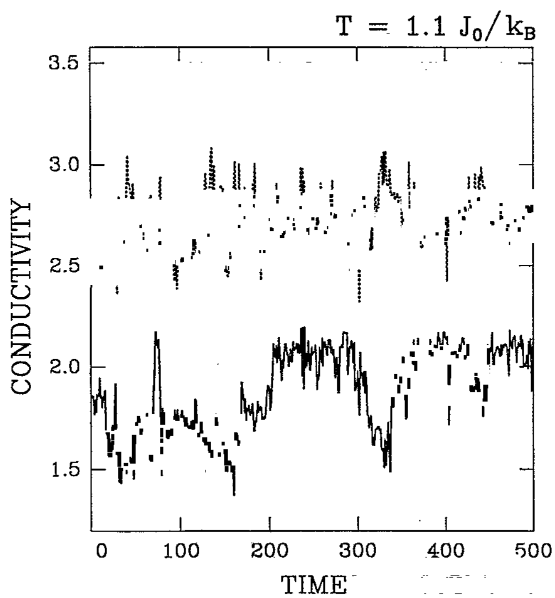

FIG. 3. Same as in Fig. 2, but for $T=1.1J_0/k_B$, i.e., just above the freezing temperature.

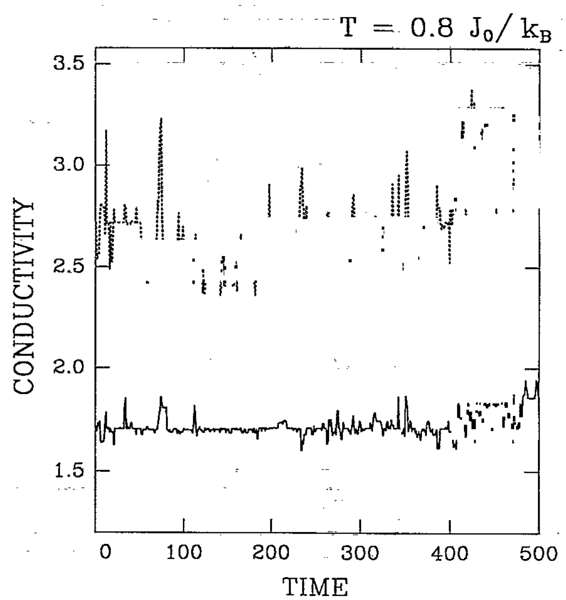

FIG. 5. Same as in Fig. 2, but for $T=0.8J_0/k_B$.

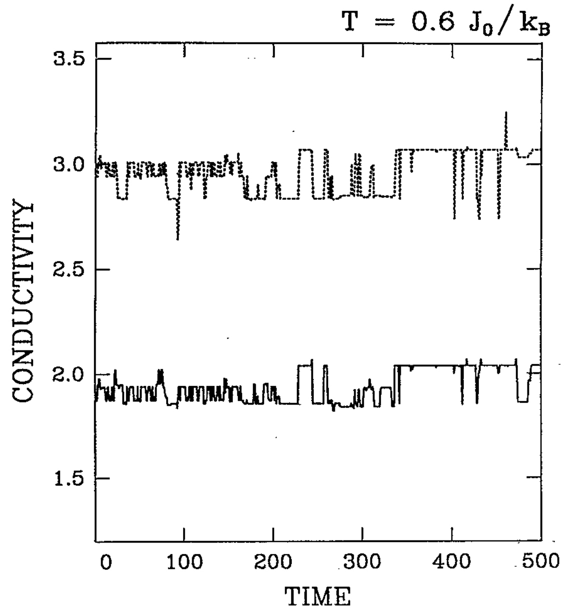

FIG. 6. Same as in Fig. 2, but for $T=0.6J_0/k_B$.

0.03).

An interesting question is what happens to the dispersion when orders-of-magnitude-longer time scales are considered. Will the asymptotic values eventually merge at each temperature? Should they become identical to the high-temperature value? The latter does not seem likely since, as we shall explain in the next section, the fluctuations are driven by flips of spins, the number of which are controlled by temperature. Thus, at low temperatures, one observes essentially small changes only, no matter how long the observation. A further study of the above issues for longer time scales than those presently

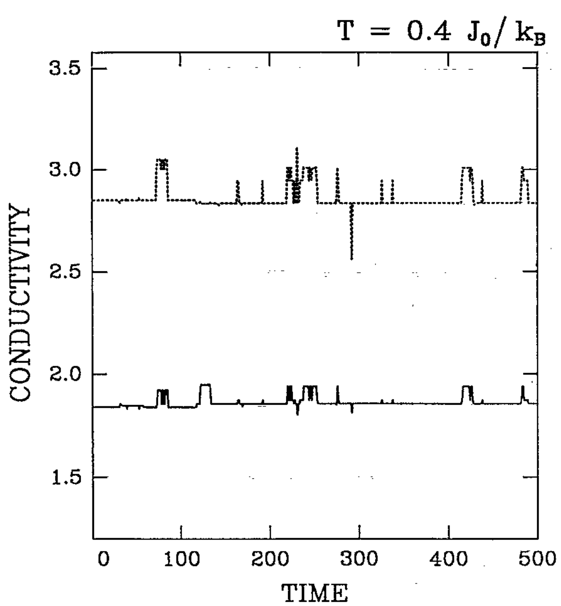

FIG. 7. Same as in Fig. 2, but for $T=0.4J_0/k_B$.

accessible to us would be valuable in view of the recent suggestion of Weissmann and co-workers[9,24] that the time evolution of resistance in small spin-glass samples can be used to choose between various theoretical models of spin-glass dynamics.

Figure 9 shows the cumulative dispersion after 1000, 1500, and 2000 Monte Carlo steps as a function of temperature for T between $0.5J_0/k_B$ and $1.5J_0/k_B$. The data points correspond to $E=0$ and are averaged over ten different runs in the same sample. The data give no evidence of an anomaly at T_c, at least for the system sizes studied. The dispersion is essentially linear in T.

The dispersion is more a measure of the amplitude of the fluctuations than measure of time separation between the events. One can see in Figs. 2–7 that the lower the temperature, the sparser the events. This should enhance the low-frequency end of the power spectra at low T. For a fixed low frequency f, one would expect an increase in the intensity of the spectrum. However, a decrease in T also reduces the amplitude of the noise, and the overall effect may not necessarily be monotonic in T. We calculated the power spectra of the noise and averaged them over 20 realizations of 4096-step-long trains of data for the same sample, corresponding to different initial conditions. Some of our results are shown in Fig. 10.

We find that in the high-T limit ($T=3J_0/k_B$) the spectrum corresponds to a white noise (f^0) at low frequencies

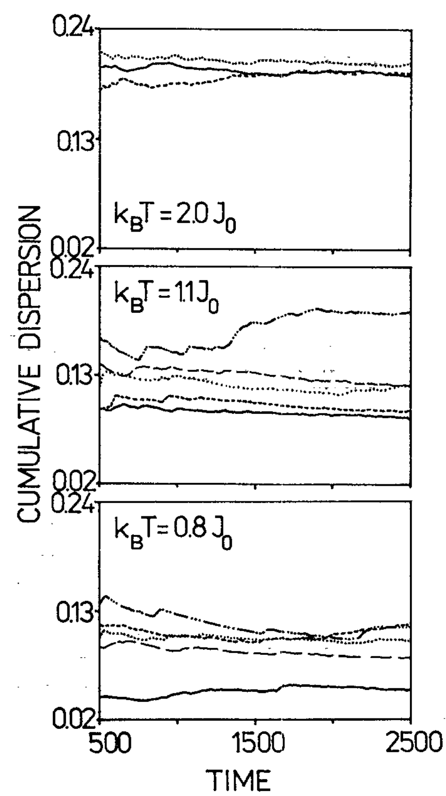

FIG. 8. Cumulative dispersion as a function of time for several starting spin configurations in the same system. The dispersion is in units of e^2/h, and the temperatures are indicated.

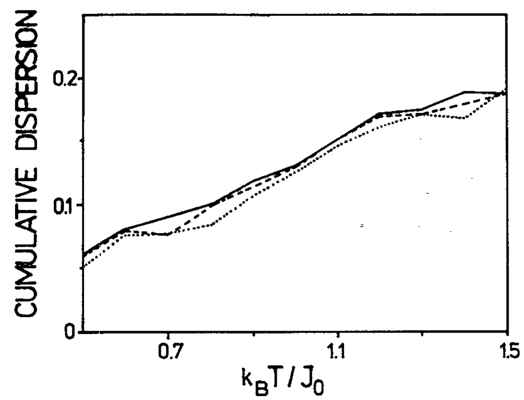

FIG. 9. Cumulative dispersion vs temperature. The dotted, dashed, and solid lines are based on 1000, 1500, and 2000 Monte Carlo steps per spin. The data points are for $E=0$ and are averaged over ten runs.

and becomes a power law with a small exponent at large frequencies. On lowering the temperature, a power-law behavior takes over all of the frequency range. In a vicinity of T_c (both above and below), the power-law exponent is close to -0.5. This is a two-dimensional analog of the $1/f$ noise encountered frequently in 3D systems. In addition to the spectra displayed in Fig. 10 ($T=3J_0/k_B$, 1.2 J_0/k_B, and $0.8J_0/k_B$), we also studied spectra at $T=1.1J_0/k_B$, $1.0J_0/k_B$, $0.9J_0/k_B$, $0.7J_0/k_B$, and $0.4J_0/k_B$. At the lowest T the exponent appears to be somewhat smaller in magnitude, but that may merely signify shorter effective time scales available in calculations compared to the higher temperatures.

We also observe that on lowering T the amplitude of the spectrum decreases monotonically. The decrease in the logarithm of the amplitude is found to be faster than linear; possibly, there is a quadratic law here. Israeloff *et al.*,[7] on the other hand, report that logarithm of the spectral density of the low-frequency resistance noise in $Cu_{1-x}Mn_x$ develops a steplike shape at T_c when plotted versus temperature. The position of the step is correlated

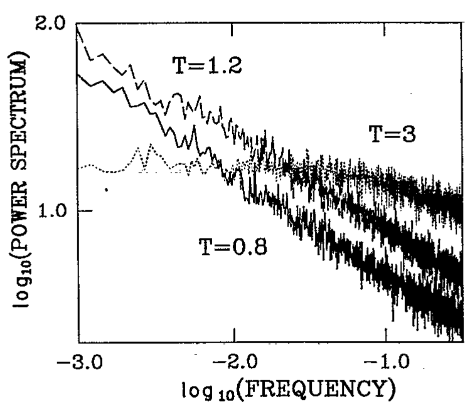

FIG. 10. Power spectrum on the log-log scale for temperatures indicated (in the usual units).

with temperature at which a cusp in the susceptibility develops, which suggests a magnetic source of the noise. Our results do not reproduce the experimental finding, and the discrepancy should be understood. There are several possibilities in way of explanation. First, our time scales studied might be too short to see the phenomenon. Second, in this experiment spins that affect conductivity occupy a 3D volume, and 3D geometry could enhance any possible anomalies. Third, our calculations of conductivity are performed at $T=0$ and thus do not take into account variations of the inelastic scattering length with T. A decrease in T increases this length which allows us to gather signal from a larger volume. This would be expected to be able to counter the diminishing trend in the noise amplitude. Actually, it is to this effect that Israeloff *et al.*[7] attribute a difference in the T dependences of the magnetization and resistance noises below T_c.

As already mentioned, there seems to be no abrupt change in the pattern of the noise on crossing the freezing temperature. However, at smaller temperatures, such as $0.4J_0/k_B$ and $0.6J_0/k_B$, and to some extent at $0.8J_0/k_B$, events in the time evolution of σ become not only sparser, but also more correlated across energies. As seen in Figs. 6 and 7, feature for $E=0$ has its counterpart for $E=0.2u$, except that the amplitude in the latter is usually larger.

VI. PHYSICAL INTERPRETATION OF THE CONDUCTANCE FLUCTUATIONS

We now focus on intepretation of the features present in the noise. Our numerical simulations imply no correlation between the noises in magnetization and in conductivity. According to Israeloff *et al.*[7] this could be expected because the latter involve spin correlations of at least fourth order. Note also that not all of the spins are coupled to the electrons. Only those spins which are in the central plane are. Figure 11 correlates numbers of spins that flipped in the central plane in each time step with the resulting change in σ. A stretch of only 120 Monte Carlo steps per spin is shown to enable a detailed inspection. We contrast two extreme cases: At $T=3J_0/k_B$, and at each time step, between 8 and 30 planar spins flip, but at $T=0.4J_0/k_B$, up to three spins are affected and most often none is. A flip of order 15 spins produces usually a large change in σ. A flip of one or two spins results typically in a smaller change. However, the number of the flips is in no monotonic relationship with the change in σ induced by it. Flipping three spins may bring about much smaller change than flipping merely one spin.

In order to explain this, we note that some spins are located in places that are temporarily pivotal in the quantum coherence of the electronic wave functions and some are off the relevant Feynman trajectories. Which places are crucial varies as a function of time since the global structure is being affected. We made a computer experiment in which we stored some initial spin configuration and then flipped one spin at systematically selected places, after which the system was restored to its initial state. The results for two spin configurations taken from

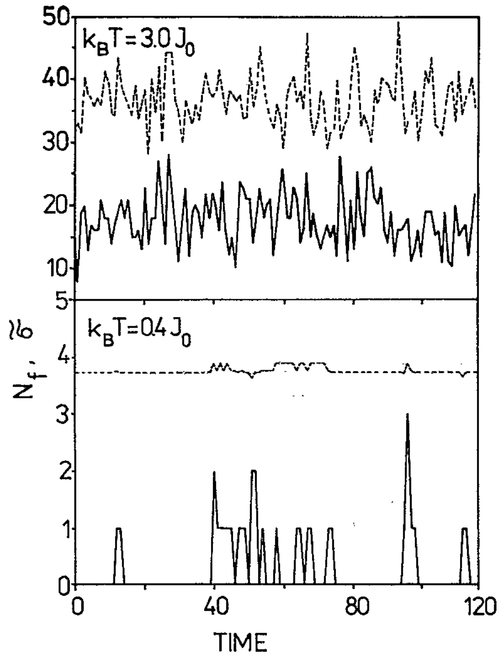

FIG. 11. Solid line shows the number of spin flips, N_f, that occurred in the central plane in a Monte Carlo step per spin at time indicated. The dashed line shows the conductivity rescaled by a factor b: $\tilde{\sigma}=b\sigma$. In the top figure, $T=3J_0/k_B$ and $b=20$. In the bottom panel, $T=0.4J_0/k_B$ and $b=2$. The conductivity is in units e^2/h.

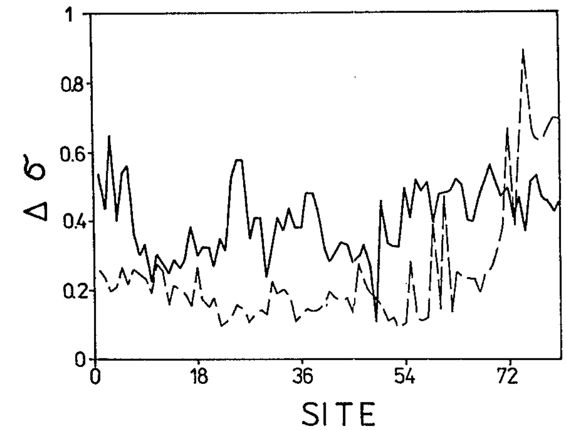

FIG. 12. Change in conductivity, relative to the situation where spins are absent, resulting from flips of single spins from a stored spin configuration. The two lines correspond to two different starting configurations generated at $T=0.4J_0/k_B$. The x axis is a label of the position, in the central plane, of a spin flipped. Conductivity is in units of e^2/h.

the evolution at $T=0.4J_0/k_B$ are shown in Fig. 12. The y axis is for the change in σ relative to a situation in which the spins are decoupled from the electrons. The x axis shows labels, in some arbitrary convention, of the 81 spin locations that are available in the central plane. Flipping a spin at a given site is seen to be inducing a change that depends more on the initial configuration than on the location of the site.

Events in the spin subsystem lead to events in the behavior of conductivity only provided the spins in the central plane are involved. Thus, in this particular geometry, we are bound to look only at a shadow of the spin dynamics. Could placing the spins and electrons in the same three-dimensional volume help in getting a better monitoring handle on the evolution of the spins? Our results suggest that arranging for a three-dimensional geometry would not help much because the extent of a spin event is usually in no simple relationship with the size of an electronic event. All we can say when observing a glitch in conductivity is that the spin configuration has changed.

There may, however, exist statistical laws that are valid and useful in predicting, say, the average size of domains that passed by. Figure 13 suggests that the noise in σ may actually have some overall relationship with the spectrum of spin-relaxation times. The figure shows the noise in σ when in the thermal cooling process and in the first 1000 time steps at $T=0.4J_0/k_B$ the spins are arrested by a strong magnetic field (of strength $g_s\mu_B=J_0/2$; the electrons themselves are artificially not influenced by the field: $g_e=0$), and then, at "time" 1000, the field is switched off. A relaxation to equilibrium is taking place, first through rapid processes and then through longer and longer global rearrangements. The behavior of the electronic noise is clearly affected by the field-switching event (a similar statement holds also at higher temperatures). The nature of the change is that in the field the glitches are sporadic, but when the field is switched off, a series of rapid fluctuations in σ is observed. The conductivity calculated for $E=0$ shows a sequence of more sparsely located glitches after the initial series of rapid fluctuations. This suggests a transition from fast to slow relaxation times. However, the conductivity for $E=0.2u$ for the same system is harder to be interpreted in terms of the spectrum of the relaxation times. Switching the field on

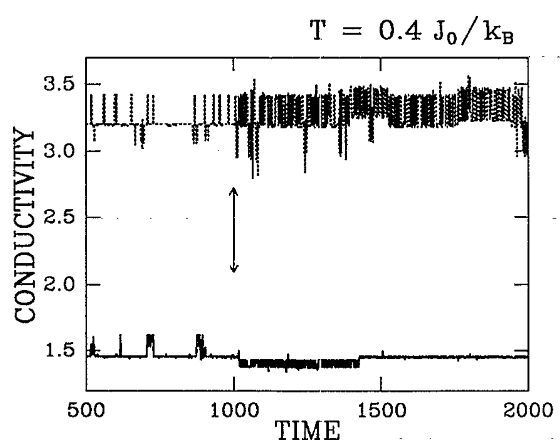

FIG. 13. Conductance fluctuations in the system perturbed by a magnetic field. The field is switched off after 1000 time units. The upper line is for $E=0.2u$, and the lower one is for $E=0$.

and off may keep shifting average values of conductivity within some range.

We have also attempted to investigate the role of the temperature-switching effects, as suggested by Feng *et al.*[6] In particular, we made the system evolve at $T=0.9J_0/k_B$ and then reduced T to $0.8J_0/k_B$. We could see nothing striking happening in the noise on the T reduction, at least within the first 1000 time steps after the reduction. Studying the effects of changes in temperature probably requires larger sizes of the spin system.

VII. CLOSING REMARKS

It is interesting to compare conductance noise in spin glasses to that in ferromagnets. The latter is shown in Fig. 14. Two differences are noticeable immediately. First, the cumulative dispersion in ferromagnets shows a broad maximum at the Curie temperature of $4.5J/k_B$. The maximum is well marked already within 1000 Monte Carlo steps. Second, at low temperatures the plot of the noise as a function of time looks like a defective comb: a line with occasional spikes. In spin glasses the spikes can point up or down from the average line. In ferromagnets, on the other hand, the spikes are predominantly downward. The reason is that the ferromagnetic ground state is uniform, which reduces the amount of disorder in the system. Thermal fluctuations destroy the uniformity and reduce the conductivity. The power spectra based on ten 4096-step-long data trains are shown in Fig. 15. Except for a very short time scale, the spectra are essentially given by a white noise with an amplitude peaking at T_c. A weak power law in f seems to be developing only at an immediate vicinity of T_c.

We conclude this paper by emphasizing that conductivity does respond to events in the spin system, but the interpretation of the noise is difficult because the response seems to be monotonic only statistically. Furthermore, low-probability spin events may have a major impact on the electronic noise pattern. In a recent report, Weissman and Israeloff[24] discuss the experimental fluctuation statistics of resistance in $Cu_{1-x}Mn_x$ to choose between various theoretical models of spin-glass dynamics and pick the hierarchical kinetics of the infinite-range model[25] over the droplet[26] and spin-density-wave[27] pictures. Further studies are needed to decide whether or not such claims are justified. The nature of the information one gets about the lattice spin dynamics from the conductance fluctuations is indirect. It is likely that properties of the noise reflect general features of the system such as whether it is anisotropic or not. We plan, for instance, to study Heisenberg spin glasses to make a comparison.

Another problem that requires investigation is characterization of the noise as a function of the magnetic field, how the presence of the spin impurities affect the noise

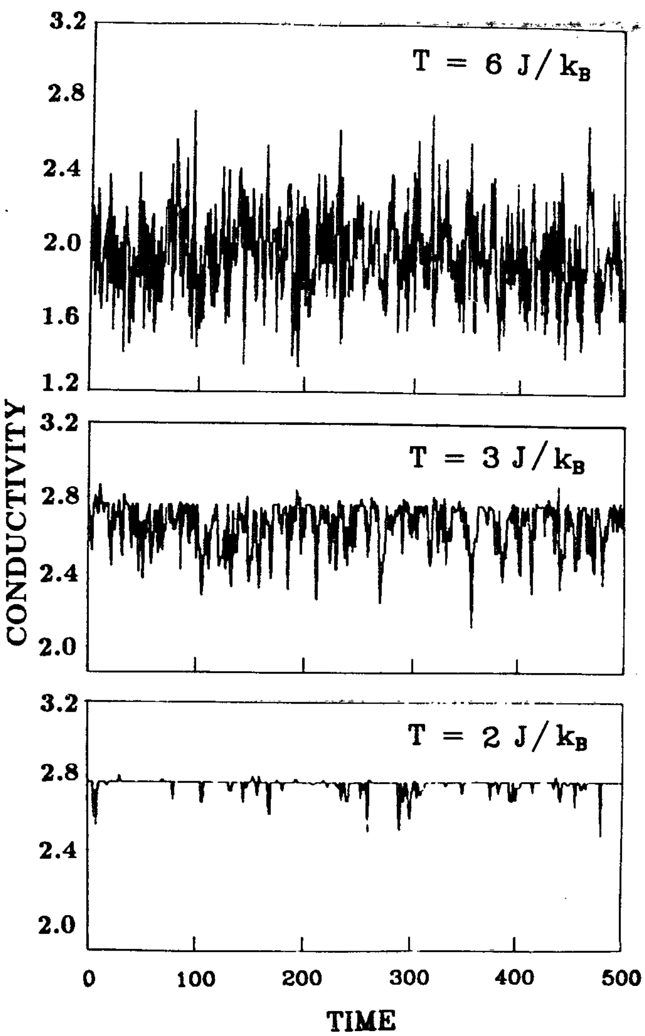

FIG. 14. Conductance fluctuations for a uniform ferromagnet characterized by the exchange constant J at temperatures indicated. The data points are for $E=0$ and are shown in a stretch of 500 time steps.

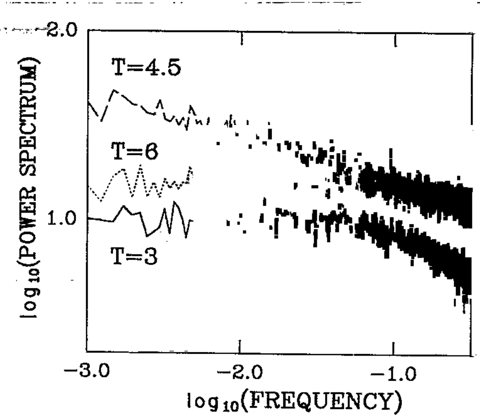

FIG. 15. Power spectrum on the log-log scale for a uniform ferromagnet at temperatures indicated (in units of J/k_B).

patterns, not as a function of time, but as a function of field-induced fluctuations in σ. By an "ergodic hypothesis" these fluctuations are expected to be similar to those induced by a random restructuring of the electronic potentials; but are they? Studying such questions requires augmenting the formulas for the conductivity of Sec. III by terms responsible for the orbital electronic magnetism. Such simulations are now under way.

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