

Bistability of Bose-Fermi mixtures

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We study the properties of the Bose-Fermi mixture from the perspective of reaching a state of a self-bound quantum droplet. The variational analysis shows that the system exhibits bistability. For weak repulsion between bosons, one of the equilibrium states, smaller in size, spherically symmetric, and with negative energy, corresponds to quantum droplet, the other with always positive energy represents the elongated droplet-like state immersed in the sea of a fermionic cloud. For stronger repulsion between bosons the bifurcation is seized and only the former state is left. Now it represents an elongated object which, for strong enough boson-fermion attraction, gets negative energy. It becomes an excited Bose-Fermi droplet when the trap is released, what is demonstrated by solving the quantum hydrodynamics equations for the Bose-Fermi system. To depict our ideas we consider the ^{133}Cs - ^6Li mixture under ideal conditions, i.e. we assume no losses.

Quantum mixtures of atomic gases have been studied already for years, both experimentally and theoretically. Of particular interest are degenerate Bose-Fermi mixtures. While initially experiments were aimed at the realization of a degenerate Fermi gas via sympathetic cooling, where the bosonic or fermionic component served as a coolant part [1–5], soon after studies of many-body quantum phenomena started.

The phase diagram of the harmonically trapped mixture of bosonic rubidium-87 and fermionic potassium-40 atoms was determined in [6, 7]. Tuning the interspecies interactions by Feshbach resonance, the experimentalists observed a collapse of the mixture above some threshold and on the attractive side of the resonance as well as the phase separation on the repulsive side. Binary mixture of potassium-41 condensate immersed in a sea of fermionic lithium-6 atoms was recently realized [8] and collective oscillations of bosonic component were studied. A breathing mode of a condensate was induced by increasing the strength of the boson-fermion forces. On the positive side of Feshbach resonance the phase separation is eventually reached and the oscillation frequencies are measured [9]. Here, temperature effects seem to be crucial to explain the experimental results, as shown in [10]. Physics of polarons was probed via radio-frequency spectroscopy of a gas of potassium-40 fermionic impurities immersed in an ultracold atomic gas of rubidium-87 [11]. The energy, spectral width, and the lifetime of Bose polaron were determined on both sides of a heteronuclear Feshbach resonance, going beyond the standard textbook description in a weakly interacting regime.

A novel quantum mixture, a self-bound system of ultracold atoms has been realized experimentally recently. First quantum droplets were observed in a gas of dysprosium-164 atoms, which possess the largest dipolar magnetic moment among atoms [12]. Interplay of short range and dipolar forces is crucial for stabilizing such systems. More dipolar droplets were created soon with erbium-166 atoms [13]. Demonstration of existence of different kind of self-bound objects, a two-component mix-

ture of bosonic potassium-39 atoms has followed [14, 15]. The origin of self-confinement both in dipolar systems as well as in two-component mixtures is due to quantum fluctuations which play essential role at the border of a collapse, as suggested in Ref. [16]. Quantum droplets in a heteronuclear bosonic mixtures were also observed [17]. New self-bound systems, the Bose-Fermi droplets have been recently elaborated theoretically [18, 19].

In Bose-Fermi mixtures, interactions among degenerate identical fermions can be changed by a contact with a Bose-Einstein condensate. For example, a mixture of ^{87}Rb - ^{40}K was used to bring a gas of fermionic potassium to collapse [20, 21]. Indeed, large enough boson-fermion attraction in rubidium-potassium mixture results in an effective attraction between fermions, responsible for the collapse. This attraction, on the other hand, could lead to fermionic superfluidity as in the case of phonon-induced attraction between electrons in superconductors. In fact, a Bose-Fermi mixture in which both the fermionic and the bosonic components are superfluid has been produced [22, 23].

In Bose-Fermi systems, fermions can mediate the interactions between bosons as well. This kind of behavior has been recently observed in an experiment with a mixture of ultracold fermionic ^6Li and bosonic ^{133}Cs atoms [24]. If degenerate lithium and condensed cesium atoms attract each other strongly enough it may result in effective attractive boson-boson interactions even though the condensed bosons alone are repulsive. This, mediated by fermions, change of the interaction character is caused by a coherent three-body scattering process. It leads to formation of trains of Bose-Fermi solitons seen in experiment [24] and predicted theoretically a long time ago [25–27].

A crude estimation of the onset of a change of the sign of interactions among bosons, given in Ref. [25], can be displayed as $g_B n_B = |g_{BF}| n_F$, where n_B and n_F are the densities of bosonic and fermionic fractions, respectively, taken at the center of the trap. The parameters g_B and g_{BF} determine the strength of contact interac-

tions. The densities can be roughly estimated assuming that the density of each component is calculated within the Thomas-Fermi approximation while the other component is not present. Then for sufficiently large Bose-Fermi attraction

$$\frac{|g_{BF}|}{g_B} > C \frac{N_B^{2/5}}{N_F^{1/2}} \quad (1)$$

the bosonic component becomes effectively attractive too. Here, $C = C_1(a_\perp^B/a_B)^{3/5}(a_\perp^F/a_\perp^B)^3\lambda_B^{2/5}/\lambda_F^{1/2}$, $C_1 = 3^{9/10}5^{2/5}\pi/16$, a_\perp is the radial harmonic oscillator length, $\lambda = \omega_z/\omega_\perp$ defines the aspect ratio of the axially symmetric (around z axis) trap, and a_B is the s -wave scattering length for the pure Bose gas related to the interaction strength via $g_B = 4\pi\hbar^2 a_B/m_B$. The interaction between bosons and fermions is characterized by $g_{BF} = 2\pi\hbar^2 a_{BF}/\mu$ with μ being the reduced mass.

To meet the condition (1) tuning one of scattering lengths via Feshbach resonance technique is required. It can be done in two ways: i) by tuning a_B to very small values or ii) by tuning $|a_{BF}|$ to large values.

We consider small a_B case first. For cesium-lithium mixture as in experiment of Ref. [24], i.e. for $a_{BF} \approx -60 a_0$ (a_0 is the Bohr radius), trapping frequencies (130, 130, 6.5) Hz for bosons and (400, 400, 36) Hz for fermions, and particles numbers as $N_B = 30000$ and $N_F = 20000$, the Eq. (1) gives the upper limit $a_B < 0.4 a_0$ (region marked by 1 in Fig. 1). In fact, the soliton trains were observed in [24] already for $a_B \approx 3 a_0$.

The other way to satisfy the condition (1) is to work close to the Feshbach resonance visible in Fig. 1, at the magnetic fields around 893 G. Across this resonance one has $a_B \approx 230 a_0$ and the Eq. (1) is satisfied for large enough $|a_{BF}|$, equal about 2000 a_0 (region marked by 2 in Fig. 1). Negative and large values of a_{BF} are explored in Ref. [28], where the observation of a degenerate Fermi gas trapped by the Bose-Einstein condensate is reported.

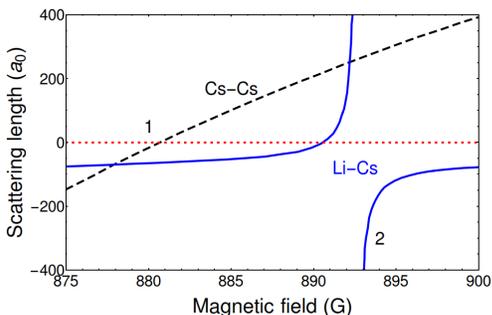


FIG. 1: Scattering lengths for Cs-Cs (dashed black line) and Li-Cs (solid blue line) collisions as functions of the magnetic field (see Ref. [24]).

In the following we will discuss a scenario of droplet formation. We consider a trapped case with fixed number of fermionic, N_F , and bosonic, N_B , atoms in a harmonic trap. Although a genuine droplet must be stable without

any trapping potential, initially atoms are always kept in a trap. Only then interaction parameters are tuned to required values and a droplet is eventually formed. To this end both kinds of atoms must be mixed in the well prescribed proportions. Typically these condition is not met in a trap with initially prepared atoms. However, droplet can still be formed but then a gaseous cloud of surplus atoms is surrounding a droplet. The cloud is stopped from expansion by the external trap. The liquid droplet is at a stable equilibrium with its vapors then.

This is the situation we are studying here. We show that Bose-Fermi mixture, spanned over parameter regions 1 and 2 in Fig. 1, shows a bistability phenomenon while trapped by external harmonic potential. For a given set of interaction parameters, there are two minima in the energy landscape, one local and the other global one. Density profiles correspond to two different droplet-like objects immersed in a vapor of surrounding atoms.

We start first with the variational calculations. The bosonic and fermionic densities are assumed to be axially symmetric distributions of the form of the Gaussian functions with two variational parameters: the axial (σ_a) and radial (σ_r) widths. The width of the bosonic cloud is the same as that of the fermionic one. No fermionic background is assumed. These assumptions are simplistic, nevertheless quite well illustrate the main idea. Hence, the densities are proportional to $\exp(-z^2/\sigma_a^2)\exp(-\rho^2/\sigma_r^2)$ and normalized to the number of fermions N_F and bosons N_B , respectively. The kinetic fermionic, including the Weizsäcker contribution [29], and kinetic bosonic energies are then given by

$$\begin{aligned} E_{kin}^F &\propto 1/\sigma_a^{2/3}\sigma_r^{4/3} \\ E_W &\propto (1/\sigma_a^2 + 2/\sigma_r^2) \\ E_{kin}^B &\propto (1/2\sigma_a^2 + 1/\sigma_r^2). \end{aligned} \quad (2)$$

At the mean-field level, the only interaction energies are $E_{int}^B, E_{int}^{BF} \propto 1/\sigma_a\sigma_r^2$ as degenerate fermions do not interact. We consider as well the quantum corrections to the intra- and inter-species interaction energies. The Lee-Huang-Yang correction [30] to the boson-boson interaction is calculated as $E_{LHY} \propto 1/\sigma_a^{3/2}\sigma_r^3$, while the quantum correction to the boson-fermion interaction [31], E_q^{BF} , is given in the Appendix A. Finally, the contributions due to trapping are as follows

$$\begin{aligned} E_{tr}^B &\propto ((\omega_a^B)^2 \sigma_a^2/2 + (\omega_r^B)^2 \sigma_r^2) \\ E_{tr}^F &\propto ((\omega_a^F)^2 \sigma_a^2/2 + (\omega_r^F)^2 \sigma_r^2), \end{aligned} \quad (3)$$

where ω_a^B (ω_a^F) and ω_r^B (ω_r^F) are axial and radial trapping frequencies, respectively, for bosons (fermions). Full formulas for all kinds of energy included in the analysis can be found in the Appendix A.

The variational analysis described above confirms the bistability phenomenon, i.e. the existence of two minima in the energy landscape of a trapped Bose-Fermi mixture. The first minimum is found to describe the large in size and axially symmetric mixture, whereas at the

size, to meet the imaging resolution requirements, one has to move from region 1 to 2 in Fig. 1, i.e. to increase the value of a_B scattering length. It simultaneously results in decrease of droplet component's densities making the particle losses less important. An example is shown in Fig. 4, where the ground state densities, still at the presence of the trap, obtained by imaginary time evolution of quantum hydrodynamics equations for $a_B = 250 a_0$ and $a_{BF} = -2.8 a_B$ are shown.

It turns out that the variational calculations for such parameters exhibit only one local minimum, corresponding to elongated clouds. See exemplary Fig. 5 confirming the existence of bifurcation in the system. For strong enough boson-fermion attraction the energy of an elongated mixture becomes negative and the trapping energy is negligibly small. It means that removing the trapping potential should not destroy the system. One should observe the oscillating Bose-Fermi droplet since its initial shape is far from being spherically symmetric. It is indeed as shown in the movies [32, 33]. For the first case, [32], we have $a_B = 250 a_0$ and $a_{BF} = -3.6 a_B$, i.e. we are deeply in the range of parameters where the droplet exists [18]. The trap is open in 1 ms. After the trap is released, the droplet survives and remains very elongated. After a further few milliseconds a rich dynamics is developed, the droplet becomes soon short in the axial and long in the radial direction. Then opposite happens and the initial shape is restored and the next cycle succeeds. Our simulations firmly prove the existence of the critical ratio $|a_{BF}|/a_B$ for the formation of Bose-Fermi droplets, see Ref. [18]. The movie [33] shows what happens with the mixture when $a_{BF} = -2.8 a_B$. After a few milliseconds the droplet seems to be formed, but eventually it explodes. For even less negative ratio, $a_{BF}/a_B = -2.0$, the mixture explodes immediately when the trap is removed, see [34].

For the Bose-Fermi droplet to be formed and persist for a longer time it is necessary to minimize losses due to three-body recombination processes. For parameters as in the region marked by 1 in Fig. 1, the atomic densities are certainly too large with respect to the losses, like in Fig. 5, the lowest frame. For the case of 2 in Fig. 1 it seems that the creation of Bose-Fermi droplets is possible. Even more favorable conditions should be accessible for systems with higher bosonic to fermionic mass ratio (see Ref. [18]) like, for example, bosonic ytterbium - fermionic lithium mixture. Another facilitation in creating Bose-Fermi droplets could be managed by introducing dipolar forces to the scene [12, 13], which can effectively act as an additional, to the contact one, attraction between atoms. In such a case bosonic dysprosium/erbium - fermionic lithium mixtures could be of interest.

In summary, we study the process of formation of a Bose-Fermi mixture and predict the bistability in the presence of the trapping potential. For weak boson-boson repulsion (region 1 in Fig. 1), the system possesses a pair of minima, the one (related to solitons) with positive and the other (related to spherically symmetric droplets)

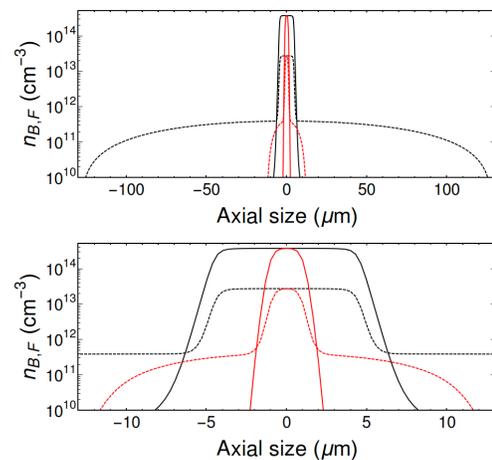


FIG. 4: Ground state densities of the Bose-Fermi mixture for the case 2 in Fig. 1. Here, $a_B = 250 a_0$, $a_{BF} = -2.8 a_B$, $N_B = 10^4$, $N_F = 10^3$, and the trapping frequencies are (130, 130, 6.5) Hz for bosons and (400, 400, 36) Hz for fermions. The figure shows bosonic (solid black and red lines) and fermionic (dashed lines) densities. The lower frame is just the zoom of the upper frame.

with negative energy. For stronger intra-bosonic repulsion (region 2 in Fig. 1) only droplets related minimum survives, which represents the axially symmetric object. For strong enough boson-fermion attraction its energy becomes negative. Then, when the trapping potential is removed, the system turns into a stable quantum Bose-Fermi droplet. It does oscillate since initially its shape is not spherically symmetric. This behavior is confirmed by quantum hydrodynamics equations based simulations. While the case 1 in Fig. 1 is better suited for studying the physics of Bose-Fermi solitons, we find that the region 2 in Fig. 1 could be considered as a scene for creating Bose-Fermi droplets.

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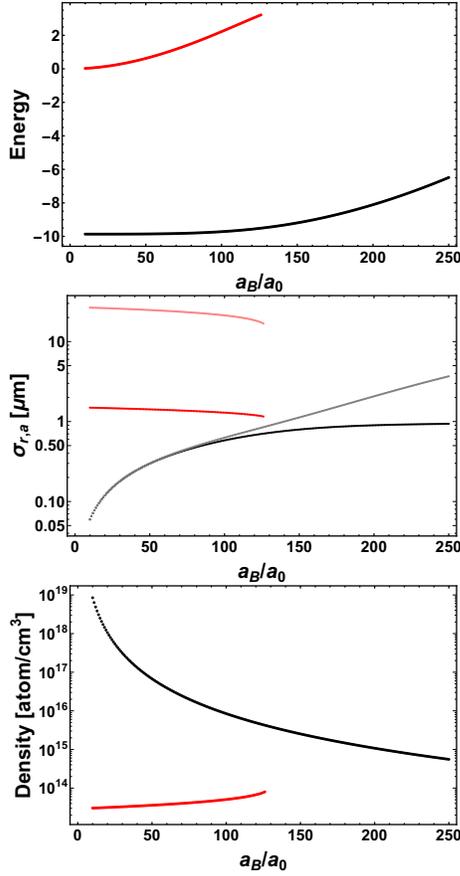


FIG. 5: Energy, radial and axial widths, and a peak bosonic density of the Bose-Fermi droplet as a function of the scattering length a_B , for $a_{BF}/a_B = -3.6$. The numbers of bosons and fermions are $N_B = 10^4$ and $N_F = 620$, respectively. Black and red curves correspond to the 'droplet' and 'soliton' minima, respectively. The lighter (darker) colors for the size of the system (middle frame) represent the axial (radial) direction.

Appendix A: Variational analysis: energy contributions

Here we assume that the densities of bosonic and fermionic components are given by

$$\begin{aligned} n_B(z, \rho) &= \frac{N_B}{\sqrt{\pi} \sigma_a \sigma_r^2} e^{-z^2/\sigma_a^2} e^{-\rho^2/\sigma_r^2} \\ n_F(z, \rho) &= \frac{N_F}{\sqrt{\pi} \sigma_a \sigma_r^2} e^{-z^2/\sigma_a^2} e^{-\rho^2/\sigma_r^2}, \end{aligned} \quad (\text{A1})$$

hence are normalized to the number of bosons and fermions, respectively, and are of the same widths. We still admit, however, axially symmetric solutions.

All energy contributions are calculated within the local density approximation. The fermionic kinetic energy due to the Pauli exclusion principle and the gradient correc-

tions (the Weizsäcker one) is

$$\begin{aligned} E_{kin}^F &= \kappa_k \int n_F^{5/3} d^3r = \frac{\kappa_k}{\pi} \left(\frac{3}{5}\right)^{3/2} \frac{N_F^{5/3}}{\sigma_a^{2/3} \sigma_r^{4/3}} \\ E_W &= \xi \frac{\hbar^2}{8m_F} \int \frac{(\nabla n_F)^2}{n_F} d^3r = \xi \frac{\hbar^2}{4m_F} N_F \left(\frac{1}{\sigma_a^2} + \frac{2}{\sigma_r^2}\right). \end{aligned} \quad (\text{A2})$$

with $\kappa_k = (3/10)(6\pi^2)^{2/3} \hbar^2/m_F$ and $\xi = 1/9$ [35, 36]. The bosonic kinetic energy is obtained as

$$E_{kin}^B = \frac{\hbar^2}{2m_B} \int (\nabla \sqrt{n_B})^2 d^3r = \frac{\hbar^2}{2m_B} N_B \left(\frac{1}{2\sigma_a^2} + \frac{1}{\sigma_r^2}\right). \quad (\text{A3})$$

We include contact interactions in the analysis. Within mean-field approximation the energies for boson-boson and boson-fermion interactions are

$$\begin{aligned} E_{int}^B &= \frac{1}{2} g_B \int n_B^2 d^3r = \frac{g_B}{2(2\pi)^{3/2}} \frac{N_B^2}{\sigma_a \sigma_r^2} \\ E_{int}^{BF} &= g_{BF} \int n_B n_F d^3r = \frac{g_{BF}}{(2\pi)^{3/2}} \frac{N_B N_F}{\sigma_a \sigma_r^2}. \end{aligned} \quad (\text{A4})$$

Degenerate fermions, we assume, do not interact. We consider the quantum corrections as well, including the Lee-Huang-Yang [30] and the Viverit-Giorgini [31] ones for bosons and for bosons and fermions, respectively

$$E_{LHY} = C_{LHY} \int n_B^{5/2} d^3r = \frac{C_{LHY}}{\pi^{9/4}} \left(\frac{2}{5}\right)^{3/2} \frac{N_B^{5/2}}{\sigma_a^{3/2} \sigma_r^3} \quad (\text{A5})$$

with $C_{LHY} = 64/(15\sqrt{\pi}) g_B a_B^{3/2}$ and

$$E_q^{BF} = C_{BF} \int n_B n_F^{4/3} A(w, \alpha), \quad (\text{A6})$$

where $w = m_B/m_F$ and $\alpha = 16\pi n_B a_B^3 / (6\pi^2 n_F a_B^3)^{2/3}$ are the dimensionless parameters, and the function $A(w, \alpha)$ has a form:

$$\begin{aligned} A(w, \alpha) &= \frac{2(1+w)}{3w} \left(\frac{6}{\pi}\right)^{2/3} \int_0^\infty dk \int_{-1}^{+1} d\Omega \\ &\left[1 - \frac{3k^2(1+w)}{\sqrt{k^2 + \alpha}} \int_0^1 dq q^2 \frac{1 - \Theta(1 - \sqrt{q^2 + k^2 + 2kq\Omega})}{\sqrt{k^2 + \alpha + wk + 2q\omega}} \right]. \end{aligned} \quad (\text{A7})$$

The coefficient $C_{BF} = (6\pi^2)^{2/3} \hbar^2 a_{BF}^2 / 2m_F$.

Finally, the trapping energies are as follows

$$\begin{aligned}
 E_{tr}^B &= \frac{1}{2} m_B \int ((\omega_a^B)^2 z^2 + (\omega_r^B)^2 \rho^2) n_B d^3 r \\
 &= \frac{1}{2} m_B \left(\frac{1}{2} (\omega_a^B)^2 \sigma_a^2 + (\omega_r^B)^2 \sigma_r^2 \right) \\
 E_{tr}^F &= \frac{1}{2} m_F \int ((\omega_a^F)^2 z^2 + (\omega_r^F)^2 \rho^2) n_F d^3 r = \\
 &= \frac{1}{2} m_F \left(\frac{1}{2} (\omega_a^F)^2 \sigma_a^2 + (\omega_r^F)^2 \sigma_r^2 \right)
 \end{aligned} \tag{A8}$$

with ω_a^B (ω_a^F) and ω_r^B (ω_r^F) being the axial and radial trapping frequencies for bosons (fermions).

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