Decay of multiply charged vortices at nonzero temperatures

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Abstract
Inspired by a recent experiment (Ryu 2007 et al Phys. Rev. Lett. 99 260401), we study the instability of multiply charged vortices in the presence of thermal atoms. We find various scenarios of the splitting of such vortices. The onset of the decay of a vortex is always preceded by an increase in the number of thermal (uncondensed) atoms in the system and manifests itself by the sudden rise of the amplitude of the oscillations of the quadrupole moment. Our calculations show that the decay time gets shorter when the multiplicity of a vortex becomes higher.

1. Introduction
Experiments with atomic Bose–Einstein condensates have already shown their peculiar response to rotation, manifested through an induced irrotational flow and leading to quantized vortices and superfluidity [1–4]. It was directly demonstrated by using an interferometric technique that the circulation is indeed quantized [5], and different aspects of dynamics of quantized vortices were investigated [2, 3, 6–9]. Recently, the persistent flow of condensed atoms in a toroidal trap was also observed [4].

So far studies of quantized vortices have been related to the single-vortex state, the vortex lattices and the multiply charged vortices. They included the analysis of the process of their nucleation as well as the stability conditions and the decay. For example, in [2] the lifetime of the single-vortex state in an axisymmetric trap is investigated at two different condensate parameters. Although in both cases the uncondensed part of the atomic cloud is almost undetectable, its influence on the lifetime of the vortex is huge. Clearly, reducing the level of thermal atoms increases the lifetime of the vortex. Surprisingly, long vortex lifetimes (up to several seconds) were observed for highly rotational triangular vortex lattices containing more than 100 vortices [3]. Multiply charged vortices have an additional degree of freedom; their lifetime is limited by the instability leading to decay into vortices having smaller charges. Pioneering work on doubly quantized vortices generated with the help of a topological phase-imprinting technique [10] showed that their lifetime is a monotonic function of the interaction strength [8, 9].

Not much experimental work has been devoted to the rotational properties of Bose–Einstein condensates at finite temperatures. In [7], the crystallization and the decay of vortex lattices are studied in the presence of thermal atoms. The decay of a lattice was observed in a nondestructive way just by monitoring the distortion of the rotating condensate, and dramatic dependence on temperature was found. It turned out that the rotational frequency of the lattice decreases exponentially; this has been attributed to the observation that the thermal cloud also rotated [7]. In contrast, when the thermal cloud is static the decay shows nonexponential behaviour [2]. In [4], on the other hand, the influence of the thermal fraction on the stable circulation of condensed atoms was investigated. It was found that in a trap with ring geometry, the persistent flow is possible even with the condensate fraction being as small as 20%.

In contrast to the case of zero temperature (for a review, see [11] and references therein; for the case of multiply charged vortices, see [12]), there is only a small number of theoretical papers considering the vortex dynamics in the presence of thermal atoms [13, 14]. In this paper, we focus on the evolution...
of multiply charged vortices generated in a way described in [4] and the role the thermal fraction plays in the splitting process.

This paper is organized as follows. In section 2 we describe the classical field approximation, an approach that allows us to treat the atomic condensate at nonzero temperatures. Section 3 gives the details of the system we investigate and some results regarding the stability of doubly charged vortices. In section 4, we discuss the dynamics of five-fold charged vortices. Finally, we conclude in section 5.

2. The classical field approximation

Experiments with dilute atomic gases are performed with thousands or millions of atoms. In the case of bosons, they display a global wave character. The analogy with electromagnetic waves is striking. In spite of consisting of photons, an intense light beam is well described by classical electric and magnetic fields. Such a beam may be only partially coherent. In this case the detailed spatiotemporal behaviour is too complicated, impractical and even unmeasurable and coarse-grained correlation functions of classical fields are appropriate. Following this analogy, many-body dynamical equations of many-body dynamical equations of Bose–Einstein condensates require the coarse graining of the thermodynamics of classical electromagnetic waves. The analogous divergence also plagues the classical fields approximation to matter waves. A short wavelength cut-off is necessary. In [15] we have reviewed our version of the lattice version of the Gross–Pitaevskii equation:

$$\overline{\mathcal{H}} = \langle \rho^{(1)}(r, r'; t) \rangle = \frac{1}{\Delta V} \int V_R \rho^*(r + R, t + t_{\text{exp}}) \rho(r, t + t_{\text{exp}}) d^3 r.$$

where $\Delta$ and $V$ are, respectively, the temporal and spatial resolutions of the detector. The averaged density matrix $\overline{\rho}$ may than be diagonalized and their dominant eigenvalue has its eigenvector which may be identified with the wavefunction of the condensate while the remainder is the density matrix of the uncondensed or thermal cloud. This way we obtain a description which, unlike most alternative ones, yields the splitting of the system into condensed and uncondensed components solely as a result of Bose statistics and interaction plus the measurement process. It neither assumes from the very beginning a two-component character of the system [17] nor does it introduce adjustable damping terms [14]. Our version of the classical fields method is well suited to describe the single realization of the experiment. Competing ones [18] corresponding to a canonical rather than a microcanonical ensemble rely on averaging over many realizations with random initial conditions subject to thermal distribution. While convenient for the thermal equilibrium, such a method fails in the case of a dynamical problem such as the decay of multiply charged vortices discussed here. Symmetry breaking observed in individual realizations would be smeared out completely by averaging over many realizations.

A remark about the averaging time and volume is in order. A simple prescription is as follows: take both directly from the experiment you want to model. In practice, the situation is different for the thermal equilibrium and different for dynamical evolutions. In the first case, a rule of thumb is $\Delta > 30 \text{ ms}$ and the space averaging volume $V$ (imitating the CCD cell volume) comparable with the distance between neighbouring spikes in the instantaneous density distribution is about $5 \mu m$. As explained in the following section, a typical column density measurement calls for integration over an axial variable and in this case a sufficient mixing is done and no additional coarse graining is required. If however we would like to do time averaging in the dynamical case, there would be of course an additional condition putting an upper limit to $\Delta$. During the averaging time a dynamical feature we want to observe, such as a core of the vortex, should not move by a distance comparable to its size (the healing length). Of course, the experiment requires the same condition for the light pulse used.
3. Dynamics of doubly charged vortices

Our numerical procedure takes the following steps. First, we find the ground state of the Bose–Einstein condensate in the toroidal trap made by combining the usual harmonic trap with the Gaussian laser beam (as in the experiment of [4]). The harmonic trap frequencies are taken as \(\omega_z = 2\pi \times 25\ \text{Hz}\) and \(\omega_\perp = 2\pi \times 36\ \text{Hz}\). The blue-detuned laser beam serves as an optical plug repelling atoms from the trap centre, and the potential from this beam is given by the Gaussian function

\[
V_0 \exp\left[-2(x^2 + y^2)/w_0^2\right] \quad \text{(the laser beam propagates in the z direction).}
\]

Here, \(V_0\) is the maximum optical potential \((V_0/h = 3600\ \text{Hz}, \ h\) is the Planck constant\)) and \(w_0 (w_0 = 15\ \mu\text{m})\) is the waist of the beam. The interaction strength \(gN\) corresponds to that as in [4] for the experiment related to doubly quantized vortices (i.e. with the scattering length for Na atoms in the \(33\Sigma_{1/2}, F = 1, m_F = -1\) state and the number of atoms \(N = 500000\)). Our calculations are performed on a rectangular grid with 27 points in each direction. The spatial step of the grid is equal to 1.1 \(\mu\text{m}\).

Next, we specify the initial condition for the classical field by putting a particular amount of energy into the system and allowing the gas to thermalize. Since the classical field fulfills the time-dependent Gross–Pitaevskii equation [15], we find its evolution and build the one-particle density matrix at each time. Then we do a coarse graining and calculate a fraction of condensed atoms, the condensate wavefunction and the distribution of thermal (uncondensed) atoms. For a typical experiment, in which the column density is measured instead of the condensate fraction obtained from formula (5) with that accuracy in this case. Second, we considered the Bose gas being in the state of thermal equilibrium and compared the condensate fraction obtained from formula (5) with that coming from the diagonalization of the time-averaged density matrix. Assuming that the time over which the density matrix is averaged is long enough, both procedures give the same results again supporting the validity of (5).

A recently developed experimental technique, based on a stimulated Raman process with Laguerre–Gaussian beams [4, 19], allows us to generate multiply charged vortices in atomic Bose–Einstein condensates. However, such vortices are unstable against the decay into a number of vortices with lower charges. It happens because (i) the energy of a collection of vortices with lower charges is smaller than the energy of a single multiply quantized vortex and (ii) the total circulation is preserved. The question that can be raised is: how is the multiply charged vortex split into a larger number of vortices?

Since the energy of various configurations of vortices changes, one might expect different scenarios of a decay.

As a first test of our method, we investigate the dynamics of a doubly charged vortex. As in the experiment of [4], we first create the circulation in the toroidal trap corresponding to \(2h\) of angular momentum per atom. It is done by multiplying the classical field by the factor \(e^{im\phi}\) with \(m = 2\) and \(\phi\) being the azimuthal angle around the \(z\) direction. The circulation can be imprinted before or after the thermal energy is pumped (if this is the case) into the system. In both cases, the system is left to reach the thermal equilibrium. Next, we remove the optical plug in 30 ms. The following evolution shows that the vortex decays into two singly charged vortices (see [12] for a detailed analysis of the stability of multiply charged vortices). However, the splitting time depends strongly on the level of the thermal noise initially present in the system. It is illustrated in figure 1, where upper and lower panels, showing densities, correspond to different initial condensate fractions. In the case of the upper panel, the system consists of approximately \(N = 290000\) atoms at an initial temperature of \(T = 41\ \text{nK}\) (of course, afterwards when the vortex decays the system gets out of equilibrium and no temperature can be assigned to it) whereas for the lower panel the number of atoms \(N = 360000\) and the temperature \(T = 54\ \text{nK}\). The temperature is calculated here just by fitting the Gaussian distribution to the density of thermal atoms obtained after the optical plug is adiabatically removed. Having the temperature, the total number of thermal and hence of all the atoms can be calculated based on the appropriate formulae valid for an ideal gas.

Comparing figure 1 with figure 5 in [4] we conclude that the initial fraction of thermal atoms in the system used for an experimental study of a doubly quantized vortex is below 0.15. Our numerics also proves that uncondensed atoms are hidden in the cores of the vortices making vortices ‘less transparent’ in the CCD camera picture in comparison with the vortex before splitting (compare figures 5(c) and 5(a) in [4]; see also [20]).
4. Dynamics of five-fold charged vortices

As already mentioned in section 3, the splitting of multiply charged vortices might proceed in different ways. Before we go into details we would like to give the reader arguments pointing out that, indeed, various scenarios of vortex decay are possible.

The way the system follows depends on how fast the plug is switched off, i.e. on the energy of the gas when the plug is off. The shorter removal time ends in a higher energy state, therefore allowing access to higher energy configurations. For example, assuming that the five-fold charged vortex was imprinted on the condensate, the configurations considered could be that with four singly charged vortices placed on a circle around the vortex located in the centre of the trap (called the ‘4+1’ configuration) or the other arrangement with five vortices settled on a circle and no vortices in the middle (‘5+0’ configuration). It can be checked numerically by using the imaginary time technique in a rotating frame of reference [21] that the ‘4+1’ structure has higher energy than the ‘5+0’ one when it rotates with large enough frequency (≥0.7 of radial trap frequency), and the opposite is true for slower rotation (even more, for slower rotation the configurations obtained are no longer at local energy minima). Therefore, depending on the time the plug is turned off the energy-related arguments suggest possible scenarios of the decaying process. Experimentalists could observe the ‘5+0’ structure for slower removal of the plug and ‘4+1’ configuration followed by the transition to a ‘5+0’ structure when the plug is taken off quickly. This scenario might get even simplified if a small fraction of uncondensed atoms is initially present in the system. In this case, because of thermal fluctuations, the system could go directly to the ‘5+0’ configuration. A note on the observation of different vortex configurations was first given in given [22].

Our numerics confirms the scenarios which have just been discussed. We first discuss the case of zero initial temperature (figures 2 and 3). Figure 2(A) shows the isodensity plots at times when the configurations ‘4+1’ (left frame; here the plug is off in 30 ms) and ‘5+0’ (right frame, with the plug taken off in 200 ms) are present. The successive rows display the density as is imaged by the CCD camera along the direction of the axis of symmetry (B), the z-integrated condensate density obtained according to the classical fields approach (C) and the density of thermal cloud (D). Frames (C) and (D) clearly indicate that thermal atoms are located in the cores of vortices. An exception is the vortex placed at the trap centre in a ‘4+1’ vortex array which is initially empty. However, it is immediately filled in with uncondensed atoms, becomes unstable and moves away from the centre forming the ‘5+0’ configuration.

Details of the dynamics of a five-fold charged vortex are given in figures 3 and 4. Figure 3 shows the fraction of uncondensed atoms appearing in the system in the case of fast (30 ms) and slow (200 ms) changes of the trapping potential. Fast removal of the inner plug means stronger disturbance of the gas and hence more effective production of thermal atoms. As was already discussed in [23], the level of the thermal
noise strongly influences the lifetime of the vortex. The larger the number of thermal atoms, the shorter the lifetime of the vortex. The five-fold vortex decays approximately 1 s after the plug is taken out (vertical line (A) in figure 3) and since the fast removal of the plug ends in a higher energy state, the higher energy vortex configurations are accessible; in our case, it is the ‘4+1’ configuration. Afterwards, the singly charged vortices settled on a circle increase their separation from the vortex located at the centre of the trap and simultaneously the vortices increase and subsequent decrease of the thermal component during the splitting of the five-fold vortex.

During the separation process, vortices are entangled (in the classical sense of the word) and bended. Therefore, in a column density picture there is always a nonzero density in vortex cores which is interpreted as additional thermal atoms. These vortices disentangle (see movies available at [24]) while they go away from the centre and according to the definition of a thermal cloud, the condensate fraction grows. Note that the above-mentioned effects are certainly present both in our calculations and in the experiment. This is the main reason of the effect displayed in figure 3 for the case of slow removal of the optical plug (dark dots) of the sudden increase of the amplitude of the quadrupole moment (arb. units) determined by the quadrupole moment.

Figure 4. Decay of a five-fold charged vortex. The upper panel shows the density at various times (from left to right): the plug is on (A), after the plug is off (which takes 200 ms) but before the vortex is split into the singly charged vortices (B), and when the vortex is clearly split into five vortices (C). The size of each frame is $73 \times 73 \mu m^2$. In the lower frame, the quadrupole moment is plotted as a function of time showing clearly the onset (0.3 s after the plug is taken off) of the decay process.

Figure 5. Quadrupole moment as a function of time for differently charged vortices (from doubly charged to seven-fold charged as indicated in the left upper corner of each frame). The vertical lines show the beginning and the end (delayed by 200 ms) of removal of the plug. Note that, in fact, a dramatic change in the behaviour of the quadrupole moment determines the onset of the decay of a vortex.

Our calculations show that the scenario of the decay of a multiple vortex depends strongly on whether initially the Bose gas has a distinguished thermal fraction. In fact, the way the system behaves determines the upper limit for a number of uncondensed atoms. We have already found that for the condensate fraction 92%, the ‘4+1’ configuration is not present irrespective of the duration when the optical plug is taken off. Figure 4 shows the evolution of the system for slow removal of the plug when the initial condensate fraction equals $n_0 = 0.92$. In addition, we plot the time dependence of the quadrupole moment defined as $\int (x^2 - y^2) |\Psi(r)|^2 d^3r$, where $\Psi(r)$ is the classical field. It turns out that the behaviour of the quadrupole moment exhibits clearly the time the multiple vortex begins to split. The decay of the vortex is accompanied by a sudden increase of the amplitude of the quadrupole moment and unambiguously allows us to determine the lifetime of the vortex for any vortex charge (see figure 5).
Finally, in figure 6 we plot the decay time of the multiple charged vortex as a function of its charge, assuming that it is the time that elapsed since the plug beam was off. Here, the initial condensate fraction equals $n_0 = 0.92$. The main frame corresponds to a slow change of the trapping potential (200 ms), whereas the inset shows the case of a fast change (30 ms). The first observation is that faster removal of the plug leads to shorter decay times. This feature can be understood based on figure 3. Faster removal produces more thermal atoms, and hence destabilization of the vortex happens in a shorter time (see also [23]). The same arguments help us to understand the dependence of the lifetime on the charge of the vortex. As is displayed in figure 7, the production of uncondensed atoms is enhanced when the topological charge of the vortex is bigger.

5. Conclusions

In conclusion, we have studied the decay process of multiply charged vortices. We show that various scenarios of vortex splitting are possible depending on the level of uncondensed atoms appearing in the system as a result of a change of the trapping potential. Initially, the number of uncondensed atoms is determined by how fast the plug supporting the toroidal trap is removed. Faster removal produces more thermal atoms and leads to quicker decay, however possibly displaying different vortex configurations later. Finally, both fast and slow removal results in the same vortex configuration. At the time the vortex decays, the large oscillations of the quadrupole moment appear. The decay time also depends on the multiplicity of the vortex and for even a tiny uncondensed fraction it gets shorter for higher multiplicity.

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[34] http://212.33.73.26/vortices/mcv_vor.html

Figure 7. Condensate fraction as a function of time for fast (30 ms) removal of the optical plug. The squares, triangles and diamonds correspond to the charge of the imprinted vortex equal to 5, 6 and 7, respectively.