Suppression of the Rabi oscillations in a cavity partially filled with a dielectric having a time-dependent refractive index

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In this paper the dynamics of wave fields in a cavity, part of which is filled with a dielectric, are investigated. The wave equation is solved approximately using the method of multiple time scales. Phase modulations of the cavity modes in an adiabatic case are given up to the first order. A simple effective field Hamiltonian operator is derived and then used to analyze the interaction of radiation with a two-level atom passing through the cavity. Corrections to the Rabi oscillations due to the time-dependent frequency of the cavity photons are investigated. It is shown that adiabatic time dependence of the refractive index of the dielectric can lead to suppression of the Rabi oscillations of atomic inversion.

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One of the most interesting problems of modern electrodynamics and optics is the behavior of the electromagnetic field and its interaction with atomic matter in time-dependent environments with nontrivial boundary conditions. A lot of work has been done recently in a fascinating subject of the dynamics of fields and atom-field interactions in a cavity with two perfectly reflecting mirrors one of which oscillates and it should be continuous, together with its derivative over $z$, at $z=d$. In the following we shall write simply $E$ instead of $E_y$. Let us use the dimensionless variables $X = k_0 z$ ($0 \leq X \leq \pi$) and $T = \omega_0 t$, where $k_0 = \pi / l$ and $\omega_0 = \pi c / l$ are the fundamental wave number and frequency of the cavity without any dielectric. Let $D$ be the dimensionless thickness of the dielectric layer, $D = k_0 d$. The dielectric function, expressed in the new variables $X, T$ can be written as

$$
\epsilon(X,T) = n_0^2(X) + \Delta \epsilon(X,T),
$$

where $n_0^2(X)$ is equal to $\epsilon$ for $X < D$ and equal to 1 for $X > D$. On the other hand, $\Delta \epsilon(X,T) = \delta g(T)$ for $X < D$ and $\Delta \epsilon(X,T) = 0$ for $X > D$. Equation (2) can now be written as

$$
\frac{\partial^2 E}{\partial X^2} = \frac{\partial^2 E}{\partial T^2} + \frac{\partial^2}{\partial T^2} [\Delta \epsilon(X,T)E].
$$

Let $f_p(X), p=1,2,\ldots$ be the orthonormal and complete system of eigenfunctions of the Helmholtz operator for $\Delta \epsilon = 0$,

$$
\frac{\partial^2 f_p}{\partial X^2} + n_0^2(X) \nu_p^2 f_p = 0,
$$

where $\nu_p$ are the eigenfrequencies of the cavity with the nonmodulated dielectric expressed in units of $\omega_0$. On expanding $E$ as

$$
E = \frac{1}{\pi} \sum_p E_p f_p(X)
$$

we easily find the following infinite system of equations for the coefficients $E_p$:

$$
E_m + \nu_m^2 E_m + \delta \sum_p u_{mp} [\ddot{g}(T)E_p + 2 \dot{g}(T)E_p + g(T)E_p] = 0.
$$

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where \( u_{mp} = \int f_m^*(X) f_p(X) dx \), an overdot means differentiation over the time \( T \), and \( g(T) = \sin(\Omega T/\omega_0) \). Let us now assume that \( \Omega \) is very small when compared with \( \omega_0 \nu_1 \). More precisely, let us assume that \( \Omega/\omega_0 = \delta \mu \), and \( \mu = O(1) \). This assumption defines adiabaticity for the purpose of this work. If we introduce two time variables \( T_0 = T, T_1 = \delta T \), expand \( E_m \) in terms of \( \delta \), and use the perturbative method of multiple scales, similarly as in [16], we shall find in the zeroth order

\[
\frac{\partial^2 E_m^{(0)}}{\partial T_0^2} + \nu^2 m^2 E_m^{(0)} = 0
\]  

and in the first order

\[
\frac{\partial^2 E_m^{(1)}}{\partial T_0^2} + \nu^2 m^2 E_m^{(1)} + \frac{2}{\partial T_0} \frac{\partial}{\partial T_1} E_m^{(0)} + \sum_p u_{mp} \sin(\mu T_1) \frac{\partial^2}{\partial T_0^2} E_p^{(0)} = 0.
\]

From Eq. (8) we get

\[
E_m^{(0)} = a_m^{(0)} e^{-i v_m T_0} + a_m^{(0)*} e^{i v_m T_0}.
\]

The requirement of the absence of any secular terms in Eq. (9) leads to

\[
\frac{\partial a_m^{(0)}}{\partial T_1} = \frac{1}{2} i v_m \mu_{mm} \sin(\mu T_1) a_m^{(0)}
\]

together with its complex conjugate. In most cases, for very small \( \delta \), the zero-order approximation with the correction to phases given by Eq. (11) provides a sufficiently accurate approximation. Thus, it is clear that the dynamics (in time \( T \)) of \( E_m \) in the zeroth order are generated by the following effective time-dependent Hamiltonian:

\[
H = \sum_m \nu_m a_m^* a_m^\dagger [1 - \delta^2 u_{mm} \sin(\delta \mu T)]
\]

with the canonical Poisson brackets:

\[
[a_m, a_m^\dagger] = \delta_{mm}.
\]

Performing a naive quantization by replacing Poisson brackets by commutators, and adding the coupling of the fundamental cavity mode with a two-level atom we get the quantized Hamiltonian of the system: one (lowest-frequency) mode plus atom:

\[
H_{A+F} = \nu_1 [1 - \delta^2 u_{11} \sin(\delta \mu T)] a_1^\dagger a_1
\]

\[+ \frac{\delta^2 \gamma}{2} (a_1^\dagger \sigma_{12} + \sigma_{21} a_1) + \nu \sigma_{22},
\]

where \( \nu = \omega / \omega_0 = (E_c - E_e) / \omega_0 \) is the difference between the energies of the atomic excited and ground states in units of \( \omega_0 \) (\( \hbar = 1 \)). The above Hamiltonian, written in the rotating-wave approximation, generates quantum dynamics in the variable \( T \). The operators \( \sigma_{ij} \) are the usual atomic lowering and rising operators. In the derivation of Eq. (13) it has been assumed that the coupling constant \( g \) between the lowest-frequency mode and the atom scales like \( \delta^{3/2} \sim 10^{-6} \), \( g / \omega_0 = \delta^{3/2} \gamma \). This seems to be reasonable; indeed, for a cavity of the size \( \sim 1 \) cm we have \( \omega_0 \nu_1 \sim 10^{10} \) Hz, and the atom-field coupling constant \( g \sim 10^8 \) Hz. Note that we assume that \( g \) is still much larger than \( \delta^2 \omega_0 \nu_1 \). Let us now additionally assume that the atom and the lowest-frequency mode of the field are perfectly tuned, \( \nu_1 = \nu \). Expanding both sides of the Schrödinger equation for the evolution operator in powers of \( \delta^{3/2} \) and using, e.g., again the method of multiple scales, we find that (cf. [16])

\[
U = \exp \left[ -i \omega N t - i \frac{\delta \omega u_{11}}{2 \Omega} (\cos \Omega t - 1) a_1^\dagger a_1 \right] U_1,
\]

\[
U_1 \approx \left[ 1 - 2 i \frac{\delta \omega u_{11}}{2 \Omega} \sum_{k=1}^{\infty} \frac{1}{k} (S_{21} e^{i(\delta u_{11}/2) - k \pi/2} + S_{12} e^{-i(\delta u_{12}/2) - k \pi/2}) J_k \left( \frac{\delta \omega u_{11}}{2 \Omega} \right) \sin(k \Omega t) \right] V_1,
\]

\[
V_1 = \cos \left[ J_0 \left( \frac{\delta \omega u_{11}}{2 \Omega} \right) g \sqrt{N t} - i (e^{i \delta u_{11}/2 \Omega} S_{21} + e^{-i \delta u_{11}/2 \Omega} S_{12}) \sin \left[ J_0 \left( \frac{\delta \omega u_{11}}{2 \Omega} \right) g \sqrt{N t} \right] \right].
\]

where we have reintroduced the physical time \( t \) and \( S_{21} = \sigma_{21} a_1 / \sqrt{N}, S_{12} = a_1^\dagger \sigma_{12} / \sqrt{N}, N \) being the excitation number operator, and \( \mathbf{1} \) the unit operator. \( J_k(x) \) are the \( k \)th-order Bessel functions of the argument \( x \). From Eqs. (14)–(16) we can infer a very special behavior of dynamics of the system when

\[ J_0(\delta \omega u_{11}/(2 \Omega)) = 0. \]  

In this case we have to do with suppression of the Rabi oscillations of the atomic inversion. This effect has been previously predicted for an atom in an external laser field with frequency modulation [15] and for an atom in a cavity with one oscillating mirror [16]. It is the author’s hope, however, that the present prediction can be much easier realized experimentally by using, e.g., an acousto-optical device to modulate \( \epsilon \). If an atom prepared in a Rydberg state is sent
through a microwave centimeter-size cavity, the dielectric function of the dielectric material is to be modulated with the frequency of the order of 1 MHz and with the amplitude of the order $10^{-4}$. The integral $u_{11}$ which appears in Eq. (17) does not change the order of magnitude of the argument of the Bessel function for reasonable values of $\epsilon$ and $D$. For instance, if $\epsilon = 4$ and $D = \pi/2$, we have $\nu_1 = 0.608\,173$ and $u_{11} = 0.632\,913$.

The cavity itself must be of very high $Q$, since the losses through the walls should be very small compared to all other quantities of the dimension of frequency, otherwise all the above analysis fails. The losses in the dielectric are to be very small as well. While the present formalism and results are quite similar to those obtained in Ref. [16] for the case of the cavity with one adiabatically vibrating mirror, one has to note an important formal difference. Here, already in the first order of perturbation expansion, we have to do with the coupling between the various modes even in the case of adiabatic or subadiabatic modulation of the refractive index (‘‘subadiabatic’’ means here that $\Omega \ll \delta\omega_0$). There is no such coupling in the case of an adiabatically or subadiabatically moving mirror.

In Ref. [12] it has been shown how the adiabatically moving mirror influences the spectra of transmitted light and of the spontaneous emission radiated by a two-level atom in the cavity. Due to the similarity of effective Hamiltonians one may conjecture that the modulation of the refractive index of the dielectric slab embedded in a cavity should give the same results regarding vacuum Rabi splitting and changes of the spectra. In a future work on a similar subject I would like to obtain corrections to the spontaneous emission and the energy-level shifts of an atom near a dielectric with time-varying refractive index in full three-dimensional electrodynamics.