Anomalous Hall effect and magnetoresistance in Ge$_{1-x-y}$Pb$_x$Mn$_y$Te cluster-glass system


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ABSTRACT

We present studies of magnetotransport properties of Ge$_{1-x-y}$Pb$_x$Mn$_y$Te bulk crystals with chemical composition changing from 0.180 < x < 0.311 to 0.019 < y < 0.136. Our samples show the presence of the cluster-glass-like transition accompanied with magnetic irreversibility at temperatures $T_{SG}$ around 90 K. The presence of strong anomalous Hall effect (AHE) with hysteresis is observed at $T < T_{SG}$. The data analysis allowed calculation of a correction to the Hall constant $R_H$ for $T < T_{SG}$ and the AHE constant, $R_S$, is around $10^{-5}$ m$^3$/C. $R_H$ and $R_S$ do not change strongly with temperature. The scaling law analysis has proved that the AHE in this system is due to the extrinsic mechanisms dominated by the skew scattering with relatively small contribution of the side jump process. Strong localization in our system at $T < T_{SG}$ and causes a presence of a minimum in the resistivity vs. temperature dependence. The magnetoresistance of our samples shows complex behavior at $T < 20$ K and interpreted as spin-dependent scattering of conducting carriers on localized magnetic moments. At higher $T$ nearly linear positive magnetoresistance is observed at temperatures from 20 K up to $T = T_{SG}$, where it reaches maximum. Positive linear magnetoresistance is related to the granular nature of our samples while the maximum near $T_{SG}$ is related to the giant spin-splitting of the valence band.

1. Introduction

Transition metal (TM) doped semiconductors are being intensively studied in view of their potential applications in spintronics for the last two decades. Carrier induced magnetic interactions are responsible for magnetic order in many conventional III–V, IV–VI, and II–VI compound semiconductors such as TM doped PbSnTe, GaAs, and other systems [1–3]. Room temperature magnetic ordering due to the presence of carrier mediated magnetic interactions is necessary for making use of diluted magnetic semiconductors in semiconductor spintronics. However, the Curie temperature of the most intensively studied and technologically mastered semimagnetic semiconductor Ga$_{1-x}$Mn$_x$As does not exceed 185 K [4], which excludes the practical application of this material. It is therefore necessary to look for alternative compounds that can operate at room temperature opening the path for practical applications.

IV–VI compound semiconductors alloyed with transition metal ions, in particular the Ge$_{1-x}$TM$_x$Te alloys, show carrier mediated ferromagnetism with high Curie temperatures reaching 200 K in Ge$_{1-x}$Mn$_x$Te with $x = 0.46$ [5]. High Curie temperatures in Ge$_{1-x}$Mn$_x$Te alloys are due to the large values of the Mn-ion conducting hole magnetic exchange constant $J_{HM} \approx 0.8$ and large carrier concentration $p > 10^{20}$ cm$^{-3}$ and caused large interest in these materials [6–8]. Moreover, IV–VI-based materials are recently perceived as important representatives of 3D topological insulators. In addition, Pb-based IV–VI crystals have the highest mobilities in this group of semiconductors. Alloying topological insulators with TM ions has been recently intensively studied in Bi$_2$Te$_3$ layers with Mn ions [9] and similar interest is observed for IV–VI materials. Ge$_{1-x}$TM$_x$Te alloys possess another important feature: are called multiferroic materials due to the simultaneous presence of ferroelectric and ferromagnetic order at $T < 200$ K.

The present paper extends our previous investigation of magnetic and transport properties of Ge$_{1-x-y}$Pb$_x$Mn$_y$Te crystals into detailed analysis and interpretation of the magnetotransport...
properties with respect to the samples having a broad range of both Pb and Mn contents [10,11]. The detailed analysis and interpretation of the anomalous Hall effect and both negative and positive magnetoresistance effects will be presented allowing the determination of the main physical mechanism responsible for the observed effects.

2. Sample characterization

For the purpose of this work a series of bulk \( \text{Ge}_{1-x-y} \text{Pb}_x \text{Mn}_y \text{Te} \) crystals was grown by the modified Bridgman method. The modifications applied to the growth technique are similar to those used by Aust and Chalmers in order to improve quality of alumina crystals [12]. A more detailed description of the growth procedure was presented in Refs. [10] and [11].

The chemical composition of our crystals was determined by the energy dispersive X-ray fluorescence technique (EDXRF). We used the Tracer Spectrace 5000 EDXRF spectrometer. The maximum relative error of the EDXRF technique does not exceed 10% of the calculated value of \( x \) or \( y \). The as-grown ingots were cut into thin slices (about 1 mm thick) perpendicular to the growth direction, mechanically polished, chemically cleaned, and etched prior to further characterization. Each slice was characterized with the EDXRF method. The obtained results indicate that the chemical composition of the slices changes continuously along the growth direction. From all our samples we selected the ones with the chemical composition showing the smallest changes in each individual slice and covering the widest range of \( x \) and \( y \) values. The chemical content of the samples selected for the current studies vary in the range of \( 0.180 < x < 0.311 \) and \( 0.019 < y < 0.136 \).

The structural quality of our samples was determined with the use of the standard X-ray powder diffraction (XRD) method. The main conclusions are that all our samples are two-phased consisting of the two NaCl structures, one cubic phase and the other phase showing rhombohedral distortion. A more detailed analysis of the XRD data can be found in Refs. [10] and [11].

Morphology of our \( \text{Ge}_{1-x-y} \text{Pb}_x \text{Mn}_y \text{Te} \) samples was studied with the use of high resolution Hitachi SU-70 Analytical UHR FE-SEM scanning electron microscope (SEM) coupled with Thermo Fisher NSS 312 energy dispersive X-ray spectrometer system (EDS) equipped with SDD-type detector. Sample surface was polished prior to the SEM/EDS measurements. A series of high resolution images of the crystal surface was done at different sample spots and magnifications. The measurements reveal the presence of micrometer-sized inhomogeneities in all our samples (see Fig. 1). There exist two types of clusters forming either spherical- or stripe-shaped geometrical forms. It might therefore have significant influence on the magnetotransport properties of the material. The detailed chemical content analysis done with the use of the EDS microprobe revealed a large difference of the chemical content between the matrix and the clusters. One can clearly see that the amount of Pb in the clusters is much higher than in the matrix. Hence it was proven that the chemical content has an influence on the magnetic exchange constant and is responsible for the two observed magnetic transitions [10,11].

Magnetic properties of our \( \text{Ge}_{1-x-y} \text{Pb}_x \text{Mn}_y \text{Te} \) samples were studied extensively before and the details can be found in Refs. [10] and [11]. The temperature dependence of the magnetization \( M \) with the sample being cooled down in the presence (FC) or the absence (ZFC) of the applied magnetic field were measured with the use of Quantum Design MPMS-XL5 magnetometer system. The temperature dependence of the ac susceptibility, \( \chi \), was measured using the LakeShore 7229 susceptometer system. The exemplary results of our measurements are presented in Fig. 2. The magnetization and susceptibility data indicate that all our samples exhibit a cluster-

3. Magnetotransport properties

Electrical properties of the \( \text{Ge}_{1-x-y} \text{Pb}_x \text{Mn}_y \text{Te} \) crystals were...
studied by means of temperature dependent and field dependent magnetotransport measurements. The magnetoresistance and the Hall effect were measured simultaneously at selected temperatures with the use of the six contact dc current technique. We have used the superconducting magnet with the maximum magnetic field \( B = 13 \) T and the sweep speed of about \( 0.5 \) T/min, equipped with the cryostat allowing the control of the temperature of the sample in the range of \( 1.4 \leq T \leq 300 \) K. The samples, cut to size of about \( 1 \times 1 \times 10 \) mm, were etched and cleaned before making electrical contacts. The contacts were made with the use of gold wire and indium solder. The ohmic behavior of each contact pair was checked prior to the measurements.

3.1. Anomalous Hall effect

The magnetic field dependence of the resistivity component perpendicular to the current and magnetic field direction, \( \rho_{xy}(B) \), was measured at several stabilized temperatures below, near, and above the cluster-glass–like transition temperatures \( T_{SG} \) (values gathered in the leg to Fig. 2) in all our samples. Our results indicate clearly that, for all \( \text{Ge}_{1-x} \text{Pb}_{x} \text{Mn}_{x} \text{Te} \) crystals below \( T_{SG} \), the \( \rho_{xy}(B) \) dependencies show large anomalous Hall effect (Fig. 3b) and the presence of hysteresis in \( \rho_{xy}(B) \) curves (Fig. 3c). The selected \( \rho_{xy}(B) \) curves visible in Fig. 3b show that the AHE makes a significant contribution to the total Hall effect in our \( \text{Ge}_{1-x} \text{Pb}_{x} \text{Mn}_{x} \text{Te} \) samples at \( T < T_{SG} \). The analysis of the magnetometric and the Hall effect hysteresis loops shows that the coercive field, \( H_C \), values obtained by using the two types of measurements coincide with a good accuracy and point into similar values of \( T_{SG} \). It is a signature that the asymmetric carrier scattering occurs in our cluster-glass-like system and can be directly linked to the magnetic properties of the \( \text{Ge}_{1-x} \text{Pb}_{x} \text{Mn}_{x} \text{Te} \) alloy.

The typical analysis of the Hall effect in a nonmagnetic conductor assumes that the magnetic field dependence of the off-diagonal resistivity component, \( \rho_{xy}(B) = R_H B \), i.e. the Hall effect is a linear function of \( B \). The temperature dependence of the Hall constant, \( R_H \), calculated from the dependence of \( \rho_{xy} \) on \( B \) is plotted in Fig. 4. As we can see the \( R_H \) does not show significant changes with the temperature at \( T > T_{SG} \) and increases with decreasing temperature at \( T \leq T_{SG} \). The carrier transport in our samples is not thermally activated and thus we do not expect any significant temperature dependence of the Hall constant. It is then evident that the increase of the \( R_H(T) \) dependence at \( T \leq T_{SG} \) is an artifact of the data analysis. In order to determine the strength of the AHE and to properly estimate the Hall carrier concentration and mobility for \( T \leq T_{SG} \) an appropriate fitting procedure must be employed.

The Hall effect in a material doped with magnetic ions shows the usual Lorentz term \( R_{HI}B \) and a second contribution, namely AHE term \( R_{HM} \), caused by the asymmetric carrier scattering. The AHE is due to the spin–orbit coupling in the presence of spin–polarization (for details see Ref. [13] and references therein). The AHE term in some cases dominates the total Hall effect below the temperature of the magnetic transition, thus making the precise estimation of the carrier concentration and mobility very difficult. The magnetic field dependence of the Hall resistivity tensor component, \( \rho_{yx} \), measured in the standard six contact Hall geometry can be expressed using the following relation:

\[
\rho_{yx} = \rho_{xy} + \rho_{xy}^0 = \rho_{HI}B + \rho_{HM}B.
\]

where \( \rho_{xy} \) is the magnetic permeability constant. Both the ordinary, \( R_H \), and anomalous, \( R_H \), Hall coefficients can be extracted from the total Hall effect with the knowledge about the magnetic field dependence of the magnetization, namely \( M(B) \) curves at given temperatures. The use of the \( M(B) \) curve (example in Fig. 3a) is crucial, especially for a system in which the magnetization does not show saturation even at relatively high fields \( B = 9 \) T, which is the case for our samples. In such a system the AHE term gives a contribution that is not constant as a function of the magnetic field over the entire measurement range. Therefore, both the ordinary and the anomalous terms of the Hall effect affect the \( \rho_{xy} \) dependence at high fields and cause it to be an increasing function

![Fig. 3. Results of magnetometric and magnetotransport measurements obtained for the selected \( \text{Ge}_{1-x} \text{Pb}_{x} \text{Mn}_{x} \text{Te} \) samples with different chemical compositions including (a) the magnetic field dependence of the magnetization necessary for the proper AHE analysis, (b–c) magnetic field dependence of the off-diagonal resistivity component \( \rho_{xy} \) shown at high fields (b) and low-fields (c), (d) the temperature dependence of the coercive field, \( H_C \), determined with the use of magnetization (open symbols) and Hall effect (full symbols) hysteresis loop data.](image)

![Fig. 4. The ordinary and anomalous Hall constants, \( R_H \) and \( R_{HM} \), respectively, as a function of temperature for the selected \( \text{Ge}_{1-x} \text{Pb}_{x} \text{Mn}_{x} \text{Te} \) samples with different chemical compositions. The Hall constant \( R_H \) is plotted without (open symbols) and with (closed symbols) the AHE contribution included in the analysis of the Hall effect data.](image)
of the applied magnetic field. Thus, a complex fitting procedure needs to be employed in order to properly determine the Hall effect parameters i.e. to calculate precisely the $R_H$ and $R_S$ at low temperatures $T < T_{SG}$. The proper determination of the strength of the observed AHE and the calculation of the Hall constant and carrier mobility requires employing a fitting procedure of the experimental data to Eq. (1). We performed a series of least square fits of the experimental $\rho_{xx}(B,M,T)$ dependence to Eq. (1) with the use of the Minuit functional minimization package [14]. Fitting procedure was divided into two steps. As a first step both $R_H$ and $R_S$ were taken as the fitting parameters. For the second series of fits we assumed a constant value of $R_H$ (averaged value calculated using the values obtained during the first series of fits). This is a reasonable result, since our samples do not show thermally activated carrier transport. During the second series of fits only the anomalous Hall constant $R_S$ was taken as a fitting parameter. The obtained values of $R_S$ as a function of temperature for the selected $\text{Ge}_{1-x-y}\text{Pb}_x\text{Mn}_y\text{Te}$ samples with different chemical compositions (see legend) are plotted in Fig. 4.

The values of $R_S$ obtained in this work are of the same order of magnitude as the values reported for other IV–VI based diluted magnetic semiconductors such as $\text{Sn}_{1-x-y}\text{Mn}_x\text{Er}_y\text{Te}$, $\text{Ge}_{1-x-y}\text{Mn}_x\text{Eu}_y\text{Te}$ and $\text{Ge}_{1-x-y}\text{Sn}_x\text{Mn}_y\text{Te}$ [15,17,16]. We found no trace of any trend between the Mn content and the value of the $R_S$. A small minimum in $R_S(T)$ is observed at $T \approx 10$ K, the effect most probably associated with the low temperature magnetic transition in our samples. Besides that effect there seem to be no significant changes in $R_S(T)$ functions for all our samples. The absence of $R_S(T)$ dependence is typical for IV–VI semiconductors [18,16]. The physical mechanisms responsible for the observed AHE in our samples are yet unknown and the proper scaling analysis of the experimental data must be employed in order to shed light on the physical origin of the AHE in this material.

Physical mechanisms responsible for the AHE can be divided into intrinsic and extrinsic mechanisms. It is a fact known in the literature, that there are two major extrinsic mechanisms leading to the formation of AHE, namely skew scattering and side jump, which have been described theoretically and distinguished by appropriate linear [19] and square [20] dependencies between the diagonal and off-diagonal resistivity components, using the relation $R_S \propto \rho_{xx}^{n_H}$, where $n_H = 1$ or 2, respectively. There also exists an intrinsic mechanism that can be responsible for the AHE. This mechanism is related to the topological effect occurring in the semiconductor. This intrinsic AHE mechanism was described theoretically based on the Berry phase theory and was used to describe the AHE in $\text{Ga}_{1-x}\text{MnAs}$ with a metallic type of conductivity [21]. The topological explanation of the AHE was also employed theoretically for IV–VI semiconductors [22,23]. The Berry phase theory predicts the square scaling dependence $R_S \propto \rho_{xx}^2$.

The scaling analysis of the AHE in our samples was based on the fitting of the experimental data using the following equation

$$\rho_{xx} = R_H B + c_H \rho_{xx} M,$$

where $c_H$ and $n_H$ are the scaling coefficients. This analysis enabled the determination of the dominant scattering mechanisms responsible for the AHE in the studied system. Scaling relation of the AHE was found for each set of magnetotransport curves by fitting the experimental results to Eq. (2). The data analysis was performed with $R_H = \text{const.}$ taken from the previous series of fits. The $c_H$ and $n_H$ constants were taken as fitting parameters. The results of the fitting procedure indicate a good agreement (with variance smaller than $10^{-12}$) between the experimental data and the theoretical curves given by Eq. (2). The values of the $n_H$ scaling coefficient was estimated for each sample and measurement temperature. The $n_H$ values obtained for our samples change from 1.06 to 1.18. The values of $n_H$ indicate that the AHE in our $\text{Ge}_{1-x-y}\text{Pb}_x\text{Mn}_y\text{Te}$ samples is mainly due to the extrinsic skew scattering process. Since $n_H > 1$ the other scattering mechanisms giving a small contribution to AHE are also present in our system. The scaling analysis of our data indicates that extrinsic skew scattering is dominant AHE mechanism in our cluster-glass-like $\text{Ge}_{1-x-y}\text{Pb}_x\text{Mn}_y\text{Te}$ samples. On the other hand, for ferromagnetic $\text{Ge}_{1-x}\text{Mn}_x\text{Te}$ and $\text{Ge}_{1-x}\text{Cr}_x\text{Te}$ layers the side jump mechanism of AHE becomes more pronounced [24,25]. The extrinsic scattering mechanism becomes more pronounced in a system with strong magnetic disorder and low electrical resistivity [26]. The strong spin–exchange interaction and low carrier mean free path can introduce a contribution from side-jump mechanism to the system. The $n_H$ values obtained for our $\text{Ge}_{1-x-y}\text{Pb}_x\text{Mn}_y\text{Te}$ samples are generally smaller than the ones obtained for $\text{Ge}_{1-x-y}\text{Sn}_x\text{Mn}_y\text{Te}$ [17] and $\text{Ge}_{1-x-y}\text{Mn}_x\text{Eu}_y\text{Te}$ [16] crystals. The $n_H$ values from Refs. [17] and [16] change from 1.1 to 1.3. Therefore, it seems that for the $\text{Pb}$-alloyed GeTe crystals the extrinsic skew scattering process is a more pronounced physical mechanism responsible for the AHE than in the the other representatives of IV–VI semimagnetic semiconductors.

### 3.2. Resistance and magnetoresistance studies

The transport studies of our $\text{Ge}_{1-x-y}\text{Pb}_x\text{Mn}_y\text{Te}$ samples included the measurements of the temperature dependence of the resistivity parallel to the current direction, $\rho_{xx}$, in the absence of an external magnetic field. The results of our measurements are presented in Fig. 5 and gathered in Table 1. The resistivity values measured at $T = 300$ K (see Fig. 5b) show a slight increase with the amount of Pb in the alloy. It is a consequence of an increasing concentration of scattering centers in the alloy. Carrier concentration and mobility do not exhibit such clear trend as a function of the average lead content. This means that the increase in resistance as a function of $x$ is most probably related to the increasing (with $x$) concentration of

![Figure 5](image-url)

**Fig. 5.** Results of the resistivity, $\rho_{xx}$, measurements for the selected $\text{Ge}_{1-x-y}\text{Pb}_x\text{Mn}_y\text{Te}$ samples with different chemical compositions: (a) normalized $\rho_{xx}(T)$ curves, (b) $\rho_{xx}(T = 300 \text{ K})$ as a function of the $x$ content, and (c) $\rho_{xx}(1/4)$ dependence obtained experimentally (markers) and fitted (using full markers) to the scaling law (lines) given by Eq. (3).
The measurements were performed over a wide temperature range below 200 K. The $\rho_{\text{Hall}}(B)$ curves were obtained by averaging the results for positive and negative current in order to remove all the artifacts due to the contact asymmetry. The $\rho_{\text{Hall}}(B)$ curves at different temperatures were normalized to the zero-field resistivity value, $\rho(0)$. We determined the relative magnetoresistance given by the formula $\Delta \rho_{\text{Hall}} / \rho_{\text{Hall}}(0) = (\rho_{\text{Hall}}(B) - \rho_{\text{Hall}}(B = 0)) / \rho_{\text{Hall}}(B = 0)$. The exemplary results are presented in Fig. 6. The experimental MR results let us speculate that there are three main contributions to the total MR observed at $T < 120$ K. Above 120 K only the classical positive MR showing $B^2$ dependence due to the orbital carrier movement in the presence of the external magnetic field is observed. In the temperature range from $T = 120$ K to $T = T_{\text{SG}}$ for all our samples, there is visible at least one additional contribution to the total MR causing an increase of the total MR with decreasing $T$ to the maximum values observed at $T = T_{\text{SG}}$ (see Fig. 7). For temperatures below $T_{\text{SG}}$ the MR decreases with decreasing $T$.

There are several different physical mechanisms that can introduce both positive and negative MR in nonmagnetic semiconductors. Moreover, there are a few mechanisms related to the presence of magnetic impurities inducing MR in diluted magnetic semiconductors. In our case we have to consider also MR related to the presence of clusters in the samples. At low temperatures, weak localization or antilocalization phenomena are responsible for the presence of negative or positive MR in the semiconductors, respectively [34]. Typically the product of the Fermi wave vector $k_F$ and the mean free path $l$, namely $k_F l$, is the quantity needed to establish the regime at which the carrier conductivity occurs in the system. For $k_F l = 1$ the system remains in the weak localization regime while for $k_F l > 1$ degenerate regime is present and localization effects should be suppressed. For most of our samples $k_F l > 1$ indicating the degenerate regime. Moreover, weak localization and antilocalization effects are suppressed by the presence of magnetic impurities in the alloy. We consider it unlikely that these effects are responsible for MR in our samples.

The presence of magnetic impurities can induce a negative MR with relatively low values at low temperatures, as in our Ge$_{1-x}$Pb$_x$Mn$_y$Te crystals, due to the formation of bound magnetic polarons. The negative MR observed in our samples at $T < 20$ K is caused by reduction in the scattering effective cross-section due to the spin alignment of the magnetic moments. This effect induces MR only at low temperatures, whereas at high temperatures spin-ordering is strongly suppressed and the positive component to the MR dominates. Moreover, this negative MR decreases with an increase of carrier concentration [35,36], as it is in our samples (see Fig. 6). The other possible mechanism, especially important at low temperatures, is related to the variable range hopping present in our samples. However, the MR at $T < 20$ K cannot be fitted with $\exp(B^{1/3})$, relation typical for hopping conduction [37]. Moreover, the negative MR diminishes at a temperature much lower than the temperature at which hopping regime is present. Thus, we believe that the negative MR present in our samples at $T < 20$ K is due to the spin-dependent carrier scattering on the localized magnetic moments.

The MR observed at $T > 20$ K changes sign to positive, scales with $B^2$ at $B < 2$ T, remains nearly linear at $B > 2$ T, and its value (measured at $B = 13$ T) increases as a function of temperature up to $T = T_{\text{SG}}$ (see Fig. 7). The temperatures at which the MR reaches a maximum (see Fig. 7) are for each our sample different than the temperatures at which a minimum in $\rho_{\text{Hall}}(T)$ is observed (see Fig. 5) indicating that the maximum in MR effect is not related to the PAHTLS effect present at low temperatures. The positive linear MR observed in our samples is most probably the superposition of a very small contribution from the classical quadratic MR and linear MR related to the granular nature of our samples. Since the

### Table 1

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>$\rho$ [10^7 cm$^{-1}$]</th>
<th>$\mu$ [cm$^2$(V s)$^{-1}$]</th>
<th>$\rho_{\text{Hall}}$ [10$^{-3}$ (cm)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.232</td>
<td>0.019</td>
<td>8.8 ± 0.9</td>
<td>15.4 ± 1.5</td>
<td>0.5 ± 0.1</td>
</tr>
<tr>
<td>0.196</td>
<td>0.055</td>
<td>6.1 ± 0.6</td>
<td>14.1 ± 1.4</td>
<td>0.8 ± 0.1</td>
</tr>
<tr>
<td>0.196</td>
<td>0.061</td>
<td>5.6 ± 0.6</td>
<td>11.7 ± 1.2</td>
<td>1.0 ± 0.1</td>
</tr>
<tr>
<td>0.180</td>
<td>0.074</td>
<td>4.6 ± 0.5</td>
<td>2.0 ± 0.2</td>
<td>13.6 ± 1.4</td>
</tr>
<tr>
<td>0.183</td>
<td>0.074</td>
<td>6.6 ± 0.7</td>
<td>12.8 ± 1.3</td>
<td>0.6 ± 0.1</td>
</tr>
<tr>
<td>0.254</td>
<td>0.074</td>
<td>6.0 ± 0.6</td>
<td>6.2 ± 0.8</td>
<td>1.3 ± 0.1</td>
</tr>
<tr>
<td>0.311</td>
<td>0.081</td>
<td>8.1 ± 0.8</td>
<td>3.5 ± 0.4</td>
<td>2.2 ± 0.2</td>
</tr>
<tr>
<td>0.294</td>
<td>0.136</td>
<td>9.9 ± 1.0</td>
<td>1.6 ± 0.2</td>
<td>3.0 ± 0.3</td>
</tr>
</tbody>
</table>

The presence of magnetic impurities and, as a consequence, the exchange interaction between the localized magnetic moments and conducting carriers is known to modify drastically the carrier transport in the solid. The exchange interaction induces a giant spin-splitting of electronic bands and shifts the critical carrier concentration of the metal-insulator transition [32]. It is most probably the case of our samples. Similar effects are known in literature for Ge$_{1-x}$Mn$_x$Te crystals [33] and for both our samples and literature data the observed minimum in the $\rho_{\text{Hall}}(T)$ dependence is interpreted in terms of PAHTLS.

The magnetic field dependence of the resistivity component parallel to the current direction, namely magnetoresistance (MR), $\rho_{\text{Hall}}$, was measured in parallel with the Hall effect measurements.
maximum MR effect occurs in our samples in the vicinity of $T_{SC}$ it is highly probable that we observe the MR effect with maximum near the Curie temperature due to giant spin-splitting of the valence band observed in many ferromagnetic semiconductors. Giant Zeeman effect observed near $T_C$ leads to a maximum in positive magnetoresistance.

4. Summary

To conclude, we have studied magnetotransport properties of bulk Ge$_{1-x-y}$Pb$_x$Mn$_y$Te crystals with chemical composition varying from 0.180 $< x <$ 0.311 to 0.019 $< y <$ 0.136. Our previous studies showed that the cluster-glass-like state appears at temperatures lower than 100 K.

The magnetic order of the alloy has a significant influence on the magnetotransport properties of our samples. A strong anomalous Hall effect with hysteresis was observed at $T < T_{SC}$ in all our samples. The data analysis indicated the ordinary Hall constant, $R_H$, at $T < T_{SC}$ does not change with temperature. The AHE coefficient was found to show small changes with the chemical composition, changing from $1 \times 10^{-6}$ to $3 \times 10^{-6}$ m$^3$/C. The scaling law analysis of the AHE shows that the extrinsic skew scattering mechanism, accompanied with the extrinsic side jump, is the main physical mechanism responsible for the AHE in the Ge$_{1-x-y}$Pb$_x$Mn$_y$Te system.

The presence of localized states influencing carrier conduction occurs in our system at $T < T_{SC}$ and is reflected in the presence of a minimum in the resistivity vs. temperature dependence. Such dependence of resistivity on temperature is characteristic of the phonon-assisted hopping through localized states. The magnetoresistance of our samples shows complex behavior at $T < T_{SC}$. The negative magnetoresistance observed at $T < 20$ K is interpreted as spin-dependent scattering of conducting carriers on localized magnetic moments. At temperatures $T > 20$ K a nearly linear positive magnetoresistance increasing as a function of temperature up to $T = T_{SC}$, where it reaches maximum, is observed. Positive linear MR is related to the granular nature of our samples while the maximum near $T_{SC}$ is related to the giant spin-splitting of the valence band. At $T > T_{SC}$ only a positive classical MR is observed.

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