Search for Jack ground states of two-body hamiltonians

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Wave functions of multiple fermions confined to the lowest Landau level, relevant for the description of fractional quantum Hall effect (FQHE), emergence of composite fermions (CFs), and other phenomena connected with condensation of two-dimensional (2D) electrons in high magnetic field into incompressible quantum liquids, have a simple mathematical structure of antisymmetric polynomials: $\Phi(\{z\}) = \Pi_{i<j}(z_i\text{-}z_j)\ \Psi(\{z\})$, where z's are complex electron coordinates on a plane and Ψ is fully symmetric.

In particular, the Laughlin ground state at filling factor ν = 1/3 has the form $\Phi_L = \Pi(z_i\text{-}z_j)^3$, and in general the factorization $\Phi_{CF} = \Pi(z_i\text{-}z_j)^{2p+1}\ \Psi$ signifies emergence of CFs – topological bound states of electrons and $2p$ attached vortices (or magnetic flux quanta) in a state $\Pi(z_i\text{-}z_j)\ \Psi$.

A special class of symmetric functions are Jack polynomials J_λ^α [1], parametrized by an integer partition {λ}, uniquely corresponding to a root orbital occupation of a given FQH state, and a real parameter α, fixed by charge-uniformity requirement. Their fermionic counterparts $\Pi(z_i\text{-}z_j)\ J_\lambda^\alpha$ (efficiently generated as eigenstates of the fermionic Laplace-Beltrami operator) describe in a unified fashion several wave functions proposed for FQHE including Laughlin, Moore-Read *Pfaffian*, and Read-Rezayi *parafermion* ground states at ν = 1/3, 1/2, and 3/5 [2].

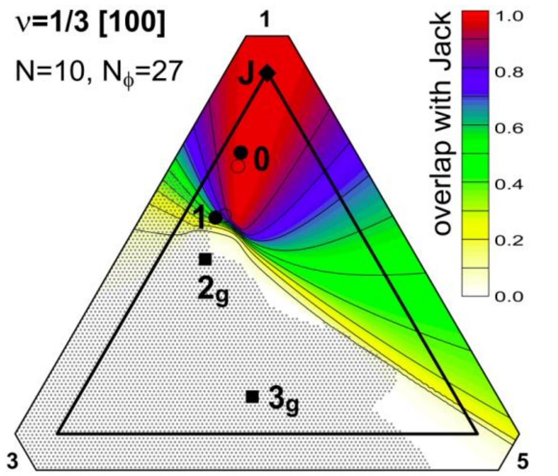

Fig.1 Each point of the map represents model interaction defined by three leading pseudopotentials (pair interaction energy for relative angular momenta m = 1, 3, 5), normalized to $V_1+V_3+V_5=1$. For each $\{V_m\}$ we computed the lowest uniform state of N=10 electrons on a Haldane sphere with magnetic flux N_ϕ=27, corresponding to ν=1/3. Color map gives the overlap of model ground state with Jack (≡Laughlin state). Symbols locate the maximum overlaps with Coulomb ground states in various Landau levels in GaAs (n) and graphene (n_g); $0_g \equiv 0$, $1_g \approx 0$. In dotted area, the absolute model ground state is non-uniform.

While the evidence for a Laughlin 1/3 ground state is widely accepted, actual realization of other Jacks in (even idealized) experimental conditions remains hypothetical.

We have mapped the ground states of arbitrary short-range two-body hamiltonians at different filling factors ν, in search of corresponding Jack ground states (e.g.: Fig. 1), and studied a possible adiabatic connection with the Coulomb ground states in different Landau levels (LLs) of GaAs or graphene.

The key results are: (i) Jacks, which are exact ground states of adequate short-range many-body repulsions, are (in general) accurately generated by suitable two-body hamiltonians; (ii) Jacks corresponding to a higher-order many-body repulsion require a longer range of the effective two-body model; (iii) the parafermion 3/5 state is a plausible candidate for FQHE in 2nd LL of GaAs, at ν = 13/5 and (by particle-hole symmetry) at 12/5, but not in graphene; (iv) several related states are hinted as candidates for FQHE, e.g., the ν = 1/3 *Haffnian* with λ = [11000011…] [3].

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