Mathematical structure of bosonic and fermionic Jack states and their application in fractional quantum Hall effect

Bartosz Kuśmierz[1], Y.-H. Wu[2], and Arkadiusz Wójs[1]

[1] *Institute of Physics, Wroclaw University of Technology, 50-370 Wroclaw, Poland*
[2] *Department of Physics, Pennsylvania State University, University Park PA, 16802*

Fractional quantum Hall effect (FQHE) is a remarkable behavior of correlated electrons in solid state. It can only be observed in two dimensional structures, in low temperatures and strong magnetic fields.

An important step in understanding FQHE was made by Laughlin. His wave functions describe the prominent FQH states which occur at Landau level filling factors $\nu = 1/q$ ($q \in \mathbb{N}_+$). Apart from the trivial Gaussian factor, this function is a Jack polynomial (i.e., a "Jack") [1-6]. Different Jacks describe also several other known FQH states: the Moore-Read state at $\nu = 1/2$ and the Read-Rezayi state at $\nu = 3/5$.

In the first part of this poster, we present the ideas needed to introduce Jack polynomials, such as the partition of a natural number, the ring of symmetric polynomials and the basic structure of this space.

Since Jacks are symmetric polynomials, they can describe only the bosonic FQH states. To anti-symmetrize Jacks one can use canonical isomorphism between rings of the symmetric and antisymmetric polynomials – multiplication by Vandermonde determinant. This isomorphism gives the so-called the fermionic Jacks [7-9].

Direct multiplication of a Jack polynomial by the Slater determinant is computationally very time consuming. An alternative and much faster method involves construction of eigenvectors of the fermionic Laplace-Beltrami operator.

Second part of the poster is devoted to the properties of the fermionic Jack polynomials, including the construction and projection onto Haldane sphere.

Finally, the exact bosonic and fermionic Jack functions will be compared with exact eigenstates of the many-electron Coulomb hamiltonian obtained by exact numerical diagonalization.

[1] L. Lapointe, A. Lascoux, and J. Morse, Elec. Jour. Combin. 7, N1 (2000).
[2] K. Sogo, J. Math. Phys. 35, 22822296 (1994).
[3] R. P. Stanley, Adv. Math. 77, 76-115 (1988).
[4] I. G. Macdonald, Symmetric Functions and Hall Polynomials, 2nd ed. (Oxford University Press, 1997).
[5] A. Hora and N. Obata, Quantum Probability and Spectral Analysis of Graphs (Theoretical and Mathematical Physics), (Springer, 2010).
[6] B. A. Bernevig and F. D. M. Haldane, Phys. Rev. Lett. 100, 246802 (2008).
[7] R. Thomale, B. Estienne, N. Regnault, and A. Bernevig, Phys. Rev. B 84, 045127 (2011).
[8] A. Bernevig and N. Regnault, Phys. Rev. Lett. 103, 206801 (2009).
[9] G. J. Sreejith, C. Töke, A. Wójs, and J. K. Jain, Phys. Rev. Lett. 107, 086806 (2011).