The Wigner Stochastic Gross-Pitaevskii Equation:

a c-field theory that includes both thermal and quantum fluctuations

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NARODOWE CENTRUM NAUKI — MARIE CURIE ACTIONS — SEVENTH FRAMEWORK PROGRAMME — Ministerstwo Nauki i Szkolnictwa Wyższego

Aim: A c-field description that includes thermal and quantum fluctuations on an even footing

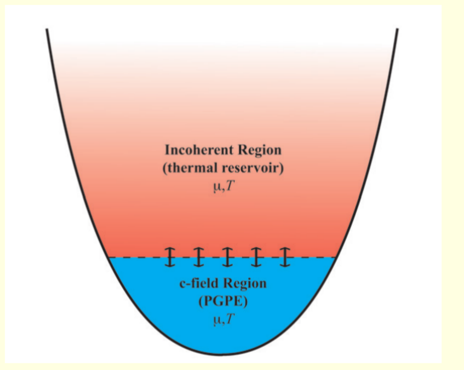
To be used for: (*) Generating a thermal ensemble of single realizations

(**) Calculating the quantum dynamics

Concept

The baseline: GPE $\hbar\dfrac{\partial\psi(\mathbf{x})}{\partial t} = -i\mathcal{L}_{GP}\psi(\mathbf{x}) \qquad \mathcal{L}_{GP} = -\dfrac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) - \mu + g\,|\psi(\mathbf{x})|^2$

- Re-derive SGPE equations from a Wigner representation of the Bose field + thermal bath, $\psi_W(\mathbf{x})$ but this time: *without explicitly assuming high occupations*
- Begin like Gardiner+Davis, *J. Phys. B* **36**, 4732 (2003).
- Still assume relatively high T: $\exp\left[\dfrac{\mathcal{L}_{GP}}{k_BT}\right] \approx 1 + \dfrac{\mathcal{L}_{GP}}{k_BT}$
- Obtain trajectories related to a ``pre-SGPE'' Fokker-Planck equation of Duine+Stoof, *PRA* **65**, 013603 (2001) that preserved fluctuation-dissipation relations, without assuming N(k) >>1.
- To our knowledge, it has not been previously simulated.



Blakie,Bradley, Davis, Ballagh, Gardiner, *Adv. Phys.* **57**, 363 (2008)

Prior candidates

SGPE (Stochastic GP Equation)

$$\hbar\frac{\partial\psi(\mathbf{x})}{\partial t} = -i(1-i\gamma)\mathcal{L}_{GP}\psi(\mathbf{x}) + \sqrt{2\gamma\hbar k_BT}\,\eta(\mathbf{x},t)$$

Stable → equilibrium; No quantum fluctuations, assumes macroscopic occupation

Truncated Wigner

$$\hbar\frac{\partial\psi_W(\mathbf{x})}{\partial t} = -i\mathcal{L}_{GP}\psi_W(\mathbf{x})$$

Stable numerically, Quantum fluctuations at short time; Unclear crossover at long time to GPE with no quantum fluctuations

Positive P $\mathcal{L}_{PP} = -\dfrac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) - \mu + g\,\widetilde{\psi}(\mathbf{x})^*\psi(\mathbf{x})$ Swislocki, Deuar, arXiv:1409.0146

$$\hbar\frac{\partial\psi(\mathbf{x})}{\partial t} = -i(1-i\gamma)\mathcal{L}_{PP}\psi(\mathbf{x}) + \sqrt{i\hbar g\,(1-2i\gamma)}\,\xi(\mathbf{x},t)\psi(\mathbf{x}) + \sqrt{2\gamma\hbar k_BT}\,\eta(\mathbf{x},t)$$

$$\hbar\frac{\partial\widetilde{\psi}(\mathbf{x})}{\partial t} = -i(1-i\gamma)\mathcal{L}^*_{PP}\widetilde{\psi}(\mathbf{x}) + \sqrt{i\hbar g\,(1-2i\gamma)}\,\widetilde{\xi}(\mathbf{x},t)\widetilde{\psi}(\mathbf{x}) + \sqrt{2\gamma\hbar k_BT}\,\eta(\mathbf{x},t)$$

Unstable numerically → equilibrium not achieveable; Full quantum mechanics while it lasts

Expectation values

$$\langle\widehat{\Psi}^\dagger(\mathbf{x})\widehat{\Psi}(\mathbf{x}')\rangle = \langle\psi_W(\mathbf{x})^*\psi_W(\mathbf{x}')\rangle_{\text{ensemble}} - \frac{1}{2}\delta(\mathbf{x}-\mathbf{x}')$$

Symmetrically ordered moments

$$\langle\widehat{\Psi}^\dagger(\mathbf{x})\widehat{\Psi}^\dagger(\mathbf{x}')\widehat{\Psi}(\mathbf{x})\widehat{\Psi}(\mathbf{x}')\rangle = \langle|\psi_W(\mathbf{x})|^2|\psi_W(\mathbf{x}')|^2\rangle_{\text{ensemble}} - \frac{1}{2}\left[\frac{1}{\Delta v} + \delta(\mathbf{x}-\mathbf{x}')\right]\langle|\psi_W(\mathbf{x})|^2 + |\psi_W(\mathbf{x}')|^2\rangle_{\text{ensemble}} + \frac{1}{4\Delta v}\left[\frac{1}{\Delta v} + \delta(\mathbf{x}-\mathbf{x}')\right]$$

WSGPE evolution equation:

$$\hbar\frac{\partial\psi_W(\mathbf{x})}{\partial t} = -i(1-i\gamma)\left[\mathcal{L}_{GP} - \frac{g}{\Delta v}\right]\psi_W(\mathbf{x}) + \sqrt{\gamma\hbar\left[2k_BT + \mathcal{L}_{GP} - \frac{g}{\Delta v}\right]}\,\eta(\mathbf{x},t) + i\sqrt{\gamma\hbar g}\,\xi(\mathbf{x},t)\psi_W(\mathbf{x})$$

Extra thermal noise at high energy

Complex thermal noise

Strange positive-P-like term with real noise from bath coupling

Terms with explicit dependence on particle number (not just gn)

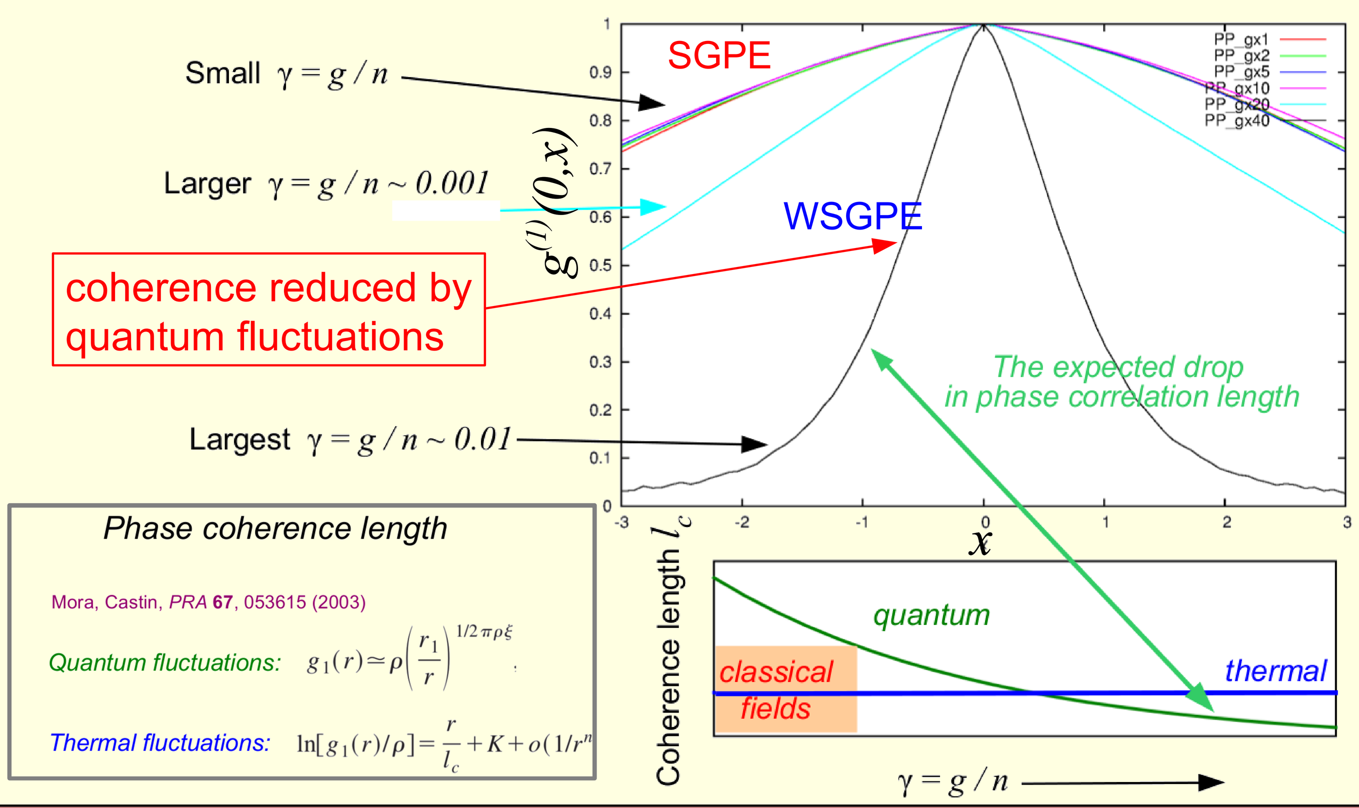
TEST CASE: Trapped 1D Bose gas $\mu = 22.4$, $T = 139 = 0.16T_\Phi$, $g=0.01$ → centrally $\gamma=4.5\times10^{-6}$ $\tau = 5.5\times10^{-5}$, $\tau/\sqrt{\gamma} = 0.026$ [cold quasicondensate]

Then, we change g, and $T\sim1/g$, to keep SGPE, μ and $\tau/\sqrt{\gamma}$ constant. Reach $\gamma\sim0.007$ and $\tau\sim0.0022$

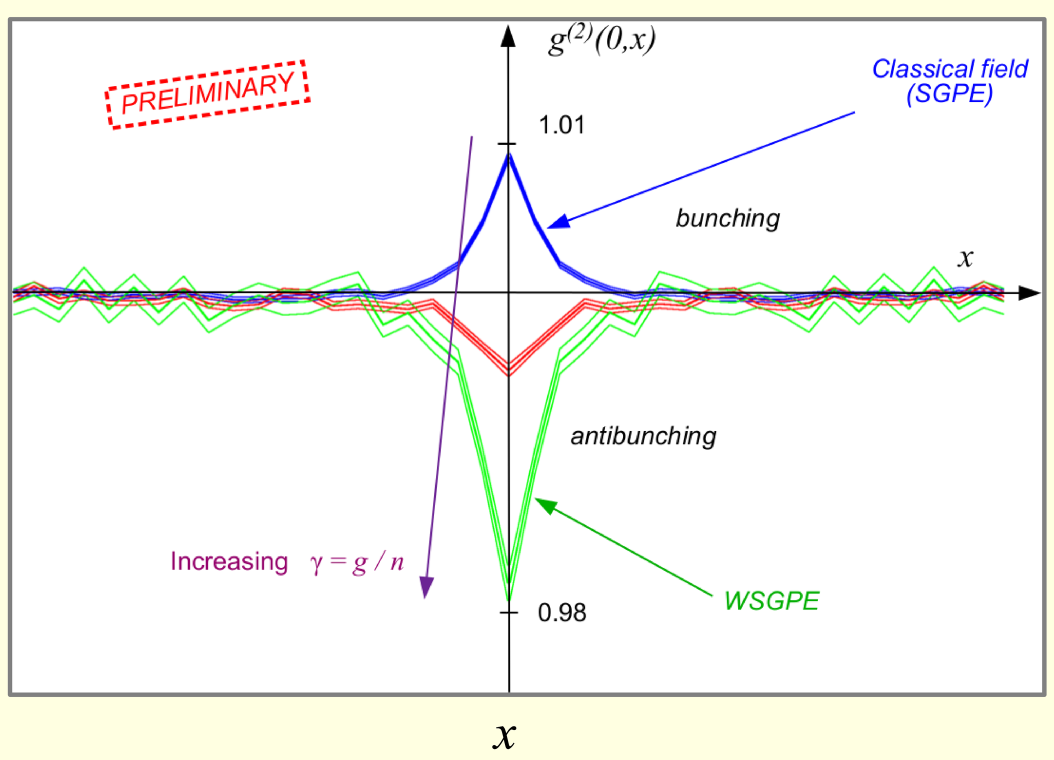
Interaction strength $\gamma = g/n$

Relative temperature $\tau = k_BT/4\pi T_d$

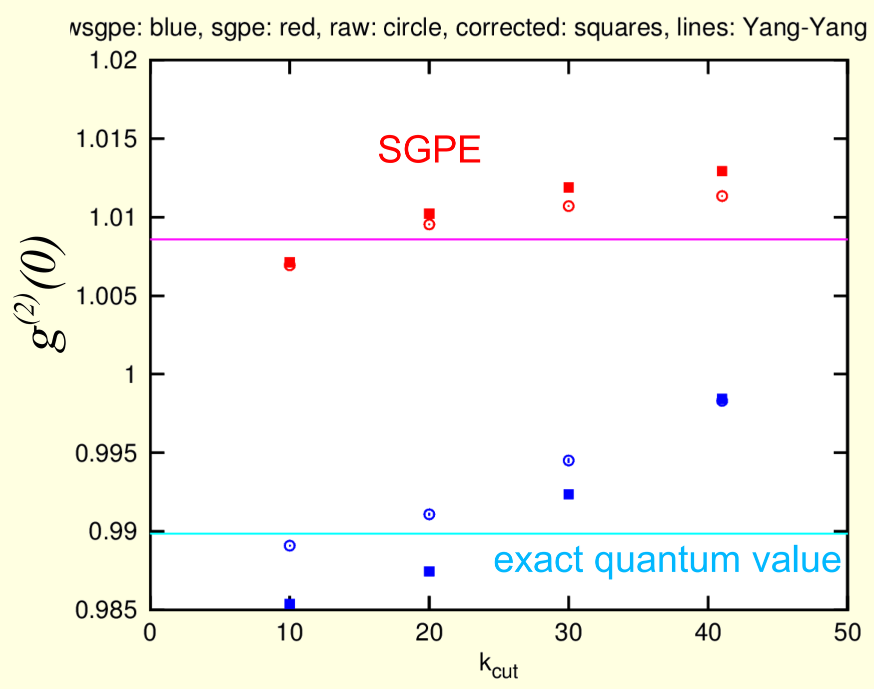
Phase correlations



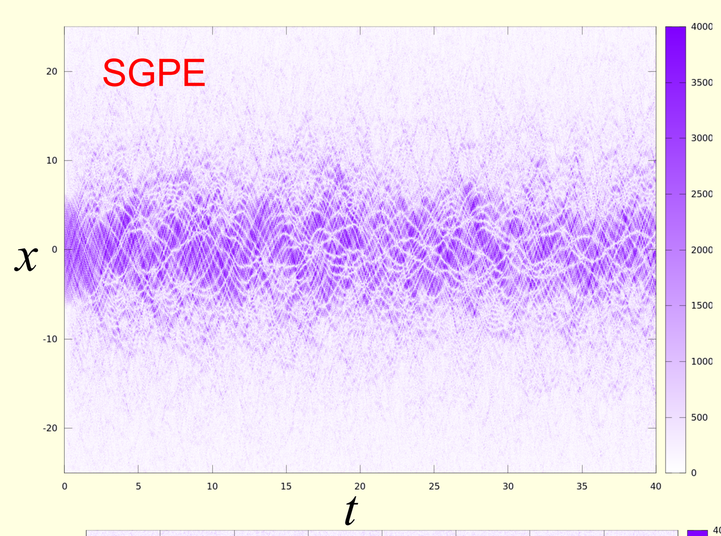
Antibunching



Cutoff dependence



Single shot dynamics



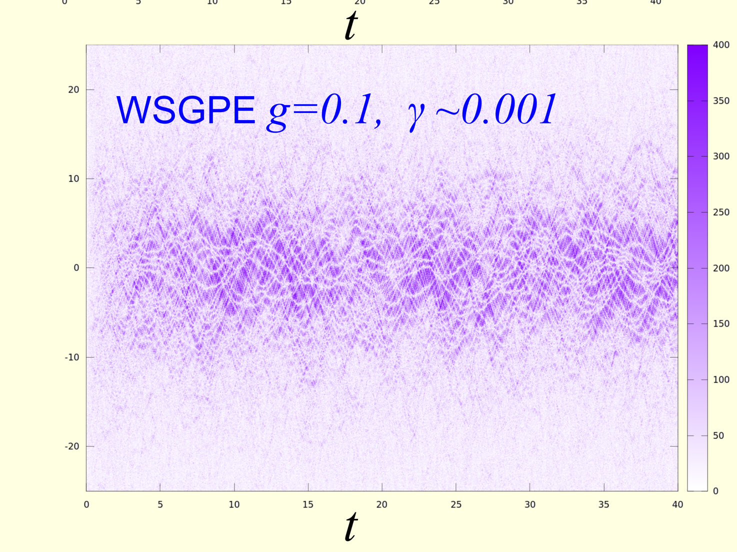
Strong soliton regime

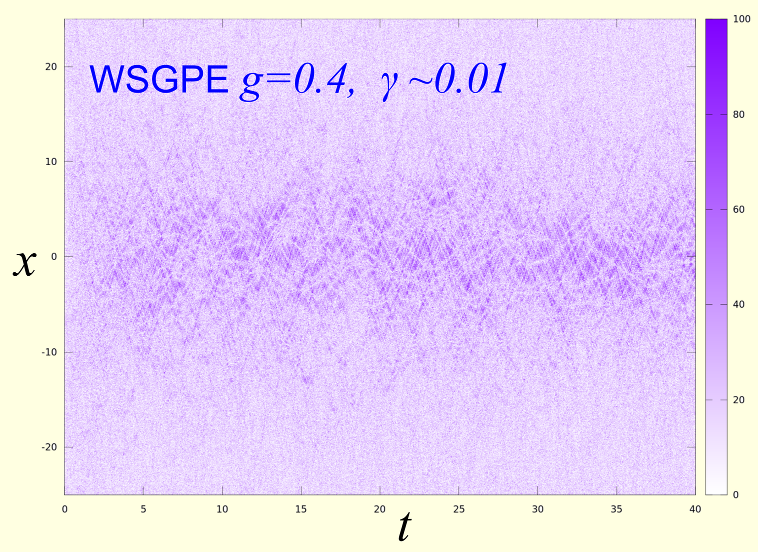
$\tau/\sqrt{\gamma}\approx1.0$, $\mu = 22.4$,

SGPE has $T = 5000 = 5.6T_\Phi$.

$g=0.01$

Large vacuum fluctuations; Solitons still survive





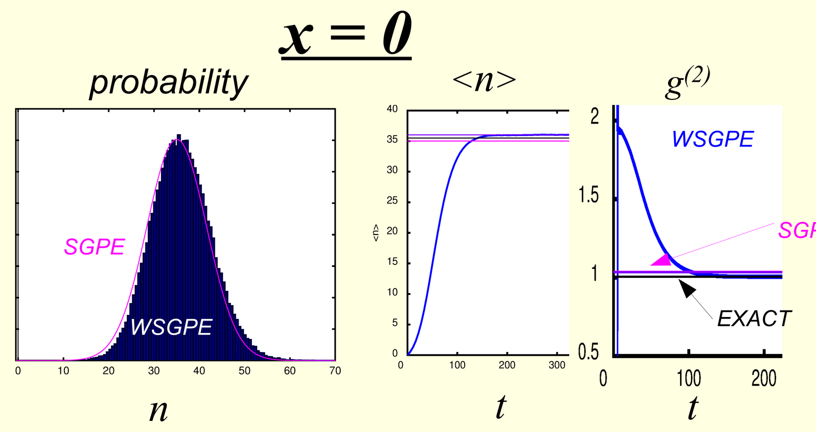
1 mode toy model

$$\mathcal{L}_{GP} = \frac{m\,\omega^2x^2}{2} - \mu + g\,|\psi_W|^2$$

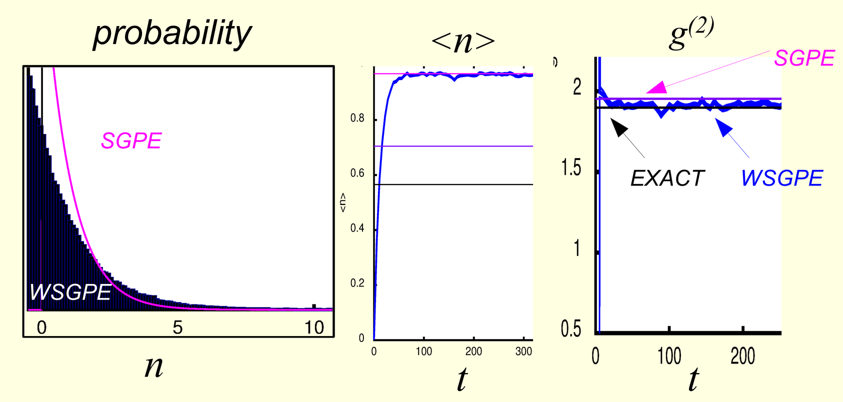
$$\hbar\frac{\partial\psi_W}{\partial t} = -i(1-i\gamma)\left[\mathcal{L}_{GP} - \frac{g}{\Delta x}\right]\psi_W + \sqrt{\gamma\hbar\left[2k_BT + \mathcal{L}_{GP} - \frac{g}{\Delta x}\right]}\frac{\eta(t)}{\Delta x} + i\sqrt{\gamma\hbar g}\,\frac{\xi(t)}{\Delta x}\psi_W$$

Long times, $\gamma=1.1\times10^{-4}$ $\tau = 2.8\times10^{-4}$ in centre

x = 0



x = 5a_ho



Questions

- What distribution is reached in equilibrium?
- What do the multiplicative phase noise terms do?
- Is the cutoff dependence the same as in the SGPE?
- At what point does Wigner truncation affect results?
- A better form of the noise can be obtained that applies also at low k_BT

We acknowledge the support of the NCN grant 2012/07/E/ST2/01389