The accuracy and applicability of classical fields to quantum systems

J. Pietraszewicz[1], P. Deuar[1]

[1]Instytut Fizyki PAN, Warszawa, Poland

NARODOWE CENTRUM NAUKI

Classical Fields

Full quantum field → Ensemble of complex-fields

$$\hat{\Psi}(\mathbf{x}) = \sum_k \hat{a}_k \, \psi_k(\mathbf{x}) \;\rightarrow\; \left\{ \sum_{k \epsilon C} \xi_k \, \psi_k(\mathbf{x}) \right\}$$

THE DREADED CUTOFF k_c

Assume highly occupied modes
Replace mode amplitude operators $\hat{a}_k$
With complex numer amplitudes ξ_k
„Quantum field theory, without discretized particles"

For many problems c-fields are the only viable method (especially when single realizations are needed) **[2],[3]**.
Perennial questions:
(*) Fine, but, are the effects real?
(**) is it quantitative or only qualitative?
(***) what was the cutoff used?

Abstract: Classical or c-fields are a way to tractably describe the thermal state of a quantum Bose gas with an ensemble of complex wave functions. They are widely used as one of the only practical methods for quasicondensates and partially condensed gases but much uncertainty over their accuracy remains. We benchmark many observables together to the exact solutions for a local density. This allows us to confidently determine the range of validity and specify good values for the cuttoff parameter that is essential for accurate results **[4].**

Units:

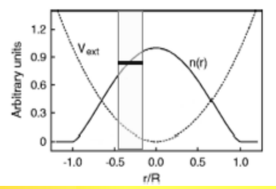
Dimensionless temperature $\tau = \frac{T}{T_d}$ (Ideal gas degeneracy temperature)

Dimensionless cutoff $f_c = \frac{k_c}{k_T}$

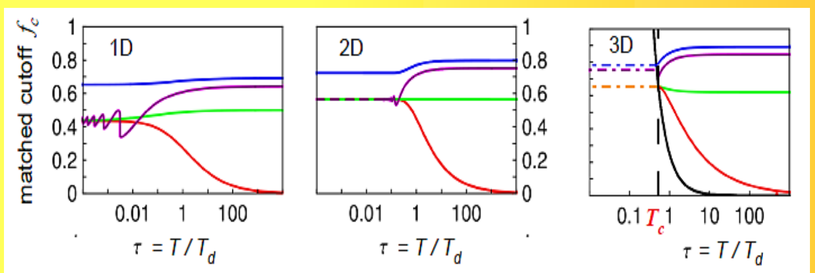
$k_T = \frac{2\pi}{\Lambda_T}$

Thermal de Broglie wavelength $\Lambda_T = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$

- Local Density approximation (LDA)
  → Grand Canonical ensemble
  (rest of gas acts as a reservoir)



Eigencutoff matched to a single observable



- E_{kin} Kinetic energy per particle
- varN / N Coarse-grained fluctuations

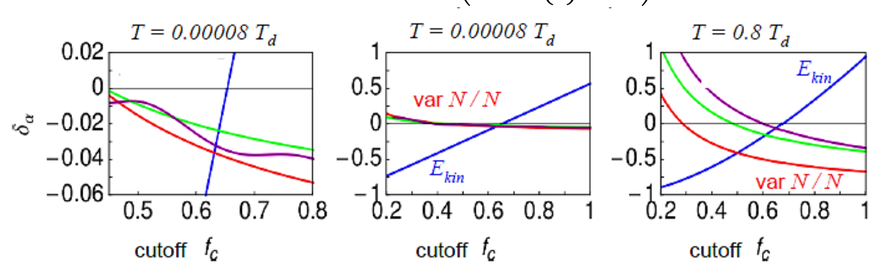
← Most extreme behaviour

- l_{pg} phase grain volume (~ coherence length l_ϕ)
- Half-width of $g^{(1)}(x)$
- ρ_0 condensate fraction

ERRORS relative and global estimator

How does a non-optimal choice of fc affect the observables, and their systematic error? This is relevant for practical considerations. For one thing, in a nonuniform system, when the cutoff is matched in one spatial region, it is good to know the sensitivity of results in other regions with a different density on this choice of cutoff fc. Furthermore, this information helps to judge how good the classical fields are in describing the system overall.

Single observable error $\delta_\alpha(\tau, f_c) = \frac{\Delta\alpha}{\alpha} = \left(\frac{\alpha^{(cf)}(\tau, f_c)}{\alpha^{(id)}(\tau)} - 1 \right)$



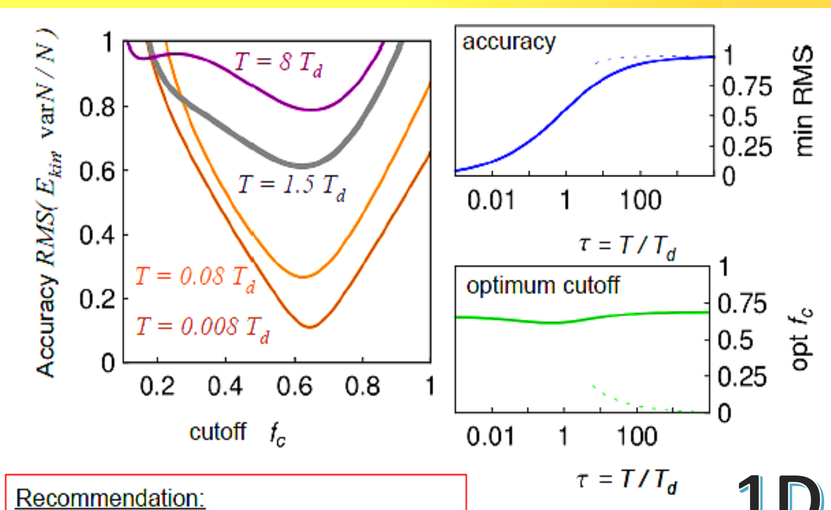
Global error $RMS_{\alpha,\beta,\ldots}(\tau, f_c) = \sqrt{(\delta_\alpha)^2 + (\delta_\beta)^2 + \cdots}$

Error in any observable will be < RMS

Kinetic energy and coarse-grained fluctuations capture most extreme behaviour

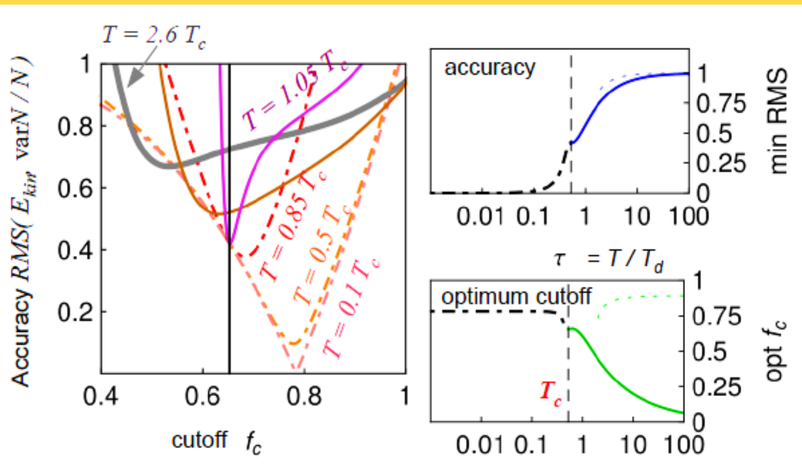
→ ***use these only***

Accuracy and optimal cutoff in **the ideal gas**



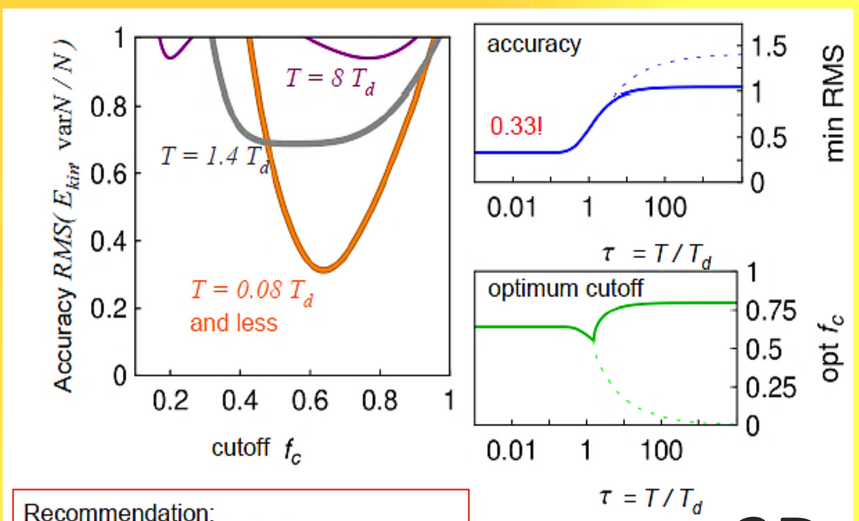
Recommendation:
Accuracy better than 10% for $T < 0.007\ T_d$
Use $f_c = 0.65$ (Energy cutoff = 1.3 $k_B T$)

1D



Recommendation:
Accuracy better than 10% for $T < 0.49\ T_c$
Use $f_c = 0.78$ (Energy cutoff = 1.9 $k_B T$)

3D



Recommendation:
Don't use classical fields,
at the least not near the ideal gas regime

2D

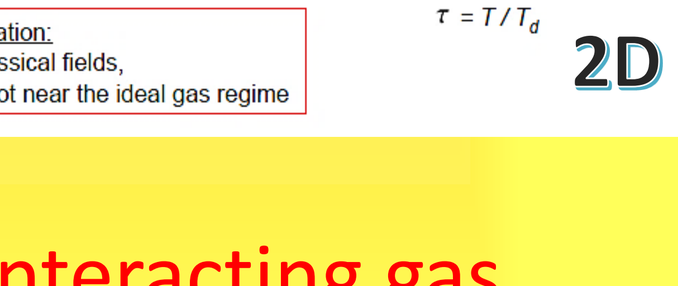
Forward to the interacting gas

(compare to exact Yang-Yang solution [1] in 1D system)

Exact γ dependence in interacting gas!

Interaction strength $\gamma = \frac{g}{n}$

Temperature $\tau = \frac{T}{T_d} = 0.0016$



Quite similar to ideal gas: 5% accuracy same energy cutoff ~1.3 k_bT

Summary:

- Cutoffs and accuracy depend strongly on the observable. *Kinetic energy and coarse-grained density fluctuations are most incompatible.*
- We found the temperatures and best cutoff for which a consistent and accurate c-field description exists in 1D and 3D. *However, the 2D ideal gas is never well described.*
- Above a crossover region near the degeneracy temperature, estimates for different quantities can no longer be reconciled, explicitly showing the way that the classical field description breaks down.
- Preliminary results in the interacting quasicondensate: *Same cutoff as ideal gas, <10% accuracy also possible.*

[1] Yang,Yang, J.Math. Phys. **10**, 1115 (1969)
[2] T. Karpiuk et al., *PRL* **109**, 205302 (2012)
[3] E. Witkowska, M.Gajda, K. Rzążewski, *PRA* **79**, 033631 (2009)
[4] J. Pietraszewicz, P. Deuar, arXiv: 1504. 06154

Support: National Science Centre grant No.2012/07/E/ST2/01389