Ultra cold atoms and moleculs

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Warszawa, 03 November, 2016

Why ultra low temperatures?

Why ultra low temperatures?

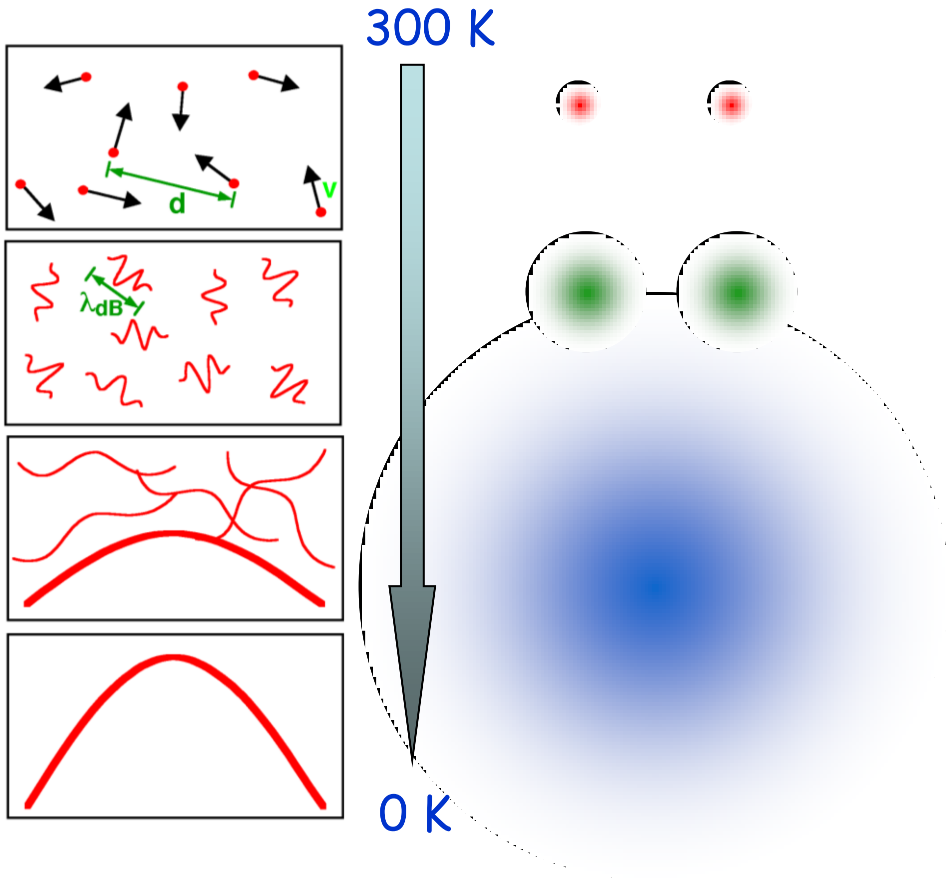
To see the quantum world
of large objects – atoms or molecules.

$$h = 6.626 \times 10^{-34}\, J \cdot s$$

For these balls here, however, this constant is much larger—about unity—and you may easily see with your own eyes phenomena which science succeeded in discovering only by using very sensitive and sophisticated methods of observation.’

George Gamov „Mr Tompkins in Wonderland”

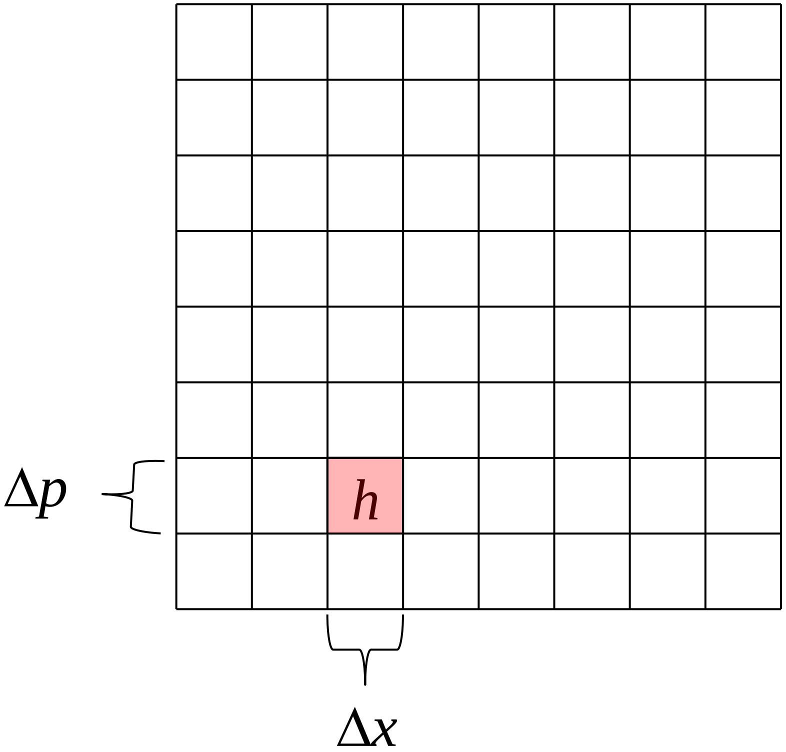
As atoms get colder, they start to behave more like waves and less like particles.

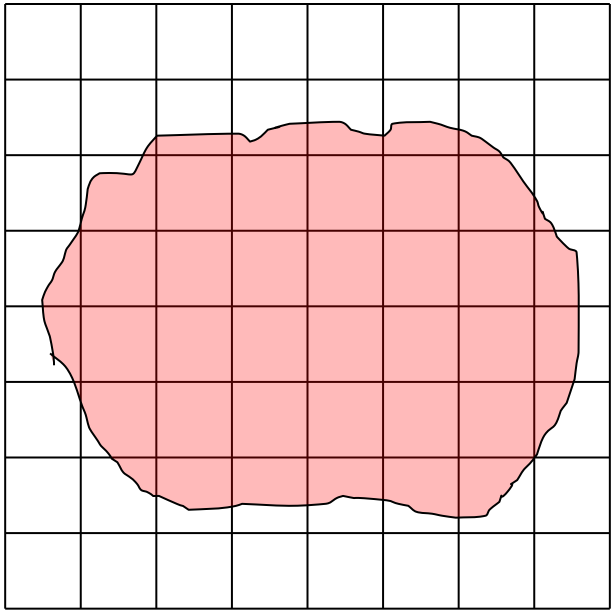


Quantum scale

Heisenberg principle

$$\Delta x \, \Delta p \geq h$$

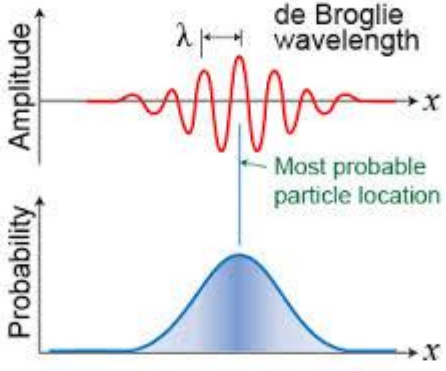




Quantum world

Heisenberg uncertainty principle:

$$\Delta x\,\Delta p \geq h \Rightarrow \lambda\,\Delta p \approx h$$



Indistinguishability:

$$\lambda \approx d = \frac{1}{n^{1/3}}$$

Quantum degeneracy temperature:

$$k_B T_{Quantum} = \frac{\Delta p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m} n^{2/3}$$

Some numbers

$$k_B T_{Quantum} = \frac{h^2}{2m} n^{2/3}$$

Electrons in metal:

$$n \approx 10^{23} \mathrm{cm}^{-3}$$

$$T_{Quantum} = 10^5 K \quad >> \quad T_{room}$$

Atomic gas of the same density:

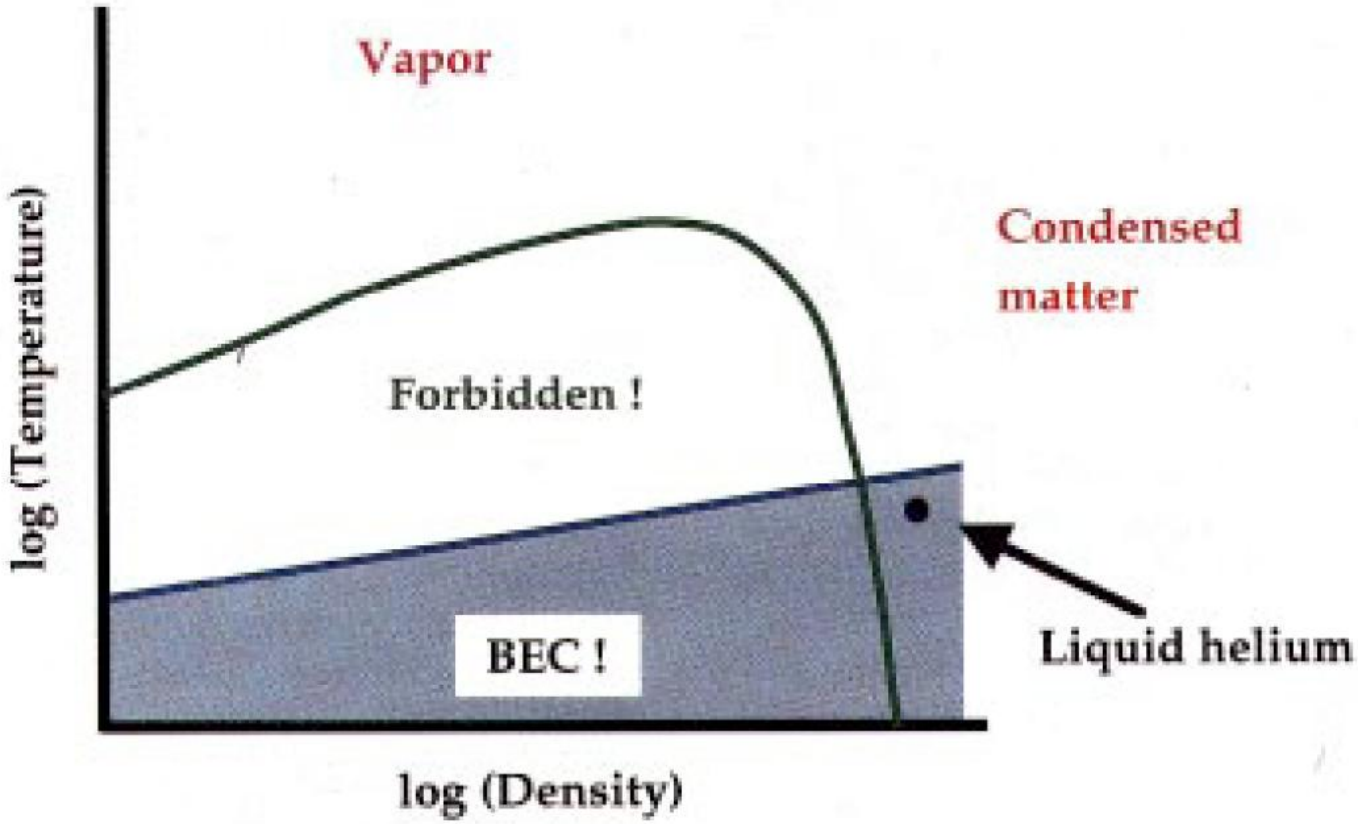
$$T_{Quantum} = 1K - 10K \quad << \quad T_{room}$$

$$k_B T_{Quantum} = \frac{h^2}{2m} n^{2/3}$$

$$T_{Quantum} = 1K - 10K \quad << \quad T_{room}$$

This estimation is WRONG – temperature has to be smaller by about 7 orders of magnitude!!!

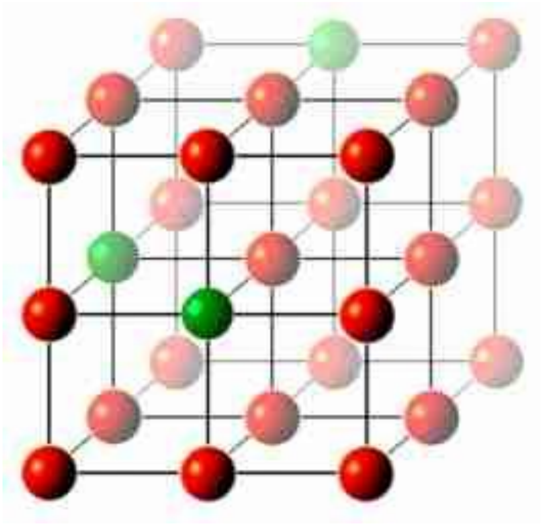
Thermal equilibrium

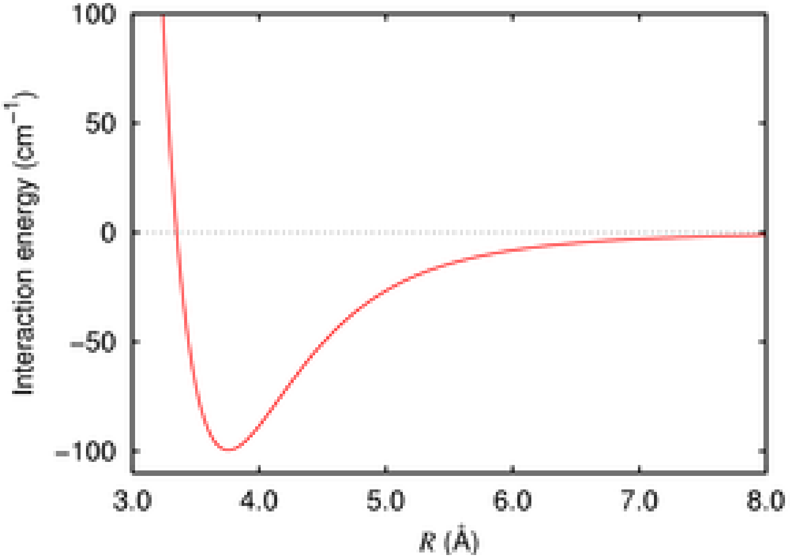


At T=1K and n=10^{23} cm^{-3}

EVERYTHING (but Helium) FREEZES

How to get to the BEC region ?





$$d = 3.5\times 10^{-8}\,cm = 3.5\times 10^{-10}\,m$$

$$d \propto \lambda = \frac{h}{p}$$

Large distance =>
=> Large thermal wavelength
=> Ultra low temperatures

$$d = 10^{-4}\,cm = 10^{-6}\,m$$

$$k_B T_C = \frac{h^2}{2m} n^{2/3}$$

Who is cool?

Electrons in metal:

$$T_{Fermi} = 10^5 K$$

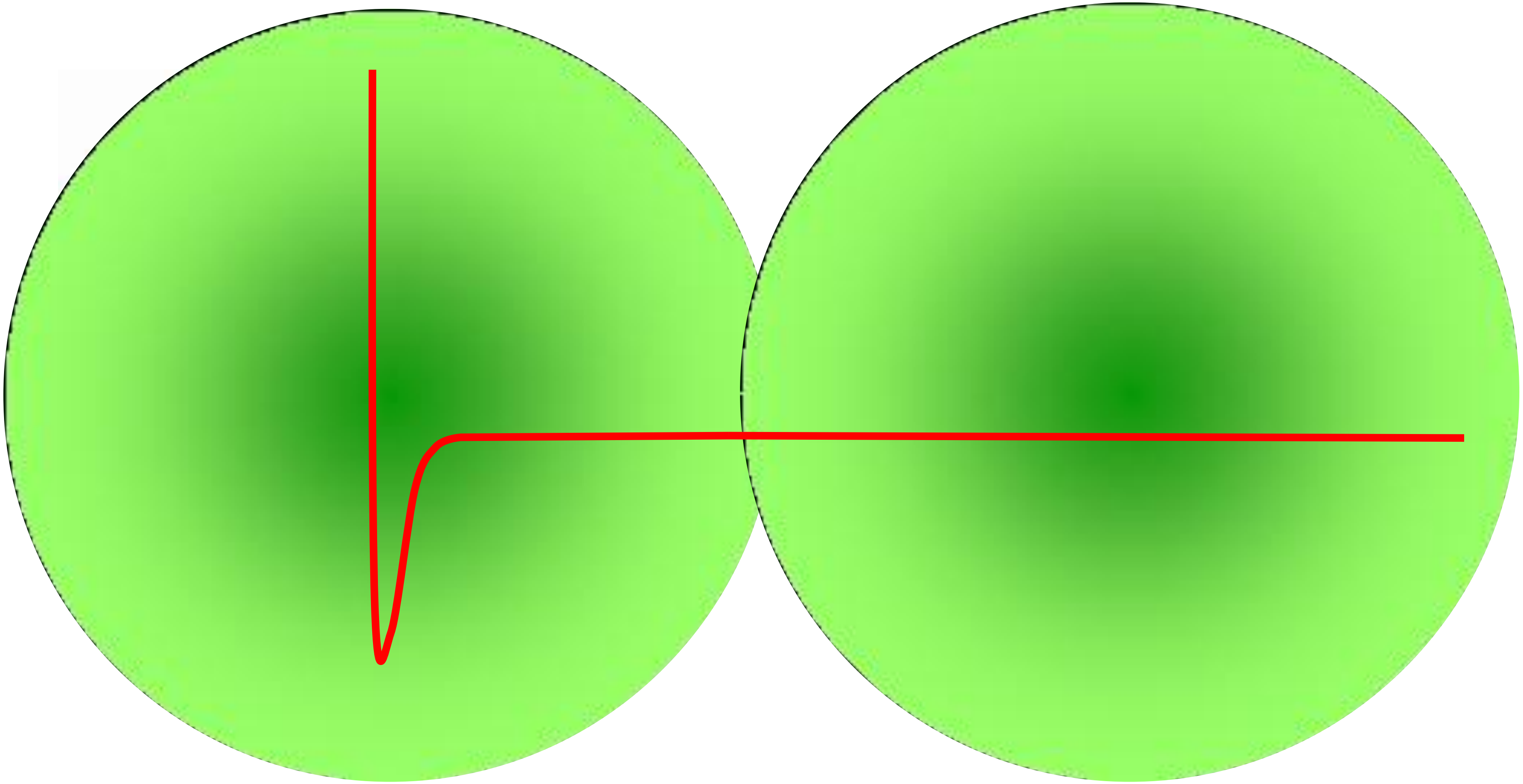
$$v_{Fermi} = 10^6 \mathrm{m/s}$$

Ultra cold atoms:

$$T_c = 10^{-7} K$$

$$v = 1 - 10 \mathrm{\ mm/s}$$

How to get to the BEC region ?



$\mathbf{d} \propto \lambda \propto 10^{-6} m$

$T \propto 100 \times 10^{-7} K$

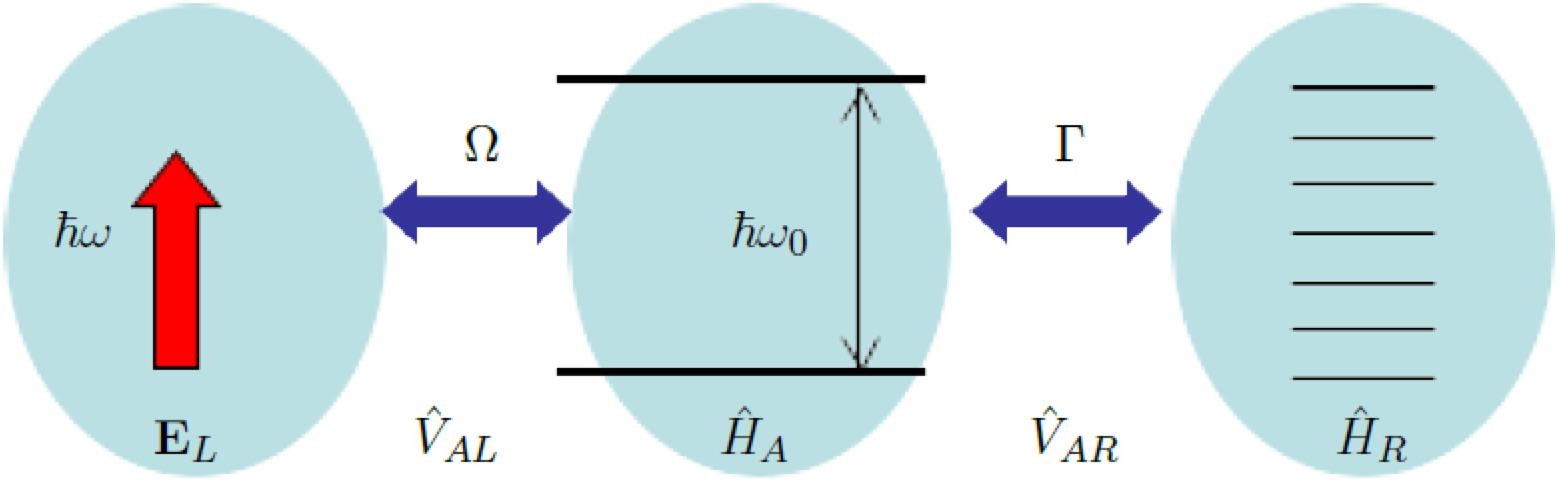
Can one bring atoms to a stop?

Force experienced by an atom in e.m. field:

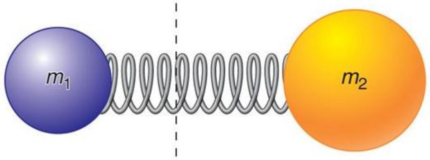
$$F_i(R) = \vec{d}(\vec{R}) \frac{\partial}{\partial R_i} \vec{E}(\vec{R})$$

$$\vec{E}(\vec{R}) = \vec{E}_L(\vec{R}) \cos(\omega t - \vec{k}\vec{R})$$

Force experienced by an atom in e.m. field:



$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}\vec{d}(\vec{R}) = -\left(\omega_0^2 + \Gamma\frac{\mathrm{d}}{\mathrm{d}t}\right)\vec{d}(\vec{R}) + \left(\frac{e^2\vec{E}_L(\vec{R})}{m}\right)\cos(\omega t - \vec{k}\vec{R})$$



Force experienced by an atom in e.m. field:

$$F_i(R) = \vec{d}(\vec{R})\frac{\partial}{\partial R_i}\vec{E}(\vec{R})$$

$$\vec{E}(\vec{R}) = \vec{E}_L(\vec{R})\cos(\omega t - \vec{k}\vec{R})$$

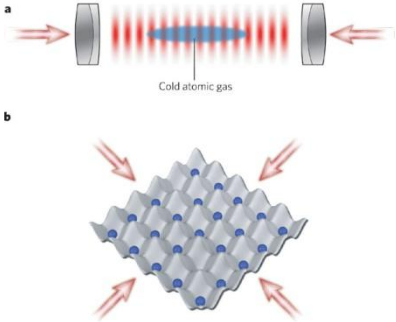
$$\vec{d}(\vec{R},t) = \alpha_1\vec{E}_L(\vec{R})\cos(\omega t - \vec{k}\vec{R}) + \alpha_2\vec{E}_L(\vec{R})\sin(\omega t - \vec{k}\vec{R})$$

$$F_i^{dipole}\left(\vec{R}\right) = \frac{1}{4}\alpha_1\frac{\partial}{\partial R_i}\vec{E}_L^2(\vec{R})$$

$$F_i^{disp}\left(\vec{R}\right) = \frac{1}{2}k_i\alpha_2\vec{E}_L^2(\vec{R})$$

Conservative force (light induced potential)

$$\vec{F}^{\,dipole}\left(\vec{R}\right)=\frac{1}{8}\left(\frac{e^2}{m\omega_0}\right)\frac{\delta}{\delta^2+(\Gamma/2)^2}\frac{\vec{\partial}}{\partial R_i}E_L^2(\vec{R})$$



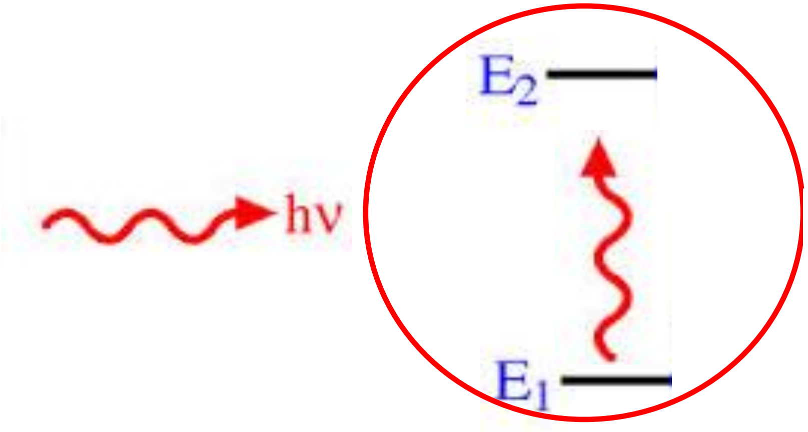
Light pressure force:

$$\vec{F}^{\,diss}\left(\vec{R}\right)=\frac{1}{4}\vec{k}\left(\frac{e^2}{m\omega_0}\right)\frac{\Gamma/2}{\delta^2+(\Gamma/2)^2}E_L^2(\vec{R})$$

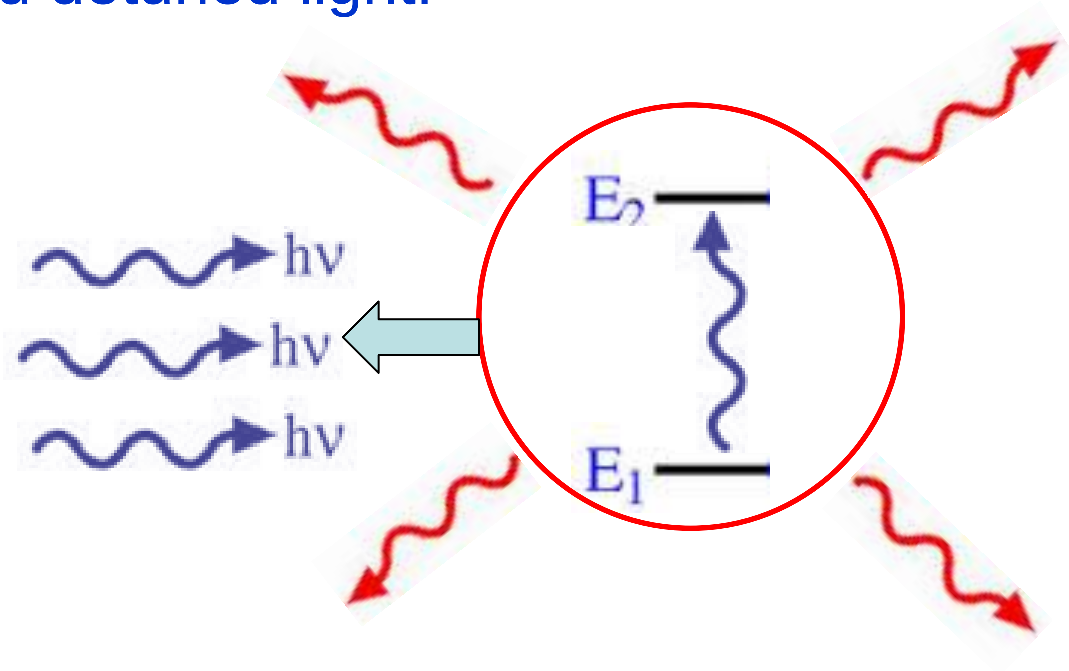
$$\delta=\omega_0-\omega$$

Slowing down atoms

Solution: Doppler effect –
use red detuned light!



Atom at rest:
Photon energy too small



$$v_{recoil} \propto 3\,\text{cm/s (Na)}$$

$$v_{Doppler} \approx 30\text{cm/s } (240\,\mu K)$$

NOBEL PRIZE 1997

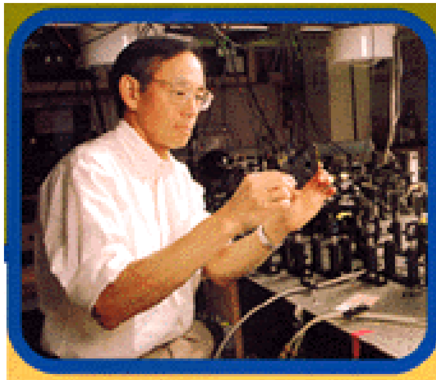


Photo: Linda A. Cicero
Stanford News Service

Steven Chu
Stanford University, Stanford, California, USA

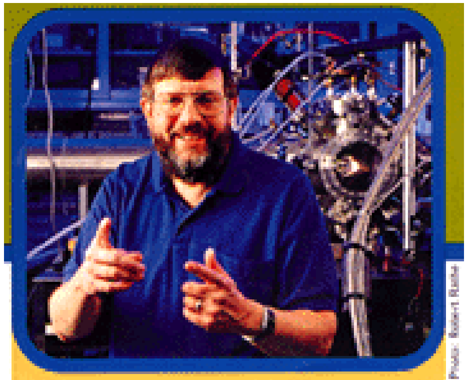


Photo: Robert Rathe

William D. Phillips
National Institute of Standards and Technology, Gaithersburg, Maryland, USA

Photo: Frédéric Reglain

Claude Cohen-Tannoudji
Collège de France and École Normale Supérieure, Paris, France

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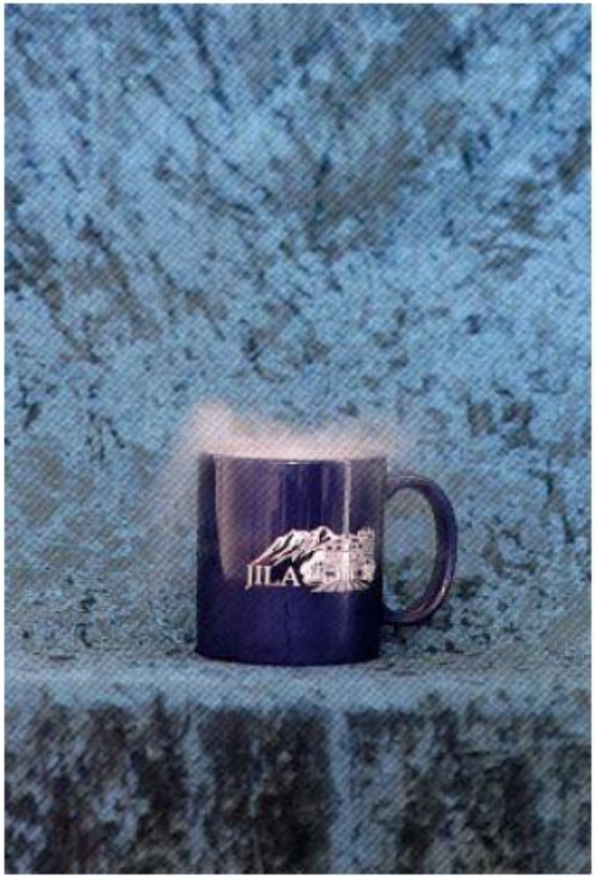
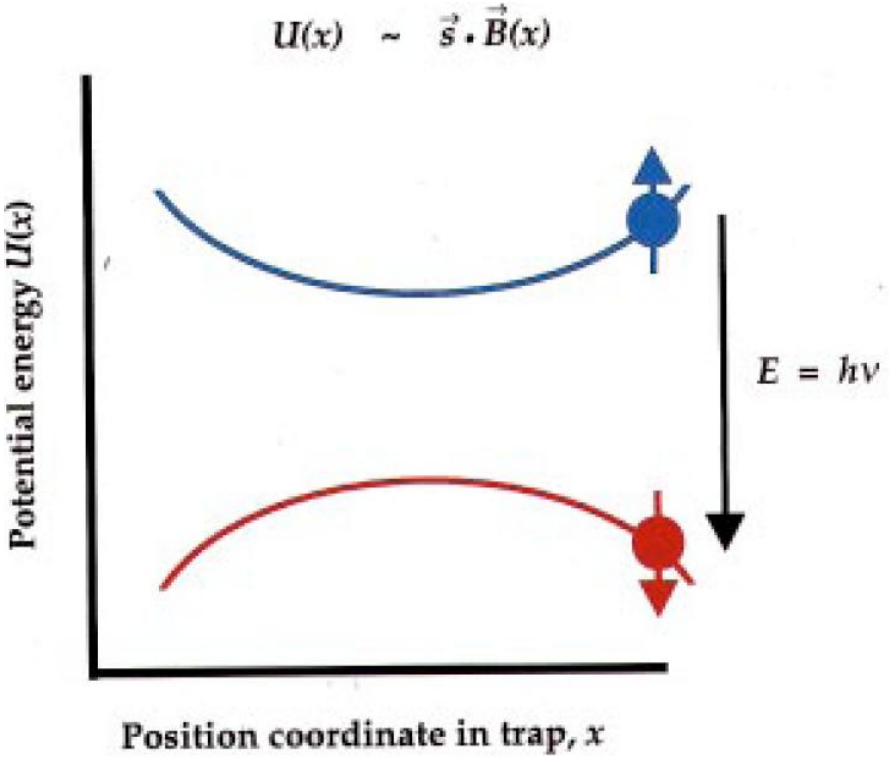
THE WHITE HOUSE
WASHINGTON
WHITE HOUSE
WASHINGTON

Evaporating cooling

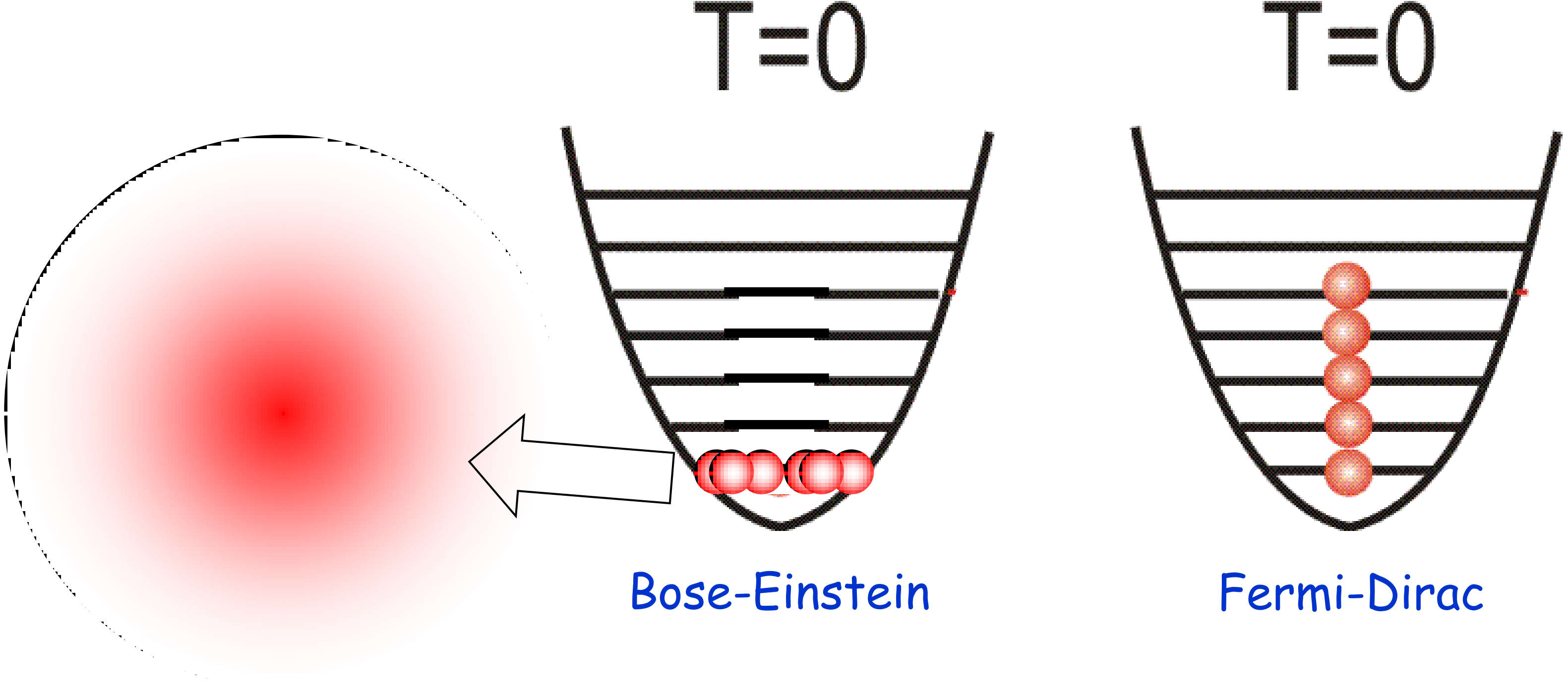




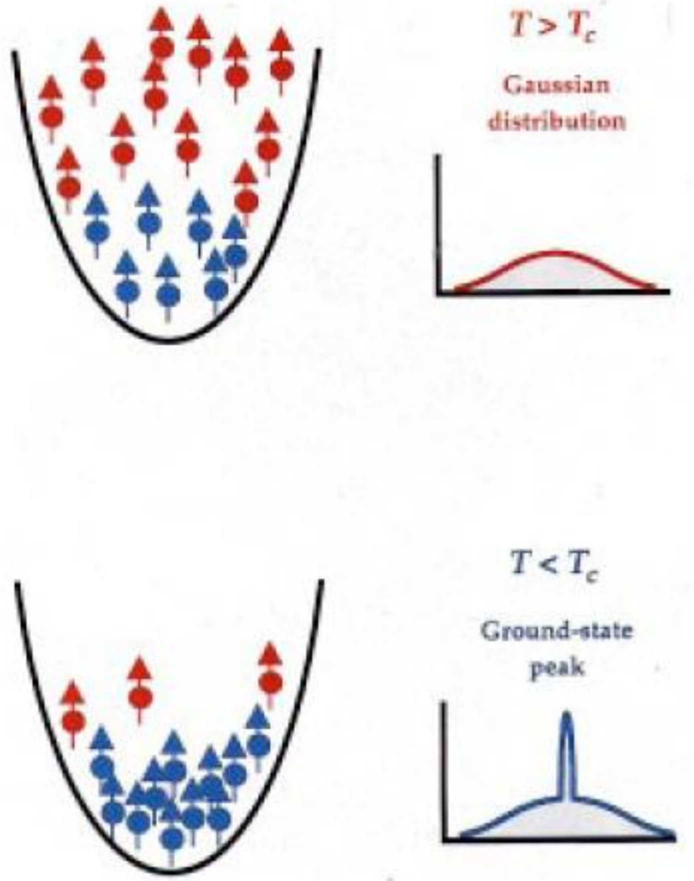
Fermions and Bosons at T=0

Character of atoms depends on the number of neutrons in the nucleus

^{87}Rb, ^{23}Na, ^{39}K - bosons
^{40}K, ^{6}Li - fermions



Bose-Einstein condensation



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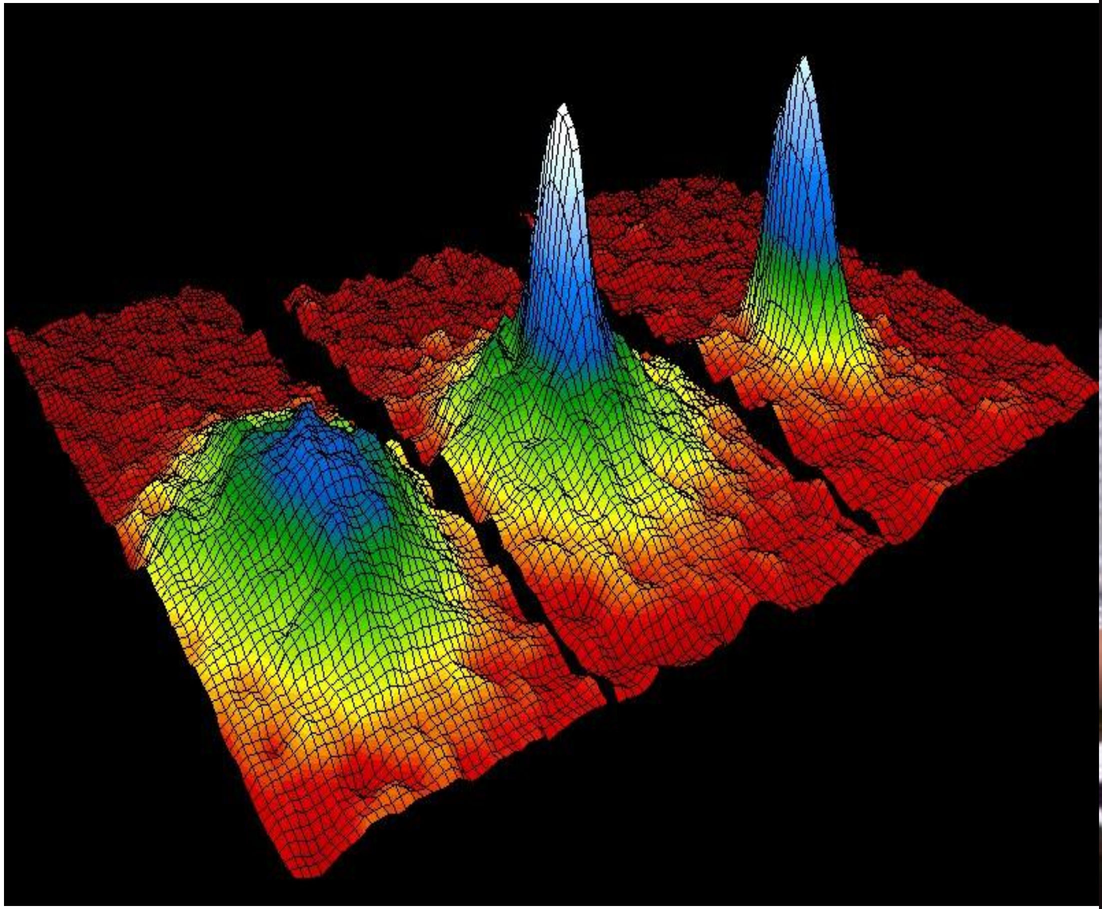
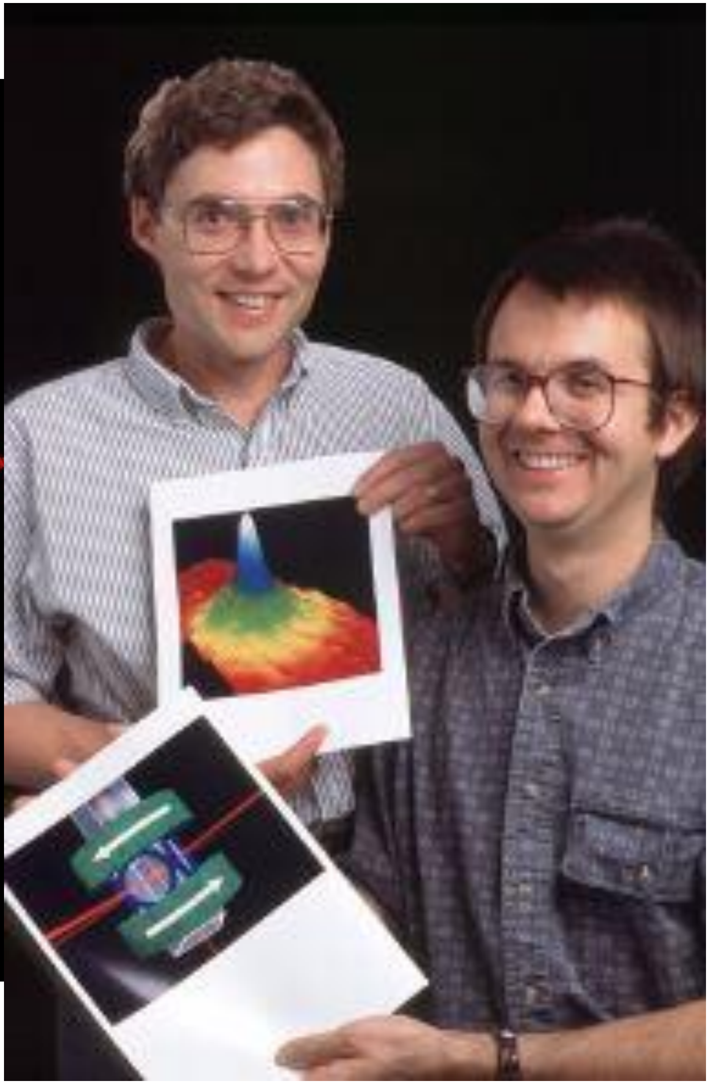
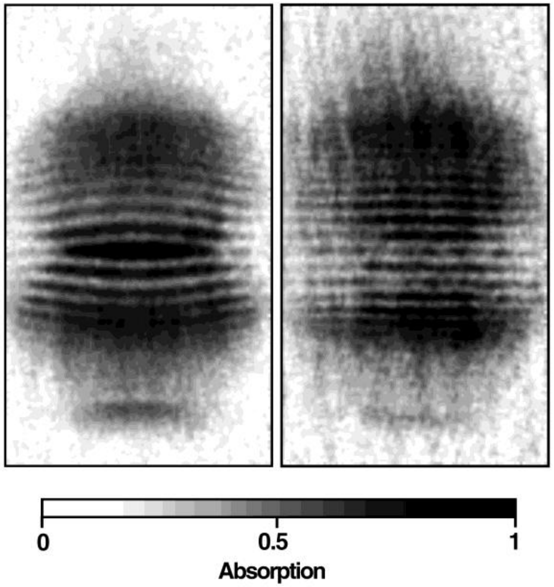




The second condensate – sodium condensate



Interferece of matter waves

The Prize Winners for 2001


University of Colorado at Boulder

E. A. Cornell (USA)
1961-
JILA and NIST
Boulder, Colorado,
USA


EPA/PRB

W. Ketterle (Germany)
1957-
MIT,
Cambridge. Ma,
USA


University of Colorado at Boulder

C. E. Wieman (USA)
1951-
JILA and UC,
Boulder, Colorado,
USA

Gross-Pitaevskii equation

$$\Psi(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \psi_0(\vec{r}_1)\psi_0(\vec{r}_2)\ldots\psi_0(\vec{r}_N)$$

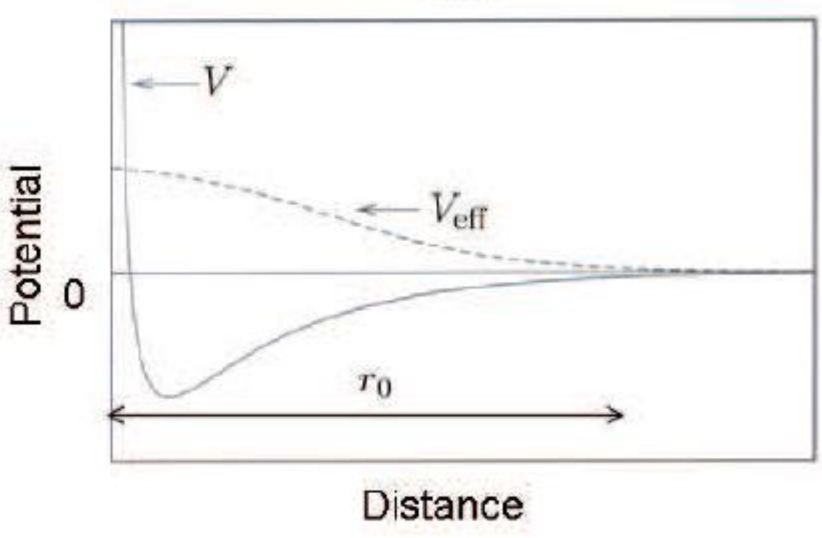
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right)\psi_0(\vec{r}) = \mu\,\psi_0(\vec{r})$$

Gross-Pitaevskii equation

$$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)=\psi_0(\vec{r}_1)\psi_0(\vec{r}_2)\ldots\psi_0(\vec{r}_N)$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2+V(\vec{r})+g\left|\psi_0(\vec{r})\right|^2\right)\psi_0(\vec{r})=\mu\,\psi_0(\vec{r})$$

$$gn\approx 10^{-15}eV-10^{-13}eV$$



Gross-Pitaevskii equation

$$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)=\psi_0(\vec{r}_1)\psi_0(\vec{r}_2)\ldots\psi_0(\vec{r}_N)$$

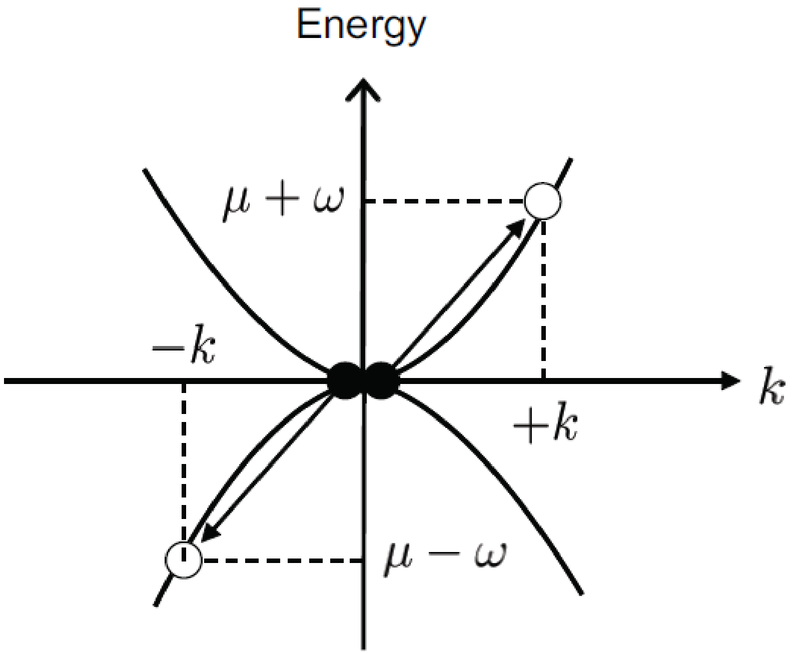
$$\left(-\xcancel{\frac{\hbar^2}{2m}}\nabla^2+\xcancel{V(\vec{r})}+g\left|\psi_0(\vec{r})\right|^2\right)\psi_0(\vec{r})=\mu\,\psi_0(\vec{r})$$

$$\mu=gn$$

$$\psi_0(\vec{r})=\sqrt{n}\,\mathrm{e}^{i\theta}$$

Bogoliubov quasiparticles

$$\psi(\vec{r},t) = e^{-i\mu t}\left(\psi_0(\vec{r}) + \psi_k(\vec{r})e^{-i(\omega t - kr)} + \psi_{-k}(\vec{r})e^{i(\omega t - kr)}\right)$$



Bogoliubov quasiparticles

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$$\begin{pmatrix} \frac{\hbar k^2}{2m} + \mu & \mu \\ -\mu & \frac{\hbar k^2}{2m} - \mu \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi_{-k} \end{pmatrix} = \omega \begin{pmatrix} \psi_k \\ \psi_{-k} \end{pmatrix}$$

$$\omega_k = \sqrt{\frac{\hbar^2 k^2}{2m}\left[\left(\frac{\hbar^2 k^2}{2m} + \mu\right)\right]}$$

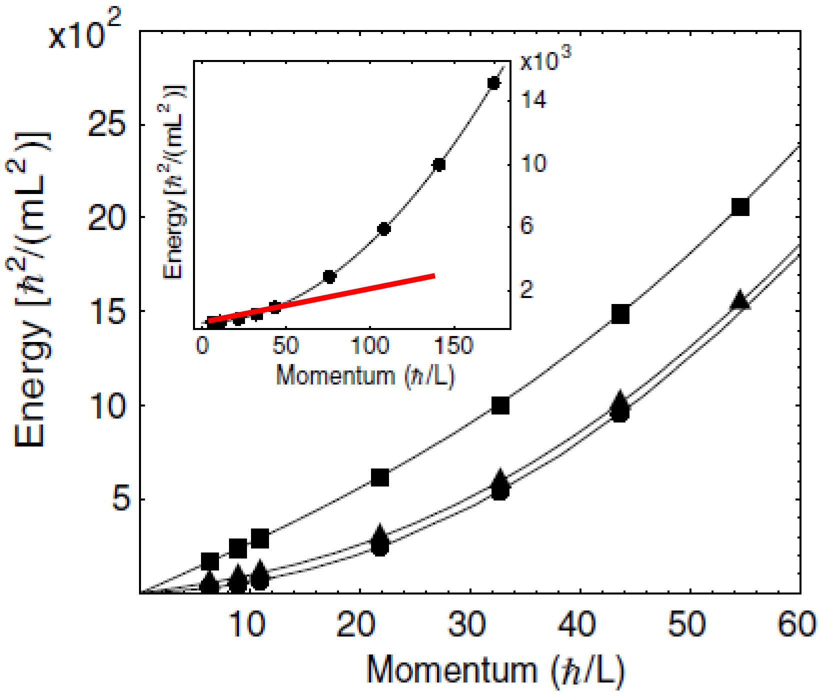
$$\omega_k = \hbar k \sqrt{\frac{gn}{2m}}$$

small k

$$\omega_k = \frac{\hbar^2 k^2}{2m}$$

large k

Bogoliubov spectrum



J. Phys. B: At. Mol. Opt. Phys. **37** (2004) 2725–2738

Landau’s criterion of superfluidity

In the rest frame of the fluid (moving with a velocity V), the excitation energy is

$$E = \mu + \varepsilon(\vec{p})$$

In the rest frame of a capillary:

$$\vec{p}' = \vec{p} + \vec{V}M$$

$$E' = E + \frac{\left(\vec{p} + \vec{V}M\right)^2}{2M} = \mu + \varepsilon(p) + \vec{p}\vec{V} + \frac{MV^2}{2}$$

Dissipation:

$$\varepsilon(p) + \vec{p}\vec{V} < 0$$

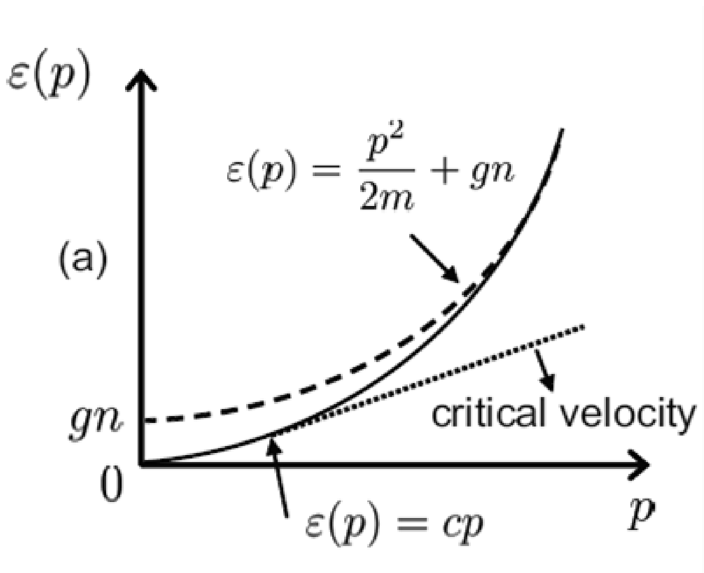
Critical velocity

Dissipation:

$$\varepsilon(p) < pV$$

$$V > \frac{\varepsilon(p)}{|p|}$$

$$V_c = \min_p \frac{\varepsilon(p)}{|p|}$$



BEC is suprfluid!

Quantized vortices

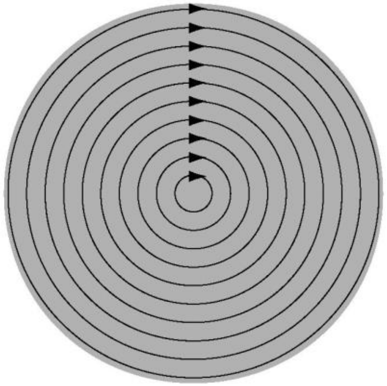
How does it rotate?

$$\Psi(\vec{r},t) = \sqrt{\rho(\vec{r},t)}\, e^{iS(\vec{r},t)} \qquad \vec{V} = \frac{\hbar}{m}\vec{\nabla}S(\vec{r},t)$$

Rigid body:

$$\vec{V}_\varphi = \vec{e}_\varphi r\omega$$

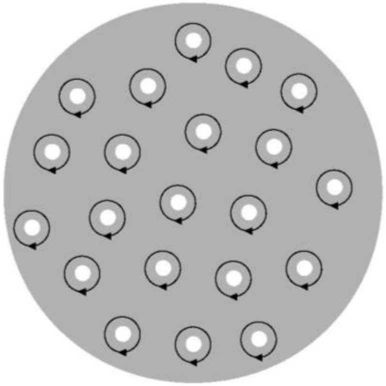
$$\mathrm{rot}\vec{V}_\varphi = \frac{1}{\pi r^2}\oint (r\omega)\mathrm{dl} = 2\omega$$



Superfluid:

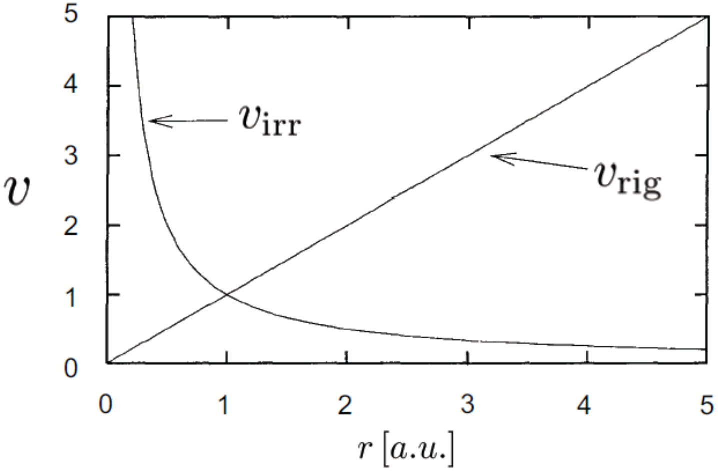
$$\mathrm{rot}\vec{V} = 0$$

$$\oint \vec{\mathrm{V}}\mathrm{d}\vec{\mathrm{l}} = \frac{\hbar}{m}\oint \vec{\nabla}S\, \mathrm{d}\vec{\mathrm{l}} = \left(\frac{\hbar}{m}\right)2\pi n$$

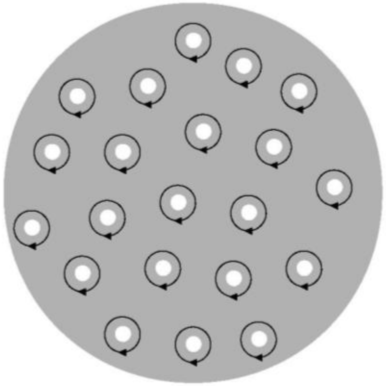


Quantized vortices

$$\Psi(\vec{r},t)=\sqrt{\rho(\vec{r},t)}\,e^{i\varphi}$$



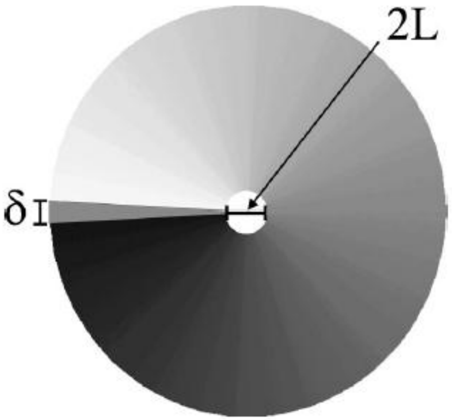
$$V_\varphi=\left(\frac{\hbar}{m}\right)\frac{1}{r}$$



At a position of a vortex a density vanishes, and a velocity goes to infinity.

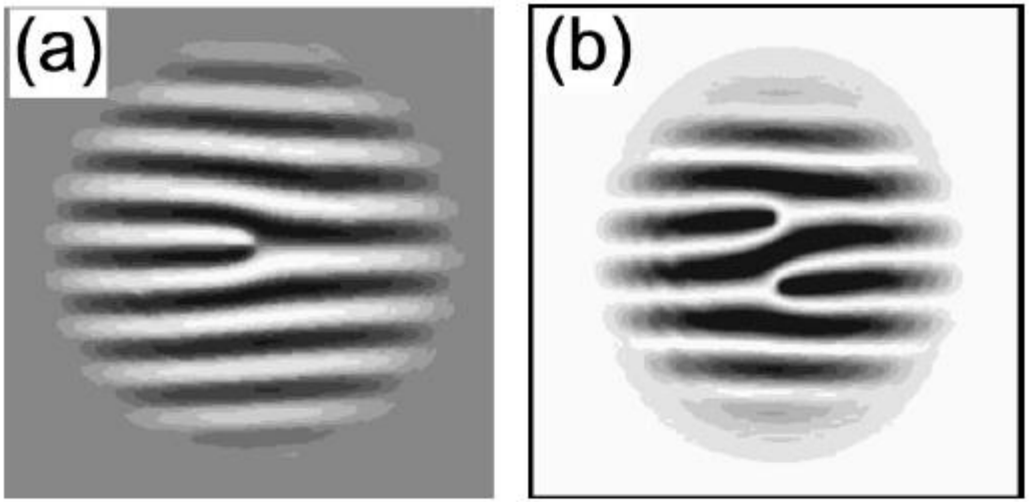
At large distances a fluid does not rotate.

Phase imprinting



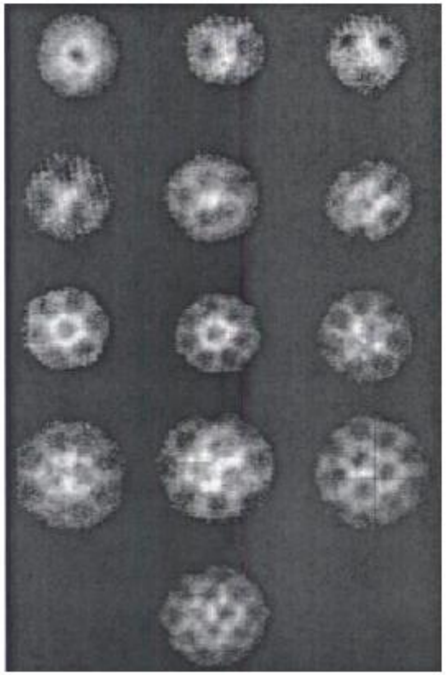
$$\psi(\mathbf{r},T)=\exp[-iTV_l(\rho,\varphi)/\hbar]\,\psi(\mathbf{r},0)$$

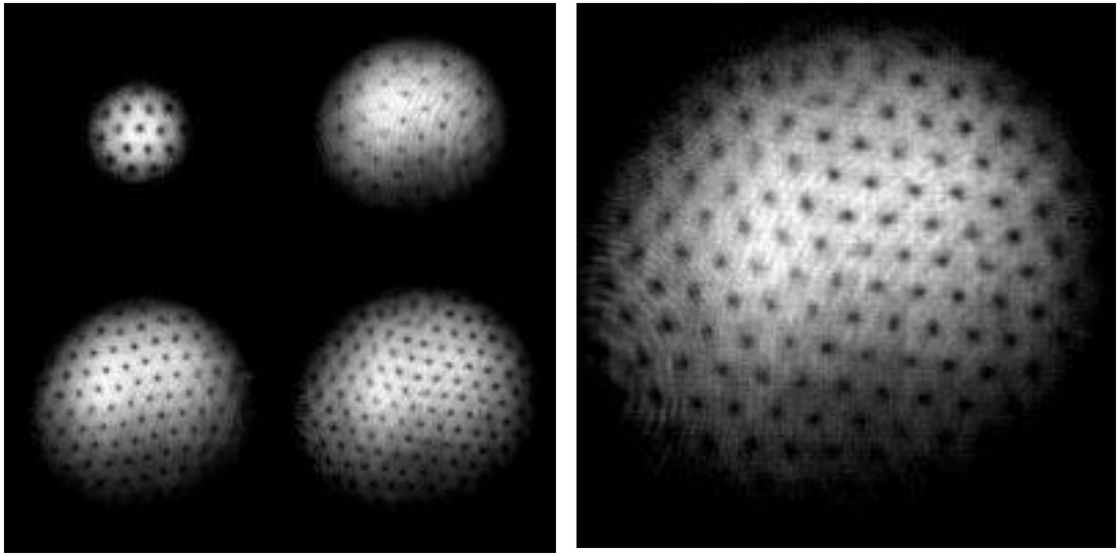
$$\left|\psi(\vec{r},t)+\alpha(r)e^{ikr}\right|^2=\left|\psi(\vec{r},t)\right|^2+\left|\alpha(r)\right|^2+2\,\mathrm{Re}\left(\psi(\vec{r},t)\alpha^*(r)e^{-ikr}\right)$$



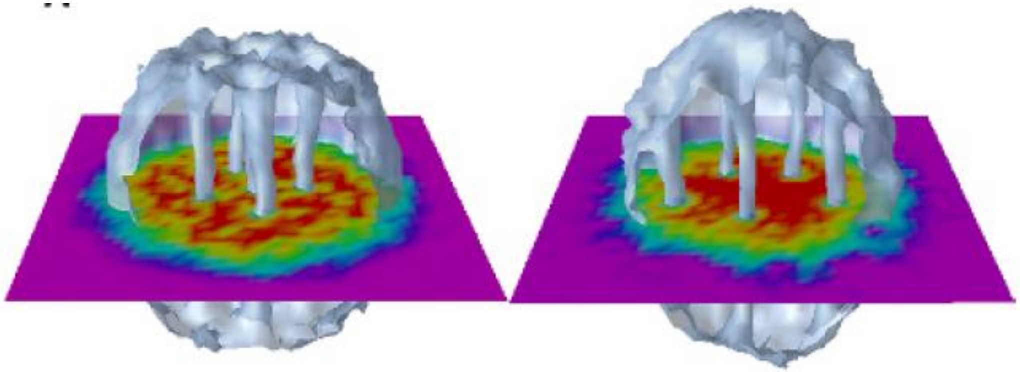
Dobrek. et. Al., PRA R3382, 1999

Quantized vortices





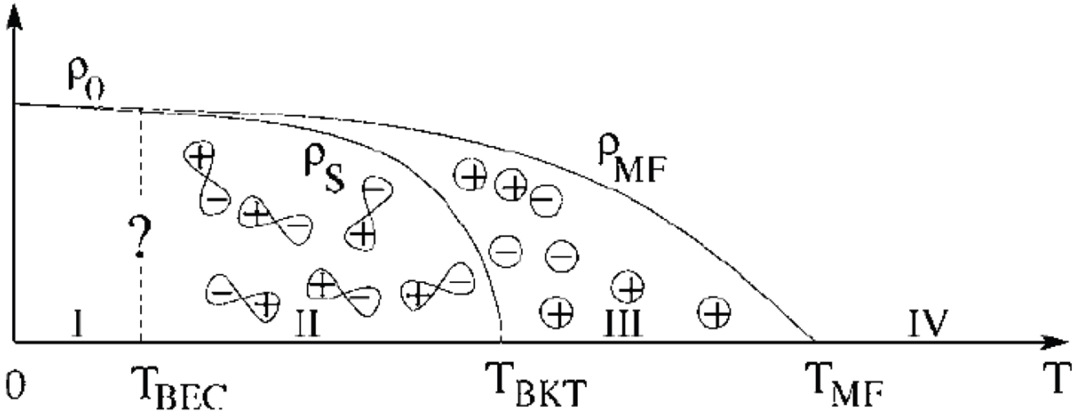
W. Ketterle, Science 2001



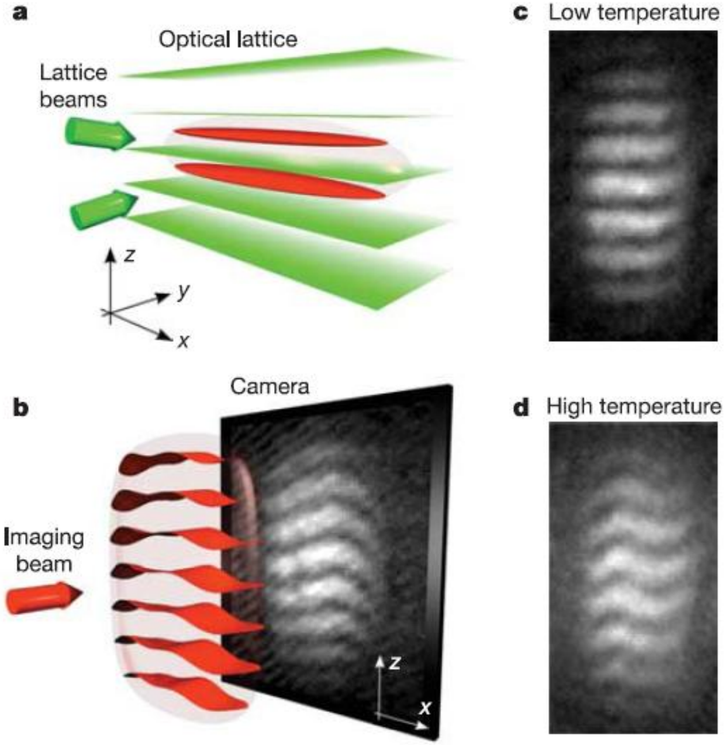
Bose-Einstein condensation in a flatland

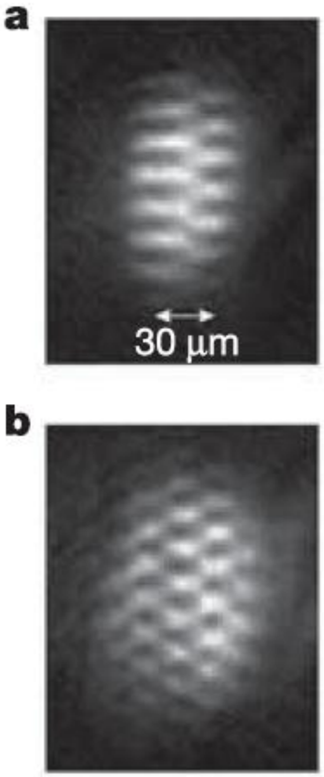
Physics in two dimenssions is intristically different
that in three! (Mermin, Wagner, Hohenberg)

Long-range order is destroyed by thermal fluctuactions at any finite temperature, however the system can become superfluid via a topological order resulting from pairing of vortices.



Berezinskii-Kossterlitz-Thouless transition in atomic gases





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Thank you!