

New approach to a small Fermi-polaron system in a harmonic trap



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ABSTRACT

The experimental realization of an ultracold mixture with a single impurity immersed in several fermions in a one-dimensional harmonic trap motivated theoretical studies of this system. The analysis is particularly challenging in the case of strong inter-component interactions. Methods used to solve this problem (such as an exact diagonalization) require an enormous amount of computational resources. Therefore they are limited to very small mixtures. Here I present an alternative numerically exact approach in which the problem is simplified using an appropriately tailored canonical transformation. The method is especially effective in the case of heavy impurity, where the problem is reduced to a weakly interacting system.

FERMI-POLARON PROBLEM

The analyzed mixture is composed of N fermions and a single impurity with intercomponent mass ratio M . Components interact through contact interactions with strength g . The mixture is trapped in one-dimensional harmonic confinement.

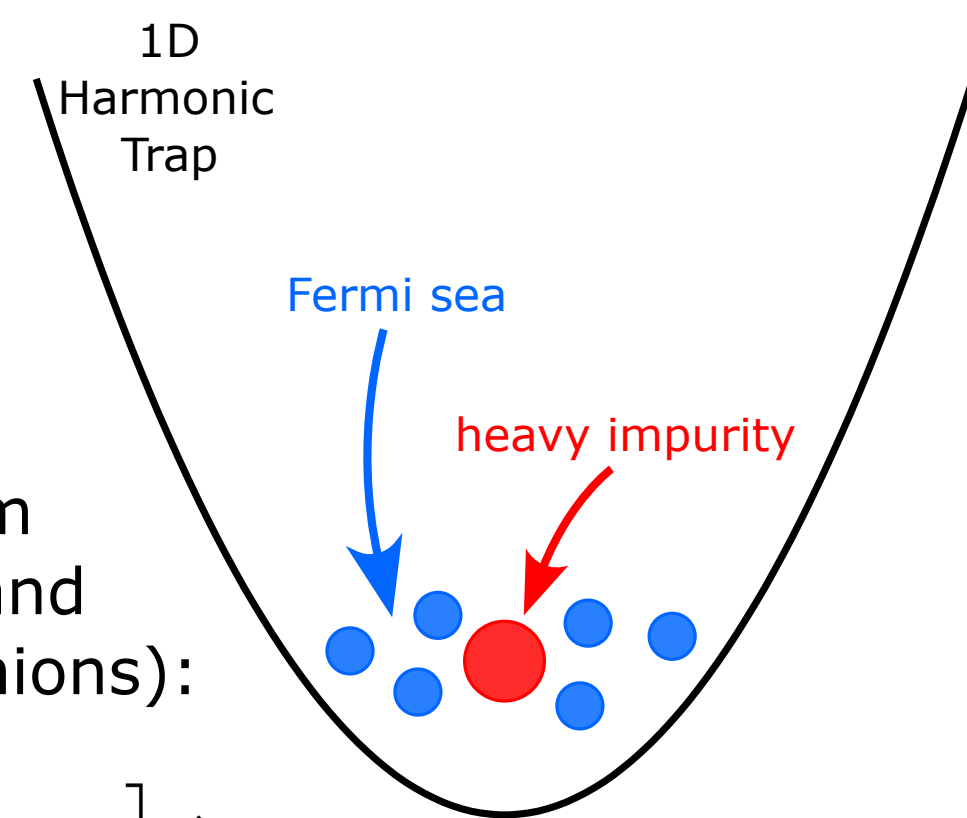
Hamiltonian of the system is described in a formalism that combine position representation (for impurity) and occupation number representation (for majority fermions):

$$\hat{H} = \underbrace{-\frac{1}{2M} \frac{\partial^2}{\partial y^2}}_{\text{impurity}} + \frac{M}{2} y^2 + \int dx \hat{\psi}^\dagger(x) \left[\underbrace{-\frac{1}{2} \frac{\partial^2}{\partial x^2}}_{\text{fermions}} + \frac{1}{2} x^2 + \underbrace{g \delta(x-y)}_{\text{interactions}} \right] \hat{\psi}(x)$$

where $\hat{\psi}(x)$ and $\hat{\psi}^\dagger(x)$ are fermionic field operators:

$$\{\hat{\psi}(x), \hat{\psi}(x')\} = 0 \quad \{\hat{\psi}(x), \hat{\psi}^\dagger(x')\} = \delta(x-x')$$

In the analysis, we focus on the case of heavy impurity ($M > 1$) in the strong-interaction regime ($g > 1$).



COORDINATE TRANSFORMATION

To simplify the analysis of the problem, we change the coordinate system using a unitary operator:

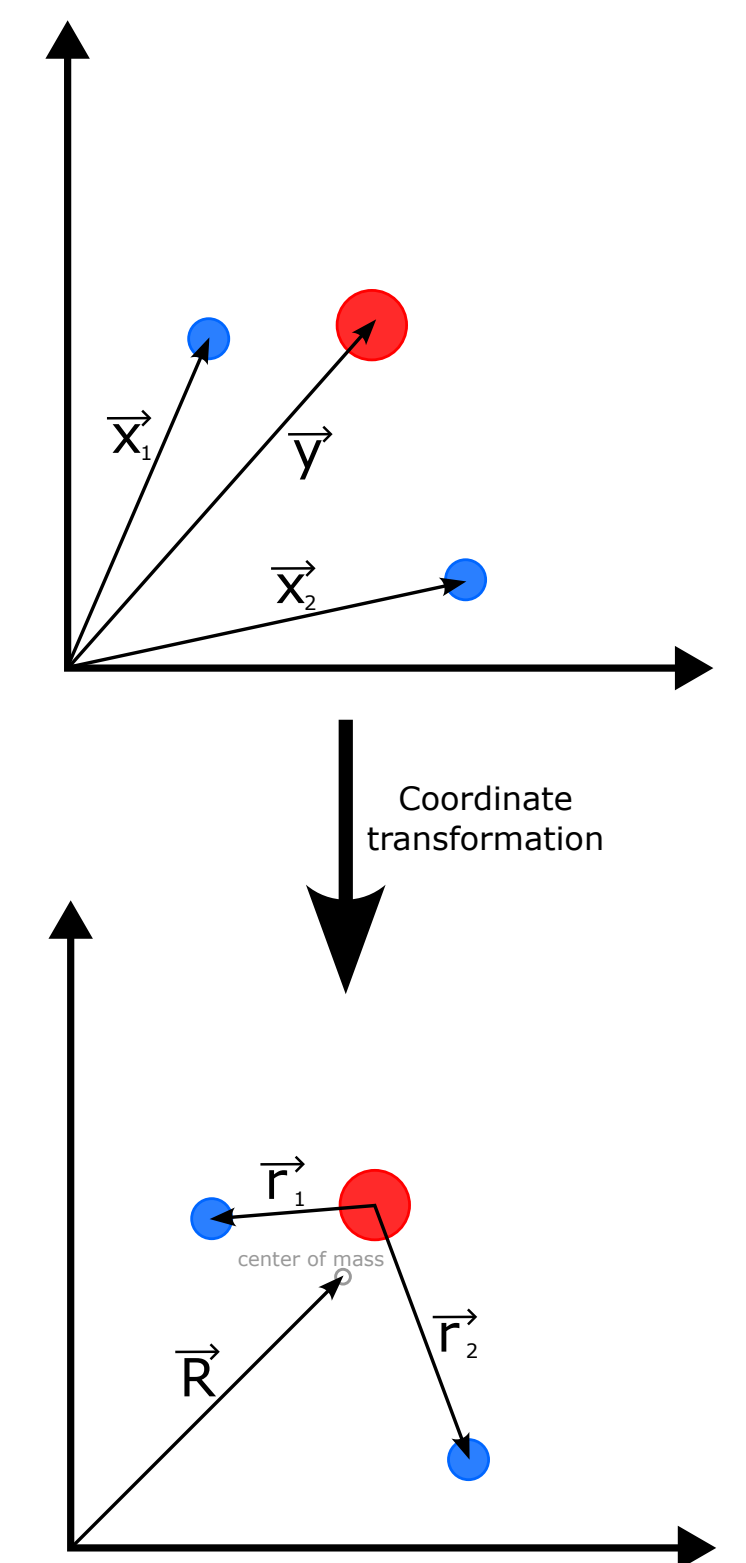
$$\hat{S}(y, \partial_y) = \exp(-iy\hat{p}) \exp\left(\frac{\hat{x}\partial_y}{M+N}\right)$$

where $\hat{x} = \int dx \hat{\psi}^\dagger(x)x\hat{\psi}(x)$ and $\hat{p} = \int dx \hat{\psi}^\dagger(x)(-i\partial_x)\hat{\psi}(x)$.

As a result of the transformation, the coordinates change to those of the center of mass and relative distances from the impurity:

$$y \rightarrow \hat{S}y\hat{S}^{-1} = \frac{M}{M+N}y + \frac{1}{M+N}\hat{x}$$

$$\hat{x} \rightarrow \hat{S}\hat{x}\hat{S}^{-1} = \hat{x} - Ny$$



CENTER-OF-MASS SEPARATION

The transformation separates the center-of-mass motion from the relative motion:

$$\hat{H} \rightarrow \hat{S}^{-1}\hat{H}\hat{S} = \hat{H}_{CM} + \hat{H}_{rel}$$

The center-of-mass Hamiltonian describes a harmonic oscillator:

$$\hat{H}_{CM} = -\frac{1}{2(M+N)} \frac{\partial^2}{\partial y^2} + \frac{M+N}{2} y^2$$

Consequently, its eigenfunctions and eigenenergies are equal:

$$\Phi_n(y) \propto H_n(\sqrt{M+N}y) e^{-\frac{(M+N)y^2}{2}}$$

$$E_n = n + \frac{1}{2}$$

where $H_n()$ is the n th Hermite polynomial.

RELATIVE MOTION

Hamiltonian for the relative motion describes the system of N interacting fermions:

$$\hat{H}_{rel} = \int dx \hat{\psi}^\dagger(x)h(x)\hat{\psi}(x) + \int dx \int dx' \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x')V(x,x')\hat{\psi}(x')\hat{\psi}(x)$$

$$h(x) = -\frac{1}{2} \frac{M+1}{M} \frac{\partial^2}{\partial x^2} + \frac{1}{2} \frac{M+N-1}{M+N} x^2 + g\delta(x)$$

$$V(x,x') = -\frac{1}{2M} \frac{\partial^2}{\partial x \partial x'} - \frac{1}{2(M+N)} xx'$$

Properties of this Hamiltonian:

- it has one less particle than the full Hamiltonian;
- contact interactions are incorporated into the single-particle term;
- the single-particle term has analytical solutions;
- effective interactions are inversely proportional to mass ratio M ;

Eigenvectors $|\Omega_m\rangle$ and eigenenergies ϵ_m are calculated using an exact diagonalization

ENERGIES

Eigenenergies:

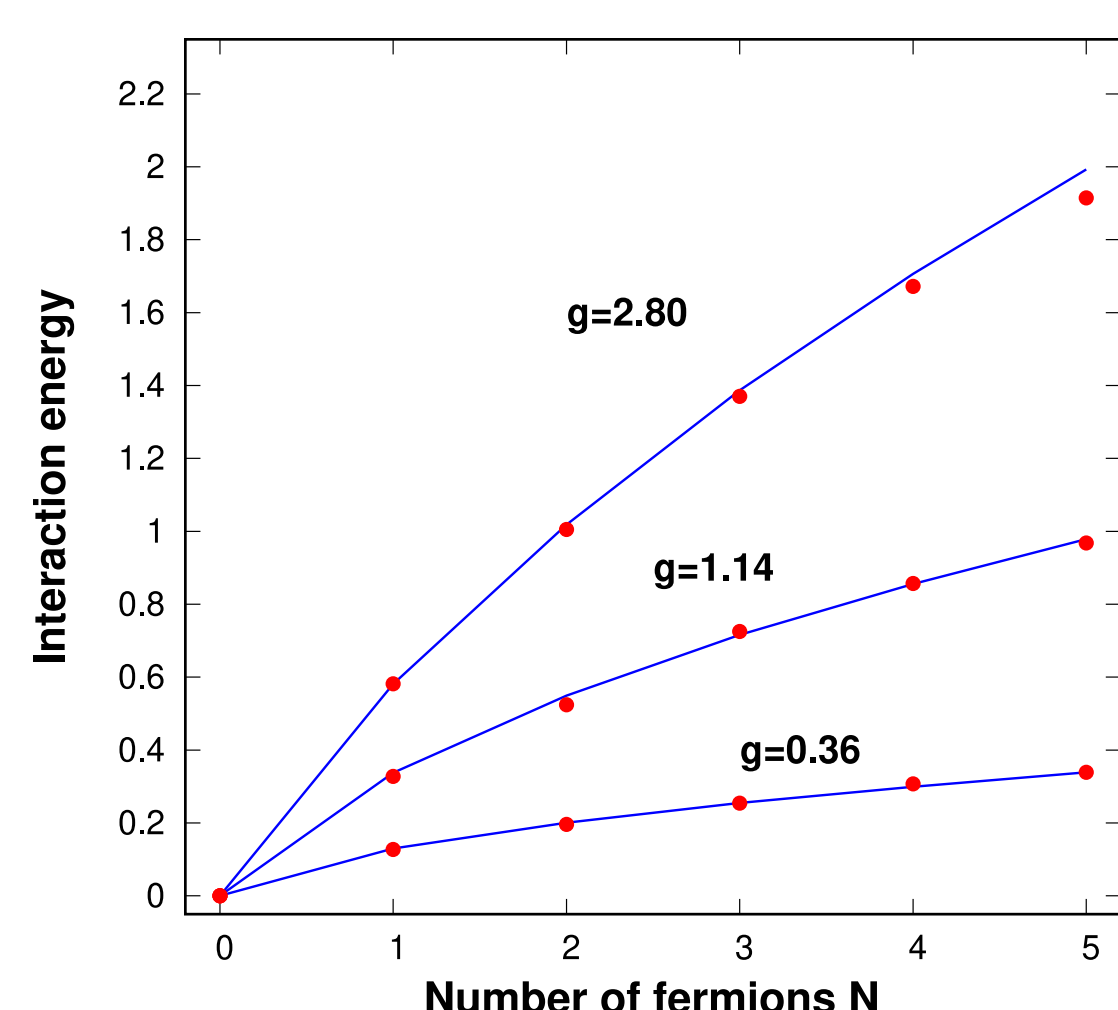
$$\mathcal{E}_{nm} = E_n + \epsilon_m$$

Interaction energy for the ground state:

$$\mathcal{E}_{int} = \mathcal{E}_{00} - \mathcal{E}_{00}^{g=0}$$

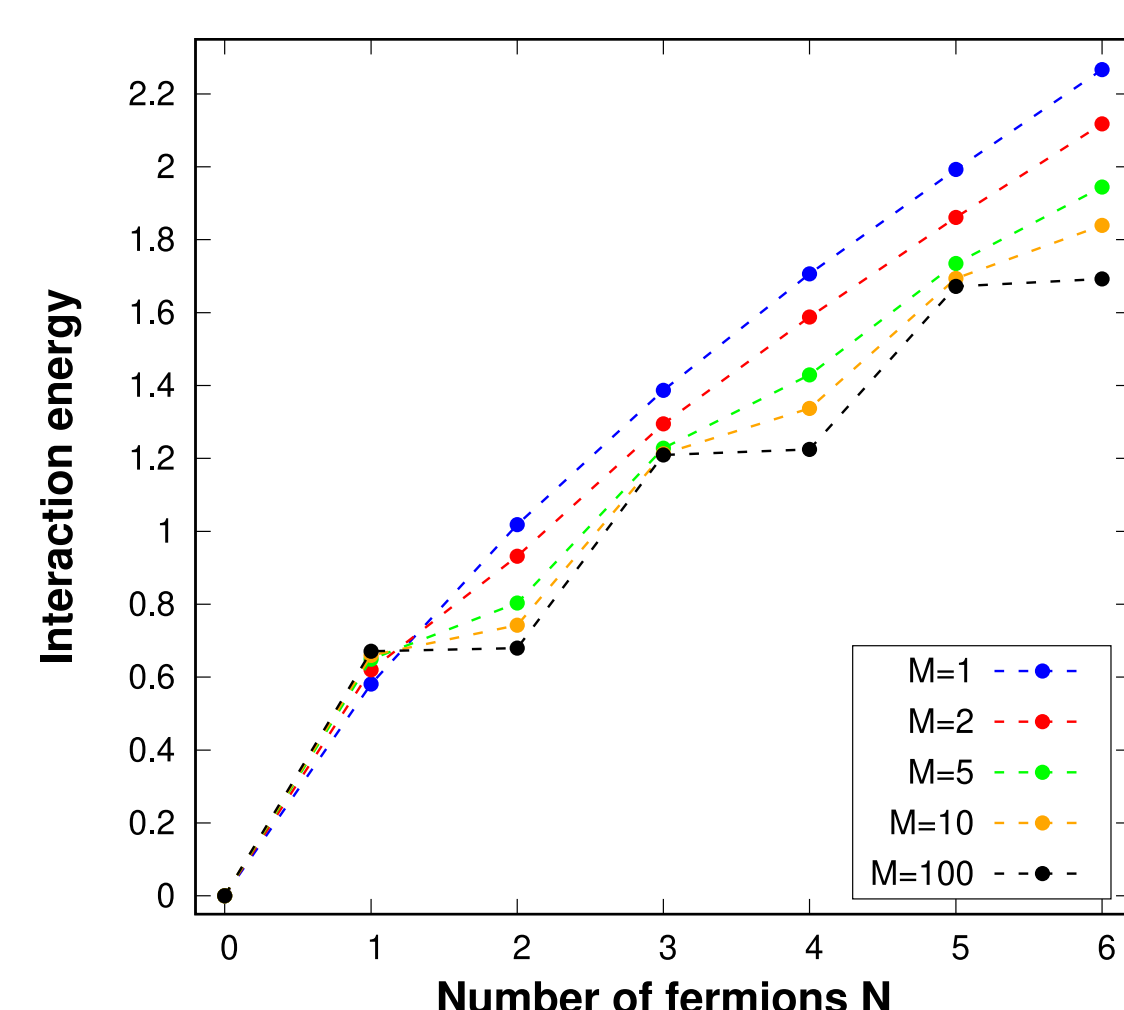
$$\mathcal{E}_{00}^{g=0} = \frac{N^2 + 1}{2}$$

Interaction energies for equal masses compared to experimental results:



Experimental data (red dots) are from A. N. Wenz, et al. Science **342**, 457 (2013)

Interaction energies for heavy impurity (with $g=2.80$):



DENSITY PROFILES

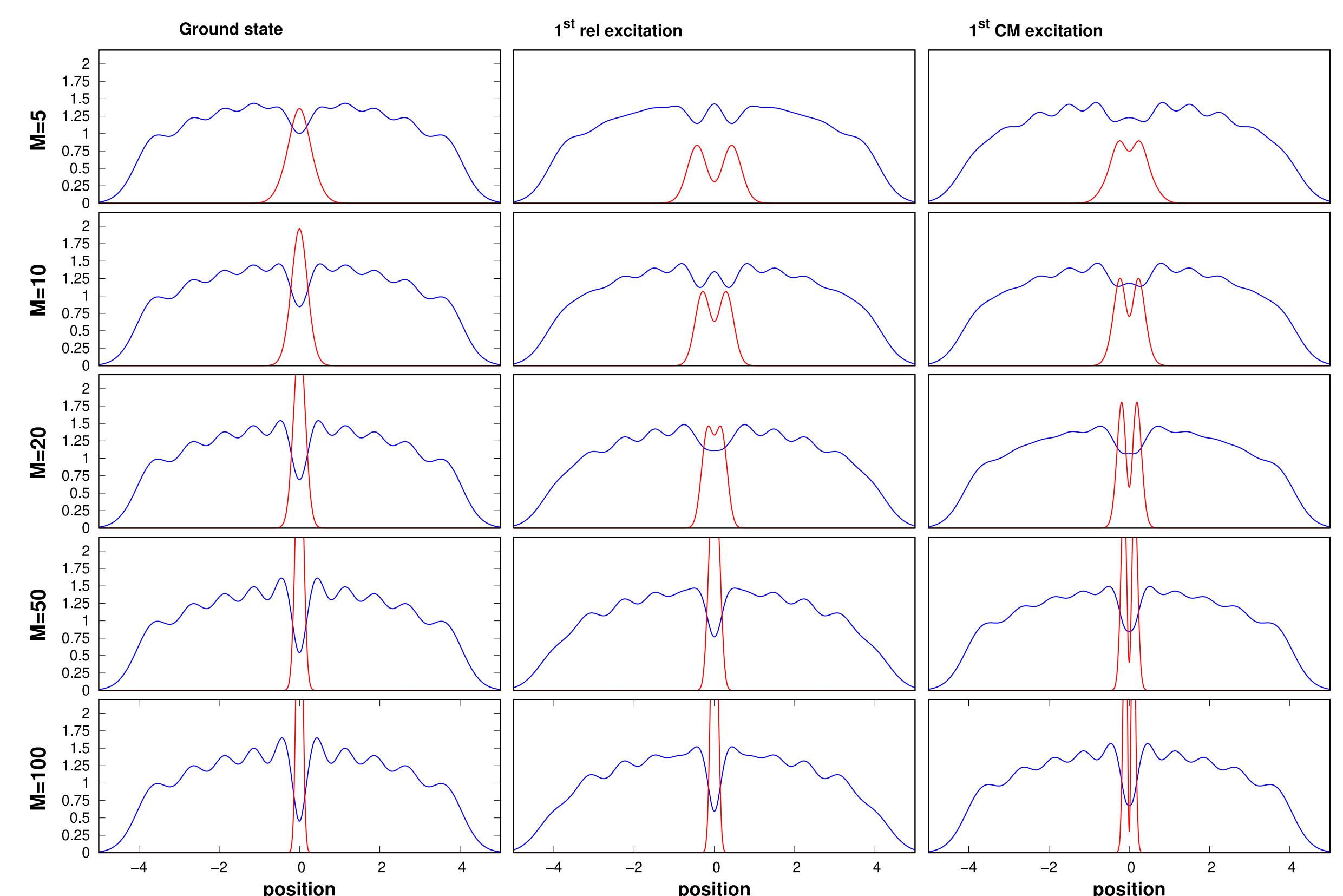
For the system in the eigenstate $|\Phi_n\rangle \otimes |\Omega_m\rangle$, the single-particle density profiles are equal:

$$n_y(y) = \langle \Omega_m | \bar{\Phi}_n^* \left(y + \frac{\hat{x}}{M+N} \right) \bar{\Phi}_n \left(y + \frac{\hat{x}}{M+N} \right) | \Omega_m \rangle$$

$$n_x(x) = \int dy \left[\langle \Omega_m | \bar{\Phi}_n^* \left(y + \frac{\hat{x}}{M+N} \right) \hat{\psi}^\dagger(x-y)\hat{\psi}(x-y)\bar{\Phi}_n \left(y + \frac{\hat{x}}{M+N} \right) | \Omega_m \rangle \right]$$

where $\bar{\Phi}_n()$ is an extension of a scalar function $\langle y | \Phi_n \rangle$ to a function of operators.

Density profiles for $N=10$ and $g=5$:



More details:

D. Włodzyński, Phys. Rev. A **106**, 033306 (2022)

