# One-axis twisting as a method of generating many-body Bell correlations

**Marcin Płodzień[1], Maciej Lewenstein[1,2], Emilia Witkowska[3], Jan Chwedeńczuk[4]**

1. ICFO - Institut de Ciencies Fotoniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels, Barcelona, Spain
2. ICREA, Pg. Lluís Companys 23, 08010 Barcelona, Spain
3. Institute of Physics PAS, Aleja Lotnikow 32/46, 02-668 Warszawa, Poland
4. Faculty of Physics, University of Warsaw, ul. Pasteura 5, PL-02-093 Warsaw, Poland

## ABSTRACT

We demonstrate that the one-axis twisting (OAT), a versatile method of creating non-classical states of bosonic qubits, is a powerful source of many-body Bell correlations. We develop a fully analytical and universal treatment of the process, which allows us to identify the critical time at which the Bell correlations emerge, and predict the depth of Bell correlations at all subsequent times. Our findings are illustrated with a highly non-trivial example of the OAT dynamics generated using the Bose-Hubbard model.

## BACKGROUND

**Twisting as a resource for many-body entangled and Bell correlated states**

Non-classical correlations, namely entanglement and Bell correlations are fundamental properties of the quantum many-body systems and crucial resources for emerging quantum technologies. The twisting protocols provide such resources, i.e. one-axis twisting (OAT) generates many-body entangled states, which are of practical interest for quantum enhancement measurement [1,2].

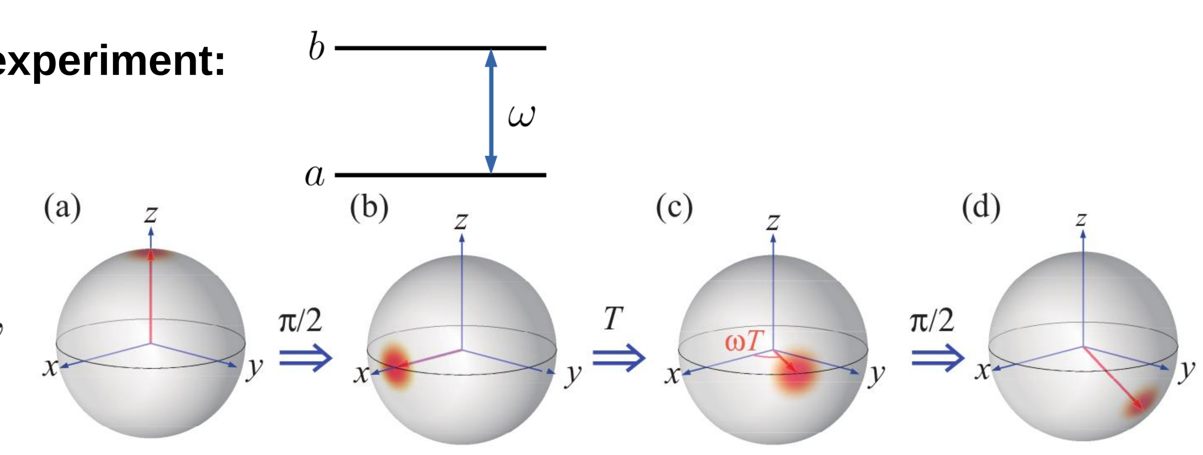
Twisting applies to systems composed of N particles in two quantum states described by a spin of quantum number S = N/2, when the final measurement is performed by spectroscopic experiments. The uncentarity of such a measurement is $\xi/\sqrt{N}$

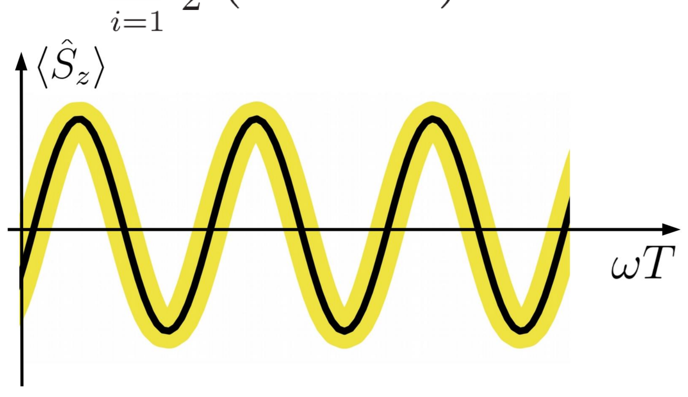
**Ramsey spectroscopy experiment:**

$$\hat{S}_x = \sum_{i=1} \frac{1}{2}\left(\hat{a}_i^\dagger \hat{b}_i + \hat{b}_i^\dagger \hat{a}_i\right),$$

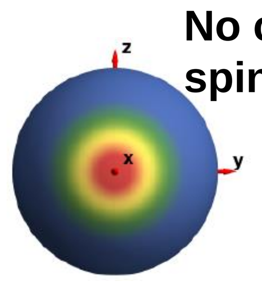
$$\hat{S}_y = \sum_{i=1} \frac{1}{2i}\left(\hat{a}_i^\dagger \hat{b}_i - \hat{b}_i^\dagger \hat{a}_i\right),$$

$$\hat{S}_z = \sum_{i=1} \frac{1}{2}\left(\hat{a}_i^\dagger \hat{a}_i - \hat{b}_i^\dagger \hat{b}_i\right)$$



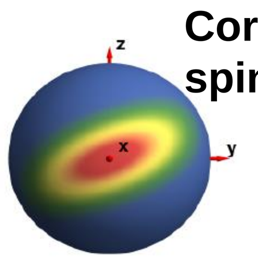




**No correlations between spins**



$$\Delta S_y^2 = \Delta S_z^2 = N/4$$

**Correlations between spins**



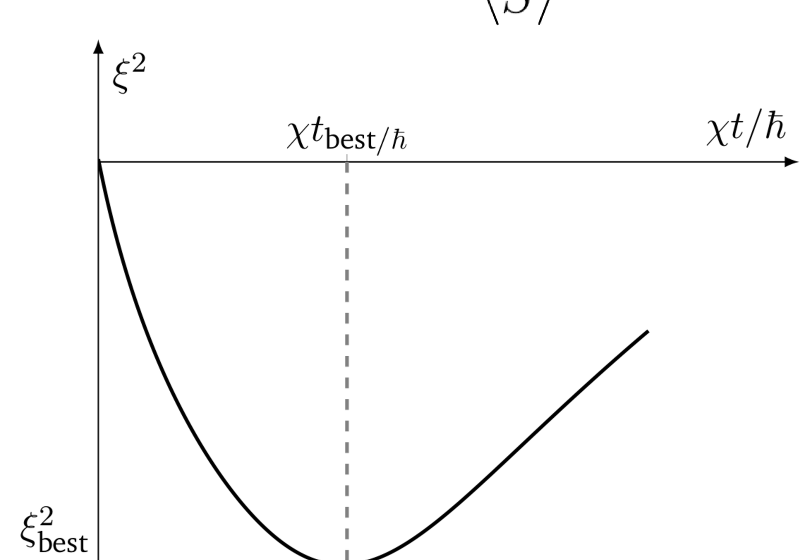
$$\Delta S_{\perp,min}^2 < N/4$$

**Spin squeezing with One-Axis Twisting**

$$\hat{H}_{OAT} = \chi \hat{S}_z^2$$

$$(\Delta\omega T)^2 = \frac{\xi_z^2}{N}$$

$$\xi^2 = \frac{N\Delta^2 S_{\perp,\min}}{\langle \hat{S}\rangle^2}$$




## CONCLUSIONS

- We provided correlator characterizing many-body entanglement and many-body Bell correlations
- We indicated critical time at which Bell correlations appear during OAT procedure
- We applied our findings to generation of many-body Bell correlations in system of two-component bosons in 1D optical lattice

## RESULTS

**Witness of the Bell correlations**

One-Axis Twisting (OAT) generates many-body entangled and Bell correlated states. To quantify extent of many-body Bell correlations generated in OAT process, we use N-body correlator witnessing the Bell correlations if the inequality is violated [3]:

$$\tilde{\mathcal{E}}_N^{(q)} = \left|\frac{1}{N!}\langle \hat{J}_+^N\rangle\right|^2 \leqslant 2^{-N} \qquad \hat{J}_+ = \hat{S}_y + i\hat{S}_z$$

Bell correlations are created in OAT procedure at critical time [4,5]:

$$\tau_{\rm crit} \simeq \frac{2}{N}\sqrt{\frac{\pi^2}{8\ln 2} - 1} \approx \frac{1.77}{N}$$

**Properties:**

a) k-qubits form a k-partite entangled state, and other N-k are separable when:

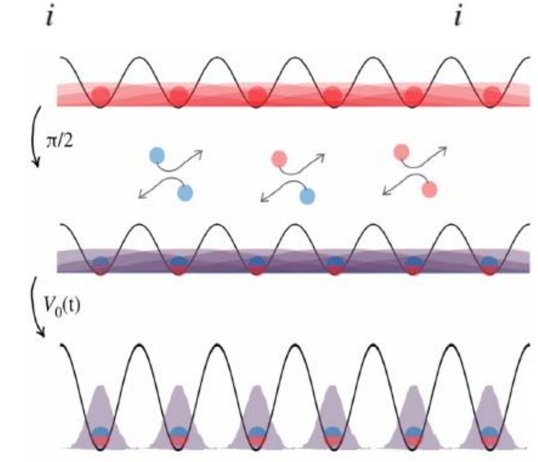
$$\tilde{\mathcal{E}}_N^{(q)} > \frac{1}{16}\frac{1}{4^{N-k}}$$

b) Bell correlations encompass at least k-qubits when:

$$\tilde{\mathcal{E}}_N^{(q)} > \frac{1}{8}\frac{1}{2^{N-k}}$$

**Physical implementation of OAT dynamics** in two component Bose-Hubbard model [5]

$$\hat{\mathcal{H}}_{\rm BH} = -J\sum_{i,j=i\pm1}(\hat{a}_i^\dagger \hat{a}_j + \hat{b}_i^\dagger \hat{b}_j) + \frac{U_{aa}}{2}\sum_i \hat{n}_i^a(\hat{n}_i^a - 1) + \frac{U_{bb}}{2}\sum_i \hat{n}_i^b(\hat{n}_i^b - 1) + U_{ab}\sum_i \hat{n}_i^a \hat{n}_i^b,$$



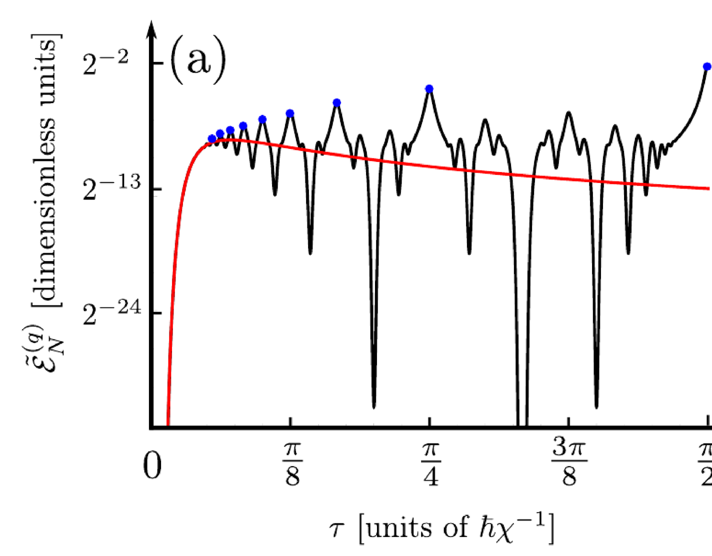




Fig 1. Time evolution of the Bell correlator for OAT (black solid line) as a function of time for N = 200 particles, compared with the short- (red solid line) and long-time (blue points) analytical approximation .

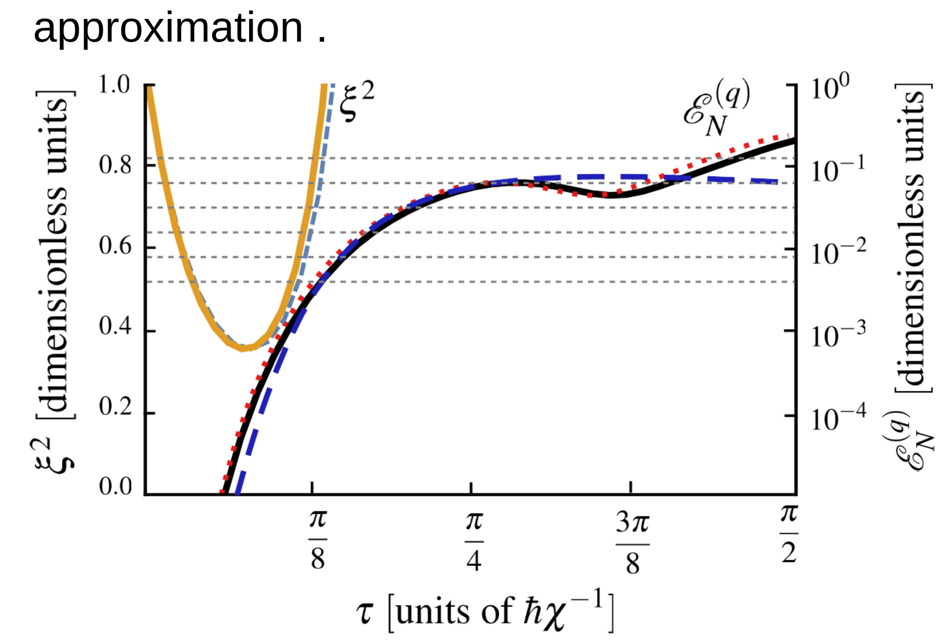



Fig 2. Left part: Time evolution of the spin squeezing parameter for OAT (dashed green line) and for two-component Bose-Hubbard dynamics (solid orange line) for N = 8 particles.

Right part: Time evolution of the Bell correlator for OAT procedure (red dashed line) and for two-component Bose-Hubbard model (black solid line). Dashed blue line corresponds to analytical predictions for short time scale for OAT model.

## REFERENCES

[1] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, Phys. Rev. A **46**, R6797 (1992).

[2] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993)

[3] J. Chwedenczuk, SciPost Phys. Core **5**, 25 (2022)

[4] M. Płodzień, M. Lewenstein, E. Witkowska, and J. Chwedeńczuk, (2022), 10.48550/ARXIV.2206.10542.

[5] M. Płodzień, M. Kościelski, E. Witkowska, and A. Sinatra, Phys. Rev. A **102**, 013328 (2020).

## ACKNOWLEDGMENTS

POLISH NATIONAL AGENCY FOR ACADEMIC EXCHANGE

M.P. acknowledges the Polish National Agency for Academic Exchange, the Bekker programme no: PPN/BEK/2020/1/00317; E.W. acknowledges QuantEra project MAQS, Grant No. UMO-2019/32/Z/ST2/00016; J.Ch. acknowledges the National Science Centre, Poland, within the QuantERA II Programme that has received funding from the European Union's Horizon 2020 research and innovation programme under Grant Agreement No 101017733, Project No. 2021/03/Y/ST2/00195