NON-ABELIAN BERRY-PHASES OF ELECTRICALLY DRIVEN HOLE-SPIN QUBITS IN WAVEGUIDE QED

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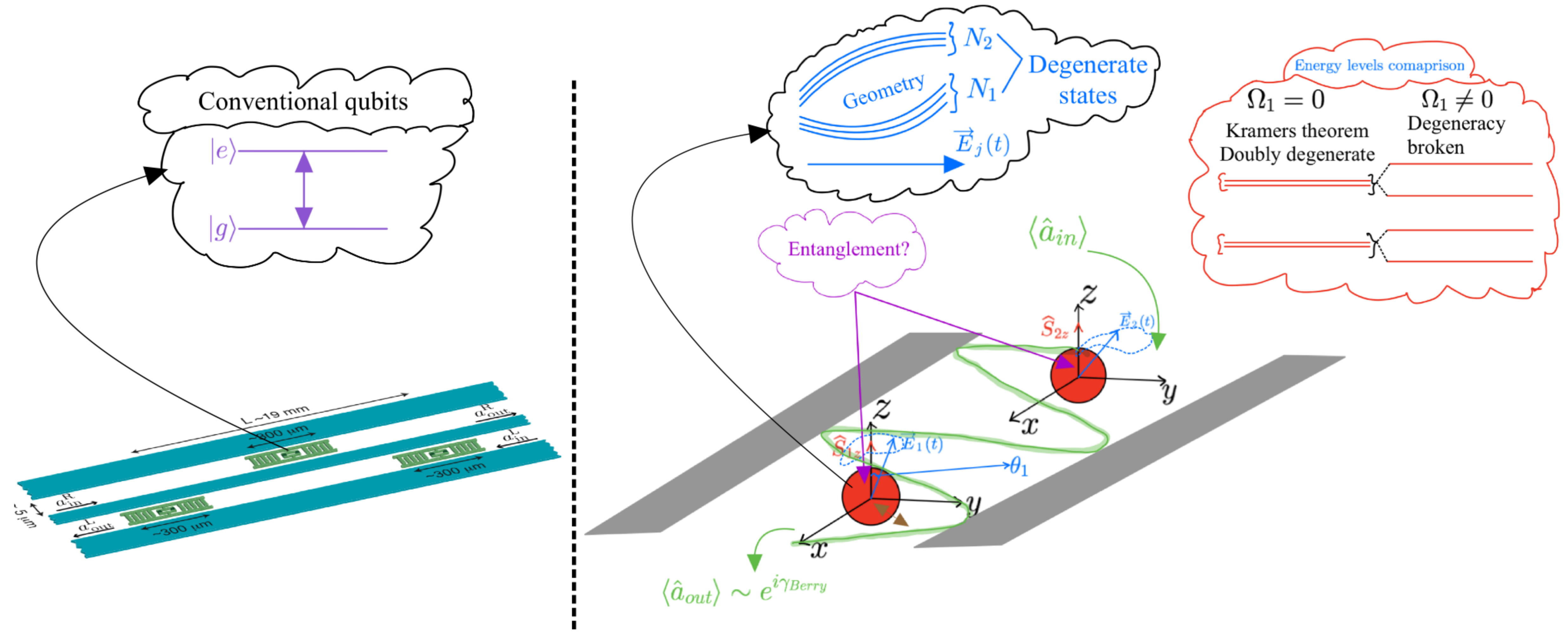
MOTIVATION & AIM

- Hole-spins are viable qubits because of their long coherence times and fast manipulation due to their strong spin-orbit coupling.
  However, usually static magnetic field are required to define the qubits → Activates relaxation channels.

  The non-abelian structure of the the hole-spin states allow only electrical manipulation and also entanglement in single-mode cavity [1].
- How is the non-abelian Berry phase imprinted and transmitted in general environments?

  Wave-guide quantum electrodynamics (QED) (artificial) atoms + coupling to a one-dimensional superconducting transmission line (bath) is a suitable candidate for this study [2].
- Simplest non-Abelian system: hole-spin qubits immersed in a waveguide cavity (photonic bath).



- Driven spin + wave-guide → open system → Born-Markov approximation + Floquet Theory
- Investigate the dynamical variables-the dressed spin trajectories, their coherence times, and the photons-mediated entanglement between distant hole-spins [3].
- Input-output theory of EM field → we calculate the photonic fluxes exiting the wave-guide and show that its transmission depends on the Berry phases stemming from the driven hole spins [3].

DRIVEN 3/2 HOLE-SPIN HAMILTONIAN IN A CONTINUOUS MODE WAVE-GUIDE

$$H_{tot}(t) = \epsilon\, \boldsymbol{E}(t)\cdot\boldsymbol{\Gamma} + H_{bath} + g\,(\Theta + \Theta^\dagger)\Gamma^1$$

$$H_{bath} = \int_0^\infty d\omega\, \omega \Big[a_R^\dagger(\omega)a_R(\omega) + a_L^\dagger(\omega)a_L(\omega)\Big],$$

$$\Theta = -i\int_0^\infty d\omega\, \sqrt{\omega}\Big(a_L(\omega) + a_R(\omega)\Big),$$

where $\boldsymbol{\Gamma} = \{\Gamma^1, \Gamma^2, \Gamma^3\}$ such that $\Gamma^1 = \frac{1}{\sqrt{3}}\{S_y, S_z\}$, $\Gamma^2 = \frac{1}{\sqrt{3}}\{S_z, S_x\}$, $\Gamma^3 = \frac{1}{\sqrt{3}}\{S_x, S_y\}$ with S_x S_y & S_z being the spin operators, $a^\dagger_{L(R)}(\omega)$ is the photon creation operator moving left (right) with frequency ω.

FLOQUET THEORY FOR AN ELECTRICALLY DRIVEN HOLE-SPIN IN WAVE-GUIDE

- For periodic drivings, $H(t+T) = H(t)$, the solutions of $i\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$ can be written as $|\Psi_n(t)\rangle = e^{-i\epsilon_n t}|\phi_n(t)\rangle$, with ϵ_n and $|\phi_n(t+T)\rangle = |\phi_n(t)\rangle$ being the Floquet quasienergy and state, respectively.
- Reduced density matrix of the driven spin → Born-Markov approximation + Floquet theory
- Usual rotating-wave approximation (RWA) may not be justified for describing degenerate levels!
- Universal Lindblad Equation (ULE) for open system [4] $\implies$ No RWA invoked, while preserving the positivity of the density matrix

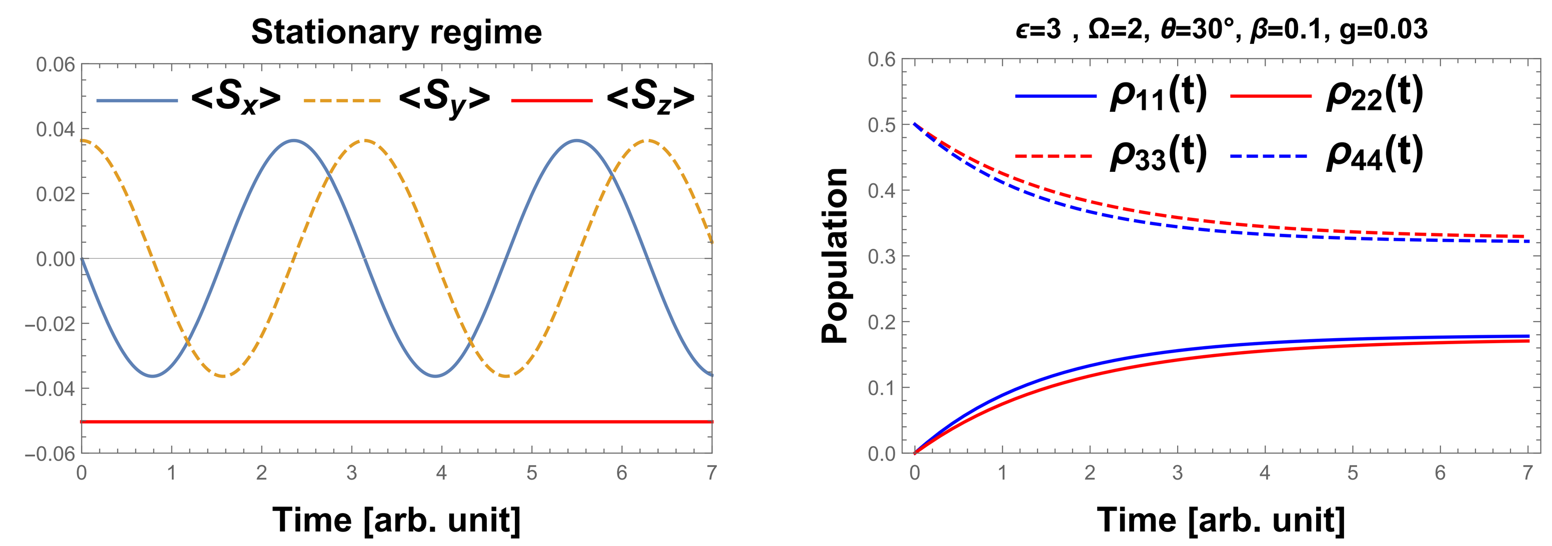
ULE → Reduced density matrix element of the driven spin (ρ_{mn}) using the periodic Floquet states as basis:

$$\dot{\rho}_{mn} = -i(\epsilon_m - \epsilon_n)\rho_{mn} + D_{mn}[\rho]\,,$$

$$D_{mn}[\rho] = \sum_{z,z'}\sum_{p,q}\left(L_{mpz}\, L^*_{nqz'}\, \rho_{pq}\, e^{i\Omega(z'-z)t} - \frac{1}{2}L^*_{pmz'}L_{pqz}\, \rho_{qn}\, e^{i\Omega(z'-z)t} - \frac{1}{2}L^*_{pqz'}L_{pnz}\rho_{mq}\, e^{i\Omega(z'-z)t}\right)$$

where $L_{mnz} = (\Omega/2\pi)\int_0^{2\pi/\Omega}\langle\phi_m(t)|\Gamma^1|\phi_n(t)\rangle\, e^{i\Omega z t} g_{11}(\epsilon_n - \epsilon_m + z\,\Omega)$, $g_{11}(\omega)$ is the bath correlation function, $\{z,\, z'\} = 0, \pm 1, \ldots$ and $\{p,\, q\} \in \{1,2,3,4\}$. In the adiabatic limit, we can write: $\epsilon_i \sim \epsilon_i^{static} + \gamma_i^B/T + \ldots \implies$ the Berry phase responsible for the degenerate levels dynamics.

In the stationary regime $\rho_{nm}(t+T) = \rho_{nm}(t)$, allowing to find the spin dynamics:



Plots for circular drive with $\boldsymbol{E}(t) = \{-\sin\theta\sin(\Omega\, t), \sin\theta\cos(\Omega\, t), \cos\theta\}$ using RWA. (Left) The driving causes the precession of the spin expectation values. (Right) The decay time of the population can be determined for gate operations.

CONCLUSION & FUTURE PLANS

- To capture the complete dynamics, we propose to use Universal Lindblad Equation [4] where master equation is in Lindblad form without any restriction on the system.
- Photon dynamics: $\langle \dot{\hat{a}}_{R(L)}(\omega)\rangle = -i\omega\,\langle \hat{a}_{R(L)}(\omega)\rangle + g\,\sqrt{\omega}\,\langle\Gamma^1(\omega)\rangle \implies$ Find the Berry-phase imprints
- Extend to two hole-spins to study the dissipative entanglement induced by the Berry phases
- Generalize for arbitrary non-abelian system

References

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[3] S. Prem, M. Wysokinski, M. Trif (in preparation).

[4] F. Nathan and M. S. Rudner, Phys. Rev. B **102**, 115109 (2020).