Mass-imbalanced mixture of a few strongly interacting fermions driven through a critical point

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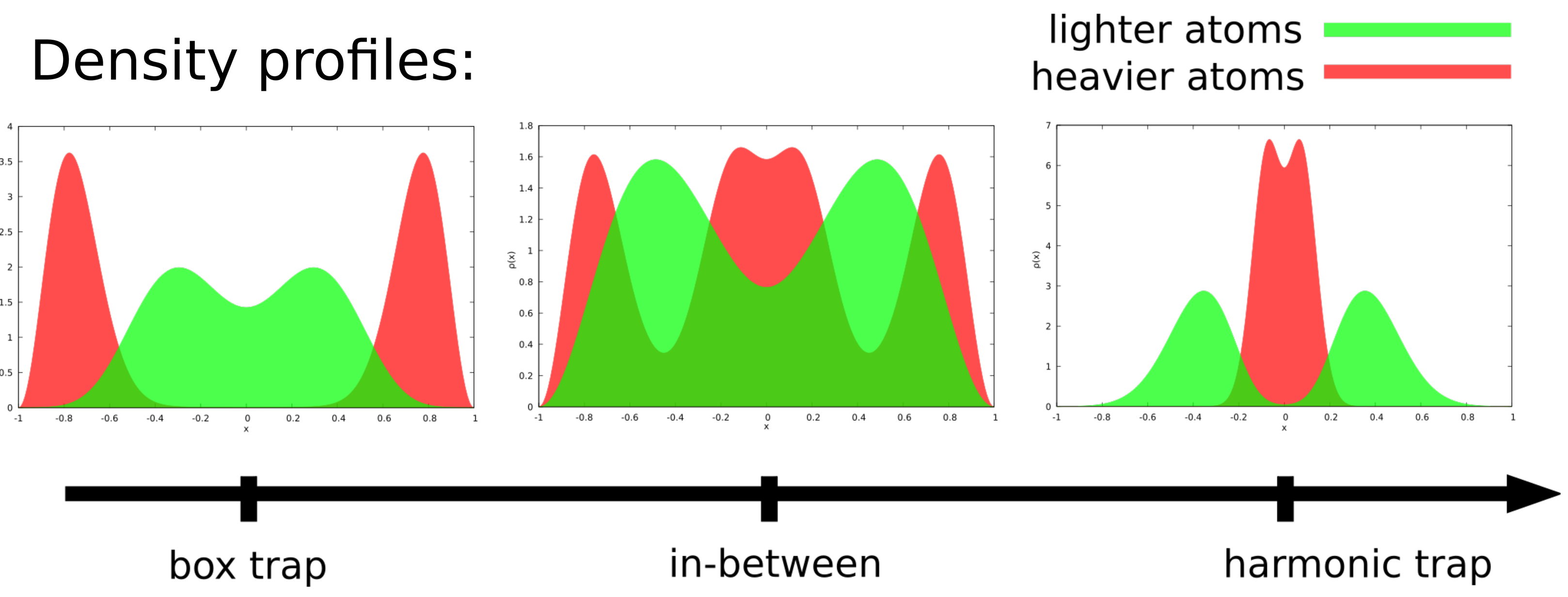
Abstract:

In a mass-imbalanced mixture of a few ultracold fermionic atoms with strong repulsive interactions, a spatial arrangement of the components depends on the shape of the external confinement. When the mixture is initially prepared in a one-dimensional box trap and then the harmonic potential is slowly turned on, the system undergoes structural transition. Finite-time quench through this transition is analyzed, using an exact diagonalization method.

Analyzed transition

- strong repulsive interactions induce the separation of the components
- in a box trap, lighter atoms are localized in the center
- in a harmonic trap, heavier atoms are localized in the center
- transition between the traps is analogical to quantum phase transition

Density profiles:



Hamiltonian of the system

$$\hat{\mathcal{H}} = \sum_{\sigma\in\{A,B\}} \int \mathrm{d}x\, \hat{\Psi}^\dagger_\sigma(x)\left[-\frac{\hbar^2}{2m_\sigma}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+V_\sigma(x)\right]\hat{\Psi}_\sigma(x) + g\int \mathrm{d}x\, \hat{\Psi}^\dagger_A(x)\hat{\Psi}^\dagger_B(x)\hat{\Psi}_B(x)\hat{\Psi}_A(x)$$

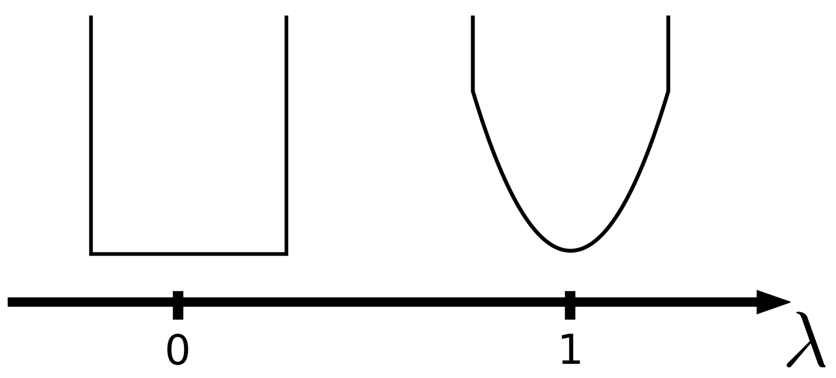
Confining potential:

$$V_\sigma(x,t) = \begin{cases} \lambda(t)\frac{m_\sigma\Omega^2}{2}x^2 & |x|<L \\ \infty & |x|\geq L \end{cases}$$

Shape parameter:

$$\lambda(t) = 2\lambda_0 \sin^2\left(\frac{\pi t}{\tau}\right)$$

Transition point — Transition time



- few-fermion mixture $(N_A+N_B=4)$
- mass-imbalanced $\left(\frac{m_B}{m_A}=\frac{40}{6}\right)$
- strong interactions $\left(g=20\frac{\hbar^2}{m_A L}\right)$

Numerical method

The exact diagonalization of the many-body Hamiltonian:

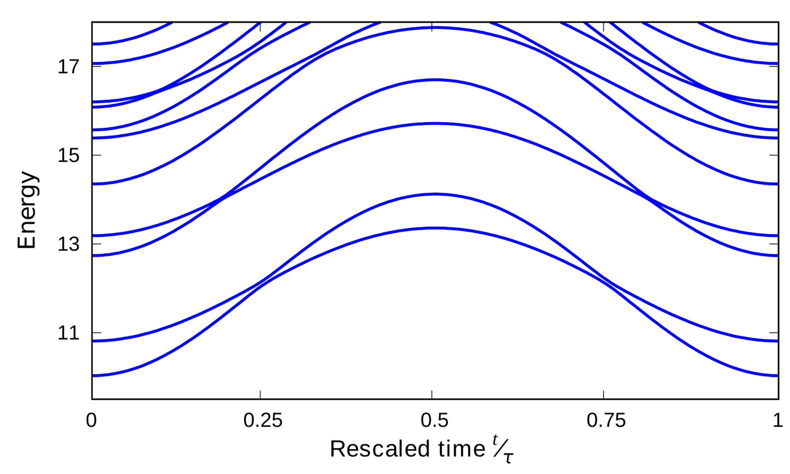
- Fock basis is constructed from eigenstates of the non-interacting system

$$\hat{\Psi}_\sigma(x) = \sum_n^M \phi_{n\sigma}(x)\hat{a}_{n\sigma}$$

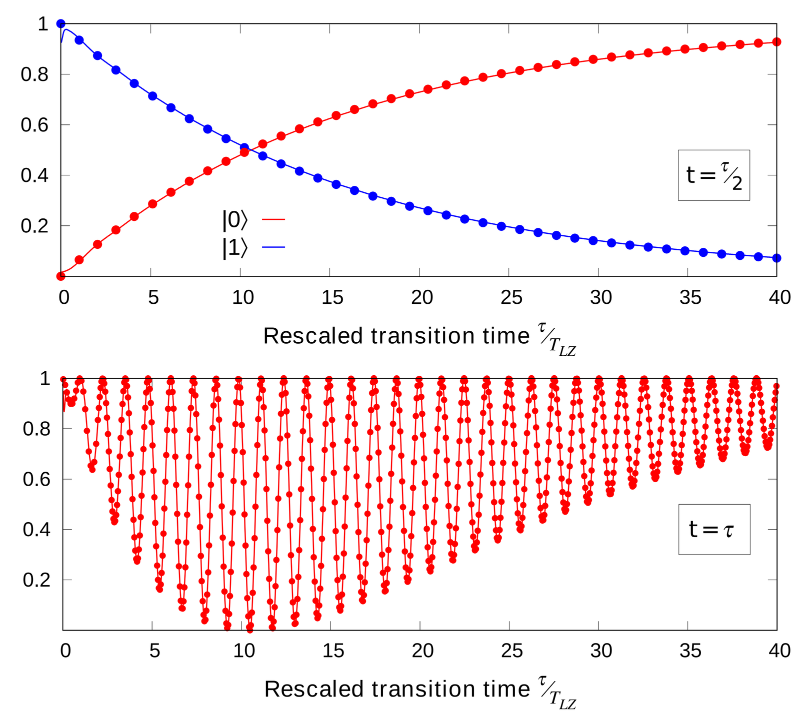
- The Hamiltonian is expressed as a matrix in this basis
- Required eigenstates are obtained (Lanczos method)

Results: 1+3 mixture

Energy spectrum:



Transition probability:



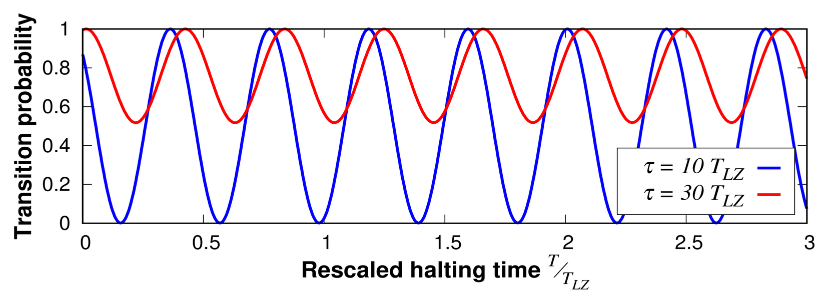
- effectively a two-level system
- can be described with the Landau-Zener model

first transition: $\mathcal{P}_1(\tau/2) = \exp(-\alpha\tau)$

second transition: $\mathcal{P}_0(\tau) = \left|\mathcal{P}_0(\tau/2)e^{2i\phi_{\tau/2}} + \mathcal{P}_1(\tau/2)\right|$

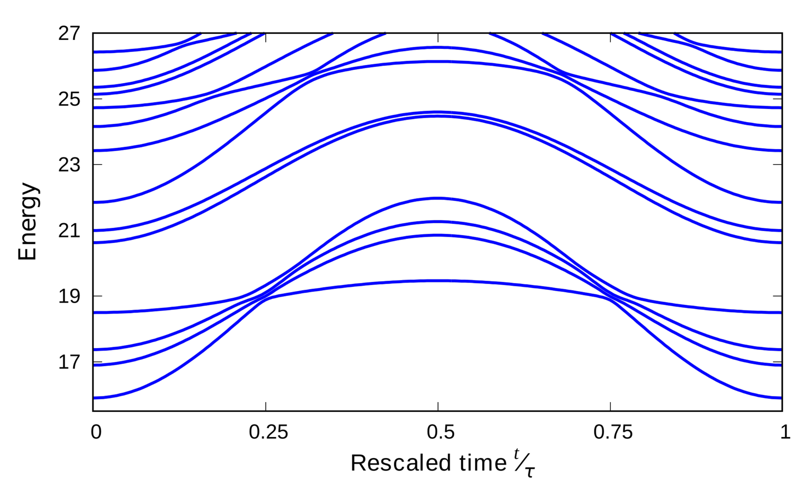
Halting protocol:

$$\phi_{\tau/2} \to \phi_{\tau/2} + \frac{\Delta\mathcal{E}(\tau/2)\,T}{\hbar}$$

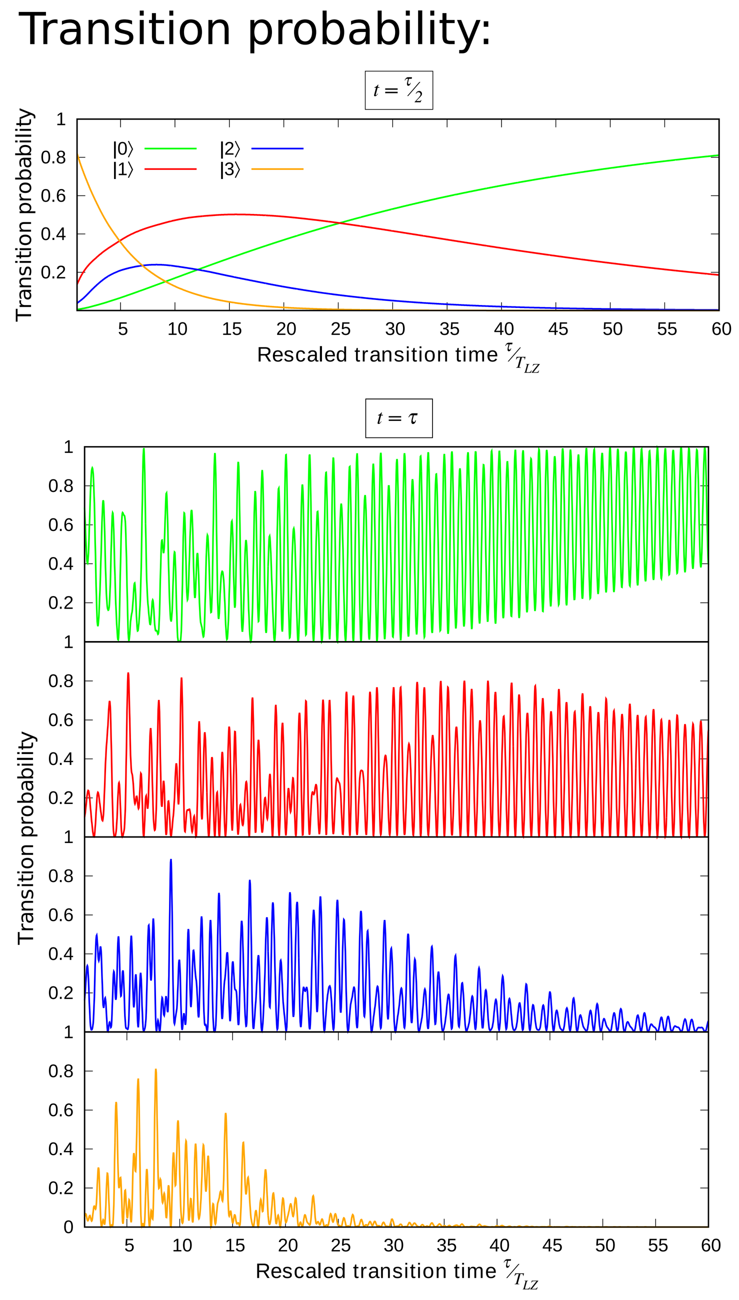


Results: 2+2 mixture

Energy spectrum:

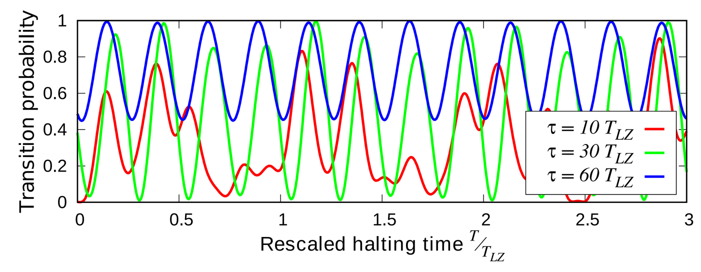


Transition probability:



- effectively a four-level system
- for long transition times effectively a two-level system

Halting protocol:



References:

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