Criticality-enhanced quantum sensing with spin-1 BECs

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Abstract: We theoretically investigate estimation of the control parameter in a spin-1 Bose-Einstein condensate near quantum phase transitions in a traverse magnetic field with a fixed macroscopic magnetization [1, 2]. We quantify sensitivity by quantum and classical Fisher information. For these different metrics, we find the same, beyond-standard-quantum-limit (SQL) scaling with atom number near critical points, and SQL scaling away from critical points [1,2]. In the particular case of antiferromagnetic condensates, the system exhibits the first- and second-order phase transition depending on the value of magnetization. We exploit both types of system criticality as a resource in the precise estimation of control parameter value. We demonstrate supersensitivity and show that the precision scales with the number of atoms up to N^4 around critically [1]. In addition, we study the precision based on the error-propagation formula providing the simple-to-measure signal which coincides its scaling with the quantum Fisher information. We find that both depletion of the m_F=0 Zeeman sub-level and transverse magnetization provide signals of sufficient quality to saturate the sensitivity scaling. To explore the effect of experimental imperfections, we study the scaling around criticality at nonzero temperature and with nonzero detection noise. Our results suggest the feasibility of sub-SQL sensing in spin-1 condensates with current experimental capabilities.

[1] S. Mirkhalaf, E. Witkowska, L. Lepori, Phys. Rev. A 101, 043609 (2020) [2] S. Mirkhalaf, D. Benedicto-Orenes, M. Morgan, E. Witkowska, Phys. Rev. A 103, 023317 (2021)

Quantum sensing

In quantum sensing and metrology* a classical parameter, e.g. externally applied field or energy level separation, is estimated from measurements on a quantum system consisting of N>1 particles. The precision is closely related to the sensitivity of the quantum state to the parameter

Different paths to quantum-enhanced sensing

using entanglement

entangled states *of N two-state particles can estimate a single-particle phase φ with variance* $\delta^2\varphi=1/N^2$ *(HL),*

non-entangled states in the same scenario can at best achieve $\delta^2\varphi=1/N$ *(SQL)*

*... but such states are very sensitive to decoherence*** *and scaling with N typically back to SQL*

other schemes

non-linear interferometer *when particles interact while experiencing a phase shift****

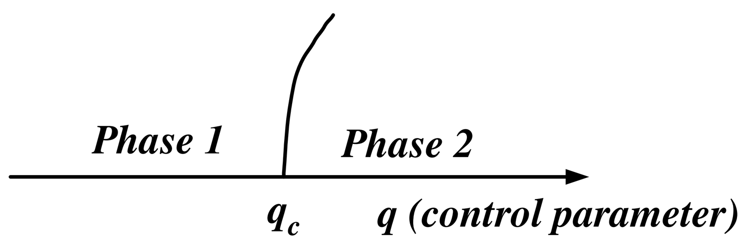
e.g. if the unknown parameter is itself the strength κ of a particular k-body interaction, it yields $\delta^2\kappa\sim1/N^{2k-1}$

quantum criticality

Quantum sensing & criticality

-purpose: quantum estimation of Hamiltonian's parameter

$$\hat H(q)=\hat H_0+q\hat H_q$$



different phases ⟶ different physical properties

The corresponding quantum states distinguishable more effectively than the states belonging to the same phase§

-resource: a family of states depending strongly on the control parameter

Formalism

The estimation of some parameter q is usually based on detection of q-dependent mean value and the sensitivity is obtained via error propagation formula†

$$\delta^2 q=\frac{\Delta^2 S}{|\partial_q\langle\hat S\rangle|^2}\qquad 1/\delta^2 q\le F_q=-4\frac{\partial^2\mathcal{F}_q}{\partial\delta_q^2}|_{\delta_q\to 0}$$

quantum fidelity-susceptibility

Scaling with the system size

Around continuous quantum phase transition the fidelity susceptibility and the inverse of error propagation formula at criticality is expected to scale as‡

$$F_q\sim N^{2/d\nu}\quad 1/\delta^2 q\sim N^{2/d\nu}$$

for measurement of the order parameter‡‡

* Giovannetti, Lloyd, Maccone, Science (2004); Pezze et al Rev. Mod. Phys. (2018)

** Demkowicz-Dobrzański, Kołodyński, Duta, Nat. Commun. (2012) *** Napolitano et al, Nature (2011)

§ Zanardi, Paris, Venuti, PRA (2008)

† Braunstein, Caves PRL (1994);

‡ Sachdev "Quantum phase transition" (1999); Contentino "Quantum scaling in many-body system" (2001)

‡‡ Pezze, Trenkwalder, Fattori, arXiv:1906.01447

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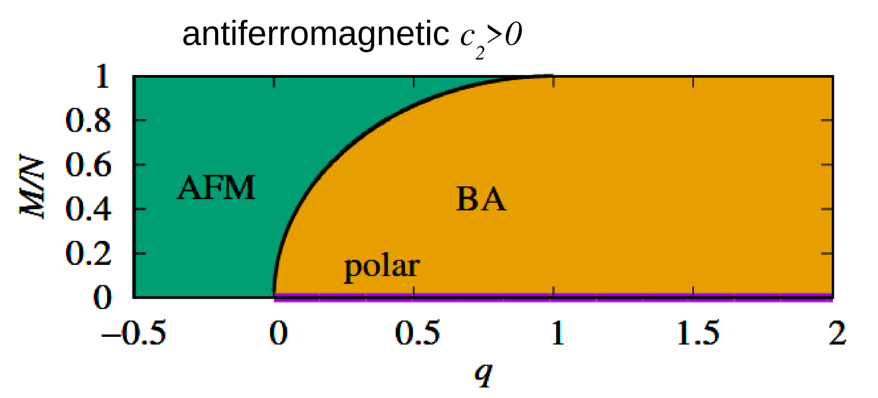
Spin-1 BECs

- a few thousand of atoms with F=1, all three Zeeman components trapped by the same trap in a magnetic field
- spin domains are energetically costly, spatial and spin degrees of freedom decoupled → SMA*

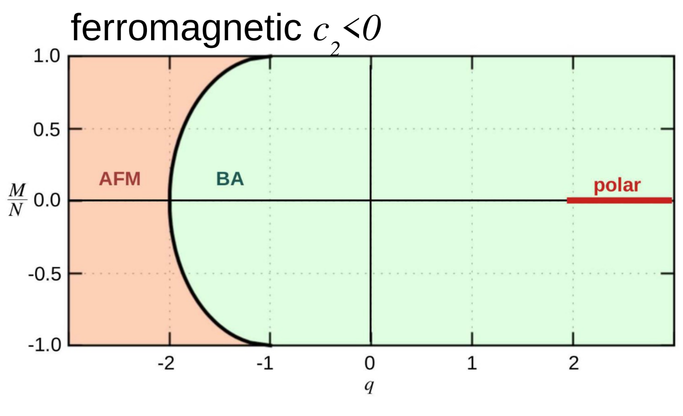
$$\frac{\hat H(q)}{c}=\frac{\mathrm{sgn}(c_2)}{2N}\hat J^2-q\hat N_0$$

Phase-diagram of ground states*

and critical points



* 1st order PT at $q=0,\ M/N=0$
* 2nd order PT for $q\neq 0,\ M/N>0$



* 2nd order PT any M/N

*Experiments e.g. Jacob et al PRA (2012) (antiferromagnetic), Stamper-Kurn et al PRL 1998 (ferromagnetic); Stamper-Kurn, Ueda Rev. Mod. Phys.(2012)

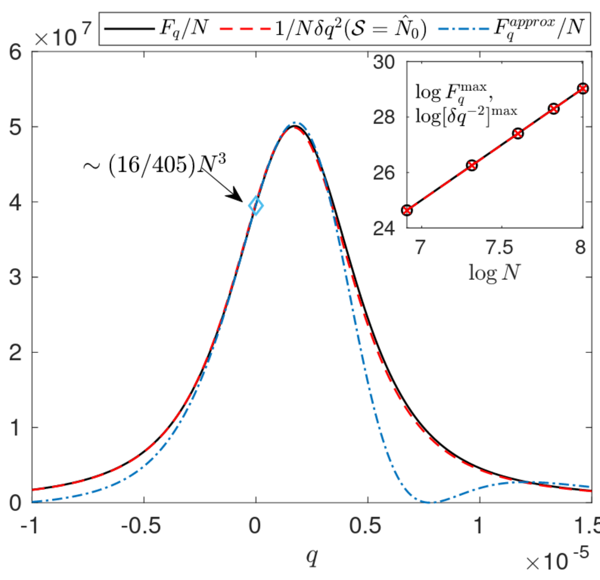
When temperature is zero

- antiferromagnetic BEC $c_2>0$ & $q_c=0$ & $M=0$

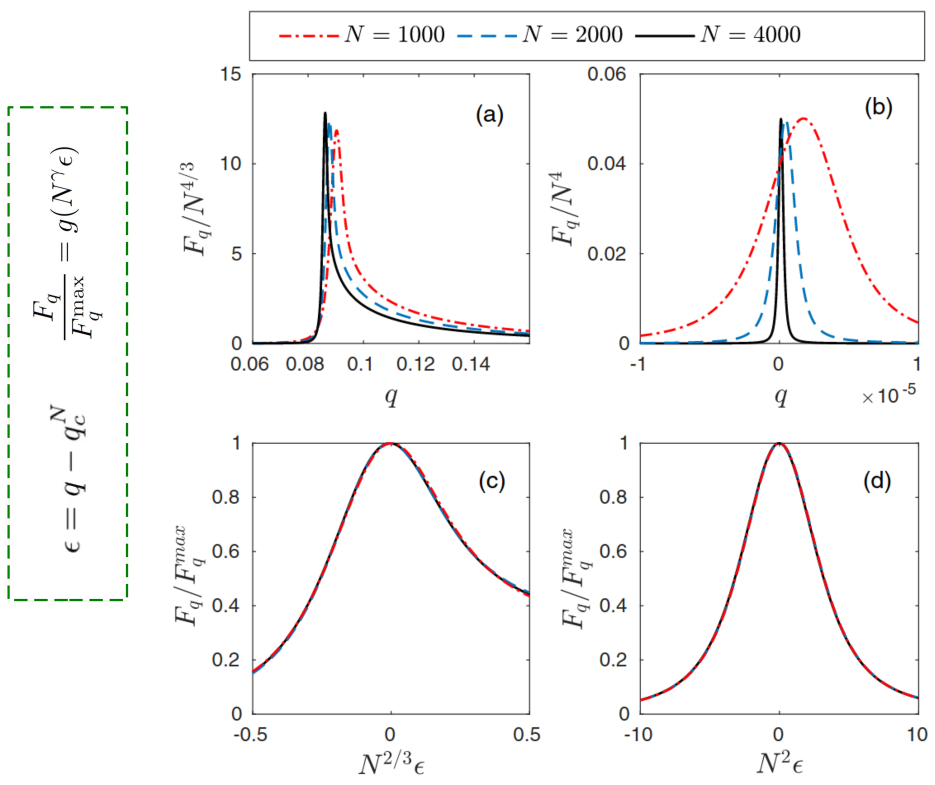
perturbation theory for q → 0

$F_q\sim N^4$

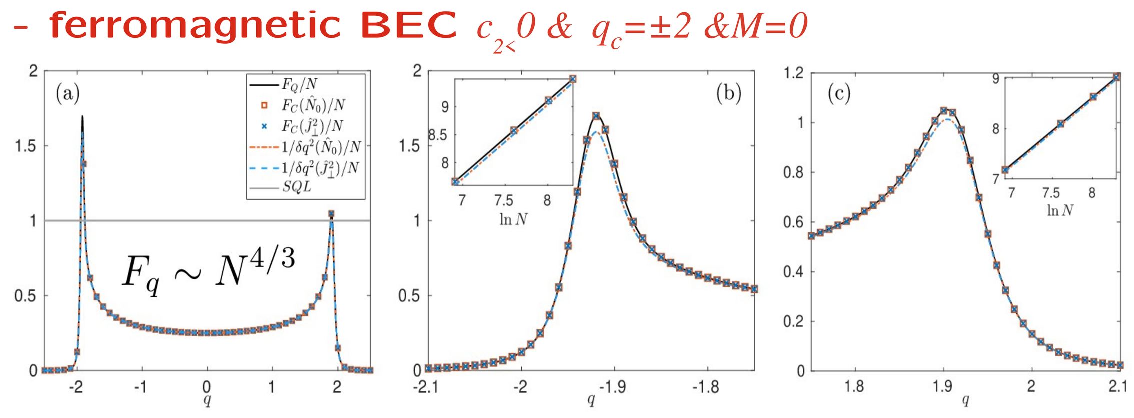
$1/\delta^2 q=F_q|_{q=q_c}$



$\epsilon=q-q_c^N$, $\frac{F_q}{F_q^{max}}=g(N^\gamma\epsilon)$



- ferromagnetic BEC $c_2<0$ & $q_c=\pm2$ & $M=0$



* S. Mirkhalaf, et al PRA (2020); PRA (2021)

Experimental imperfections

Nonzero temperature

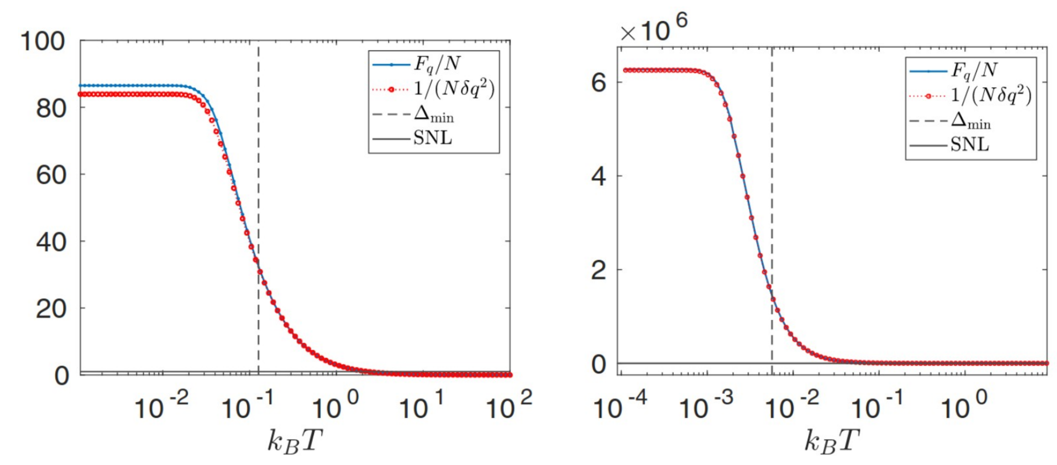
$$\hat\rho(q,T)=\sum_\alpha\frac{e^{-E_\alpha(q)/k_BT}}{Z}|\psi_\alpha\rangle\langle\psi_\alpha|$$

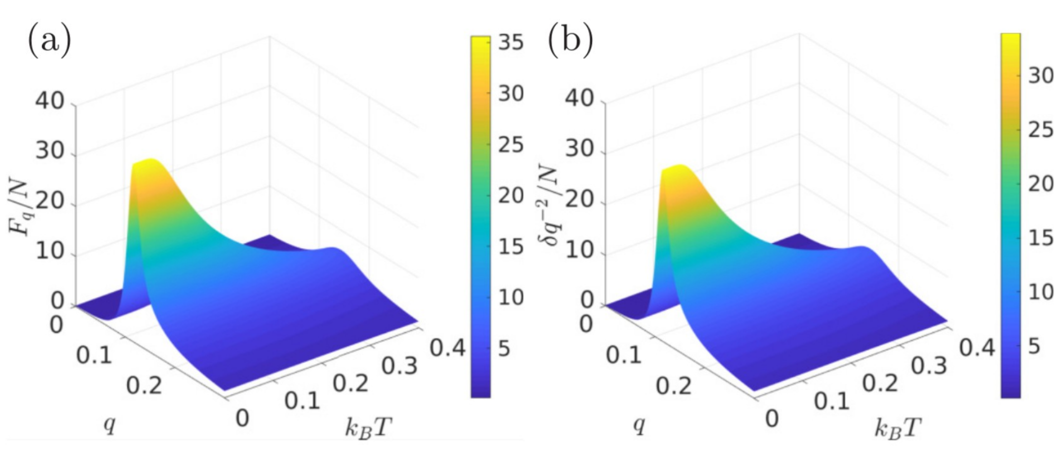
Three different regimes can be distinguished depending on the temperature value compared to energy gap

i) quantum $k_BT\ll\Delta_{\min}$

ii) intermediate $k_BT\approx\Delta_{\min}$

iii) classical $k_BT\gg\Delta_{\min}$





Imperfect detection

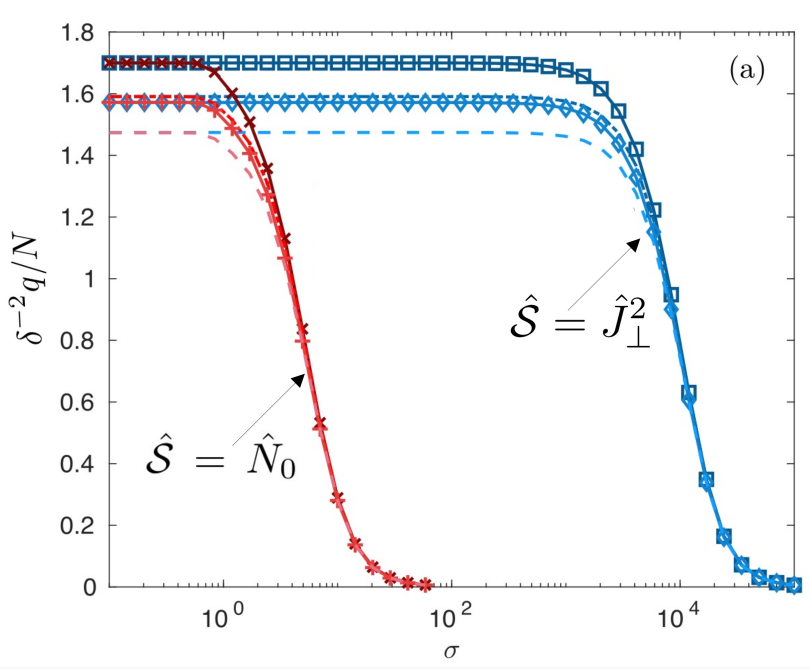
Gaussian blurring of the probability distribution

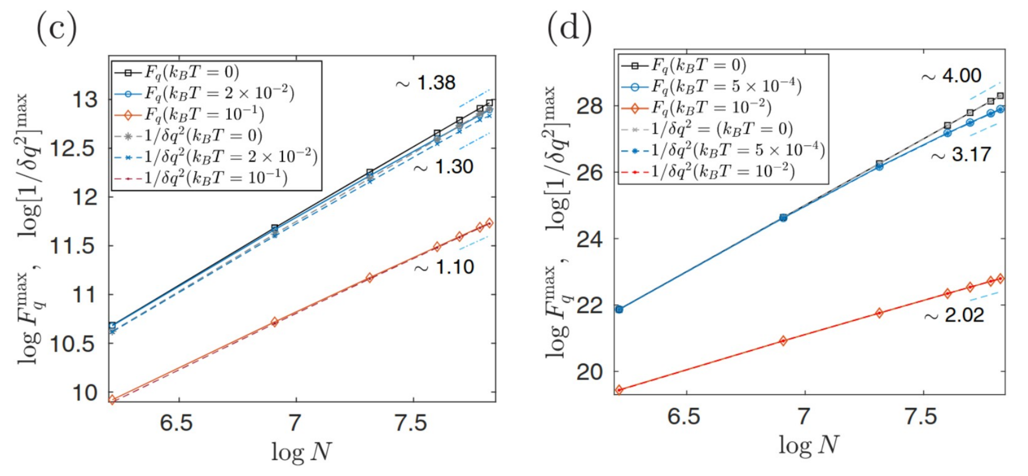
$$P(s|q)=\langle s|\hat\rho|s\rangle\to\tilde P(s|q)=\frac{1}{N}\sum_{s'}e^{-\frac{(s-s')^2}{2\sigma^2}}P(s'|q)$$

$$\langle\hat S^k\rangle_{dn}=\sum_s s^k\tilde P(s|q)$$

To get sub-SQL sensitivity:

- atom number counting $\sigma<6$ - detection noise $\sigma<10^4$ (reported* $\sigma\sim10^3$)





* Palacios et al. New J. Phys (2018); Gomez et al PRA (2019), PRL (2020)